Estimation Characteristics of Input Voltage Waveform of Single-Phase PFC Converter for Both Sinusoidal and Rectangular Input Voltages

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Keywords: single-phase PFC converter, rectangular-waveform distribution system, estimation of input voltage

A lot of industrial equipment and household electric appliances have diode rectifier circuits for the dc voltage source of the inverter circuits. However, the input current waveform of the diode rectifier circuit is distorted. To avoid this problem, PFC converter have attracted considerable attention. However, the system configuration is complication compared with that of the diode rectifier circuit.

On the other hand, the single-phase or three-phase distribution system with the rectangular voltage waveform has been proposed. The distribution system with the rectangular voltage waveform is useful for the equipment with the dc voltage source because the high input power factor is obtained by using the diode rectifier circuits whose system configuration is simple.

This paper presents a single-phase PFC converter for both the distribution systems with the sinusoidal or rectangular voltage waveform. Also, this paper presents an input voltage estimation because it is necessary to judge between the sinusoidal and rectangular voltage waveforms and to obtain the high input power factor.

Figure 1 shows the rectangular input voltage waveform. The function $\text{rec}\theta$ is defined as the rectangular waveform of which the

Fig. 1. Rectangular input voltage waveform

Fig. 2. System configuration of single-phase PFC converter

amplitude E_{amp} equals 1.

Figure 2 shows the system configuration of the single-phase PFC converter for both sinusoidal and rectangular input voltage waveforms. The control purpose of PFC converter is the realization of the unity effective power factor and the stable output dc voltage.

The amplitude $E_{RM}(n)$ and the phase angle $\theta_{RM}(n)$ of the rectangular input voltage can be estimated with the estimation error of the input voltage $\Delta e_R(n-1)$ in (1) and (2)

$$E_{RM}(n) = E_{RM}(n-1) + K_{RE}\Delta e_R(n-1)\text{rec}\theta_{RM} \dots \dots \dots (1)$$

$$\theta_{RM}(n) = \theta_{RM}(n-1) + \omega_R T_c - K_{R\theta}\Delta e_R(n-1)\text{rec}(\theta_{RM} - \pi/2) \dots \dots \dots (2)$$

where K_{RE} and $K_{R\theta}$ are the gains for the estimation of the amplitude and the phase angle, respectively. ω_R is the angular frequency of the input voltage and T_c is the control period.

Figure 3 shows the convergence characteristics for the phase angle disturbances of $-180 \sim +180$ deg.

Using the proposed scheme, the input voltage estimation algorithm can converge the estimation errors within 40 ms of the input voltage for the amplitude and phase angle disturbances. The power factors are 0.995 and 0.949 when the sinusoidal and rectangular waveforms are used as the input voltages, respectively.

Fig. 3. Convergence characteristics for phase angle disturbances

Estimation Characteristics of Input Voltage Waveform of Single-Phase PFC Converter for Both Sinusoidal and Rectangular Input Voltages

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The distribution system with the rectangular voltage waveform is useful for the electrical equipment with the dc voltage source because the high input power factor is obtained by using the diode rectifier circuits. This paper presents a single-phase type AC/DC converter using a novel power factor correction (PFC) technique to realize the unity effective power factor and stable output dc voltage for both the distribution systems with sinusoidal or rectangular voltage waveform. Also, this paper presents the estimation and judgment of the input voltage waveform and its stability and the design of the estimation gains. The new control strategy has a notable advantage that it is flexible for both sinusoidal and rectangular waveforms to realize the good characteristics. The effectiveness of the proposed control scheme has been verified by experiments.

Keywords: single-phase PFC converter, rectangular-waveform distribution system, estimation and judgment of input voltage waveform, design of gain

1. Introduction

A lot of industrial equipment and household electric appliances such as motor drive systems, lighting systems and air conditioners have rectifier circuits for the dc voltage source of the inverter circuits. Diode rectifier circuits with simple configuration are popularly used for the equipment. However, the input current waveform of the diode rectifier circuit is distorted and the harmonic current causes low power factor and harmonic interferences. To avoid this problem, PFC converters have attracted considerable attention⁽¹⁾. However, the system configuration is complicated compared with that of the diode rectifier circuit.

On the other hand, the dc power distribution system has been proposed for the equipment with the inverter circuits⁽²⁾. However, the isolation and the over-current protection in the dc power distribution system are difficult to realize compared with that in the ac power distribution system. Also, the ac distribution systems with the square-waveform and the trapezoidal-waveform are proposed⁽³⁾⁽⁴⁾. But the distribution system with the square-waveform in reference (3) cannot be applied to the three-phase system. The system configuration to generate the trapezoidal-waveform is complex. The single-phase or three-phase distribution system with the rectangular voltage waveform has been proposed⁽⁵⁾. The distribution system with the rectangular voltage waveform is useful for the equipment with the dc voltage source because the high input power factor is obtained by using the diode rectifier circuits

whose system configuration is simple.

This paper presents a single-phase PFC converter for both the distribution systems with the sinusoidal or rectangular voltage waveform. Also, this paper presents an input voltage estimation because it is necessary to judge between the sinusoidal and rectangular voltage waveforms and to obtain the high input power factor. The voltage estimation algorithm is based on the estimation of the amplitude and phase angle of the input voltage waveform⁽⁶⁾⁽⁷⁾. The stability of the proposed estimation algorithm and the design of the estimation gains are shown. Using the proposed estimation algorithm, the conventional PFC converter for the sinusoidal input voltage can also work for the rectangular input voltage. The effectiveness of the proposed estimation and control scheme has been verified by experiments.

2. Proposed Control Scheme

2.1 Definition of Sinusoidal and Rectangular Input Voltage The sinusoidal input voltage $e(n-1)$ at the sampling point $n-1$ is given with the effective value E and phase angle $\theta(n-1)$ in (1).

$$e(n-1) = \sqrt{2}E \cos \theta(n-1) \dots \dots \dots (1)$$

Figure 1 shows the rectangular input voltage waveform. The relationship between the effective value E and the amplitude E_{amp} is given in (2).

$$E = \sqrt{\frac{2}{3}}E_{amp} \dots \dots \dots (2)$$

The function $\text{rec}\theta$ is defined as the rectangular waveform of which the amplitude E_{amp} equals 1 in Fig. 1. $\text{rec}\theta$ is obtained

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Fig. 1. Rectangular input voltage waveform

Fig. 2. System configuration of single-phase PFC converter

in (3).

$$\text{rec}\theta = \begin{cases} 1 & (0 \leq \theta < \frac{\pi}{3}, \frac{5\pi}{3} \leq \theta < 2\pi) \\ 0 & (\frac{\pi}{3} \leq \theta < \frac{2\pi}{3}, \frac{4\pi}{3} \leq \theta < \frac{5\pi}{3}) \\ -1 & (\frac{2\pi}{3} \leq \theta < \frac{4\pi}{3}) \end{cases} \quad (3)$$

From (2) and (3), the rectangular input voltage $e(n-1)$ at the sampling point $n-1$ is given in (4).

$$e(n-1) = \sqrt{\frac{3}{2}} E \text{rec}\theta(n-1) \quad (4)$$

2.2 Voltage and Current Control⁽¹⁾ Figure 2 shows the system configuration of the single-phase PFC converter for both sinusoidal and rectangular input voltage waveforms. The control purpose of PFC converter is to realize the unity effective power factor and the stable output dc voltage. To realize the stable output dc voltage, the amplitude of the reactor current reference I_{amp}^* is determined through a PI controller by using the voltage difference Δv_{dc} between the dc voltage reference V_{dc}^* and the detected output voltage v_{dc} . To realize the unity effective power factor, the reactor current reference i^* is calculated from the amplitude of the reactor current reference I_{amp}^* and $|\cos\theta|$ or $|\text{rec}\theta|$. Then the on-time of the FET is determined by the current control from the difference Δi between the reactor current reference i^* and the detected reactor current i . In the current control, the estimated input voltage is used to obtain the high frequency response.

2.3 Estimation of Input Voltage Waveform The sampling delay causes the low power factor. In case of the rectangular input voltage, the power factor PF is obtained in (5)

Fig. 3. Block diagram for estimation of input voltage waveform

$$PF = 1 - \frac{3}{2\pi} \Delta\theta \quad -\frac{\pi}{3} < \Delta\theta < \frac{\pi}{3} \quad (5)$$

where $\Delta\theta$ is the phase difference between the input voltage and current. Under the frequency of 400 Hz and the sampling delay of $136\mu s$ ($\Delta\theta = 0.342$ rad), the power factor PF is 0.837 in (5). To obtain the high input power factor, it is necessary to estimate the input voltage without the sampling delay.

Figure 3 shows the block diagram for the estimation of the input voltage waveform.

The sinusoidal estimated voltage $e_{SM}(n-1)$ and the rectangular estimated voltage $e_{RM}(n-1)$ are given with the estimated amplitude values E_{SM} and E_{RM} , and phase angles θ_{SM} and θ_{RM} in (6) and (7), respectively.

$$e_{SM}(n-1) = E_{SM}(n-1) \cos\theta_{SM}(n-1) \quad (6)$$

$$e_{RM}(n-1) = E_{RM}(n-1) \text{rec}\theta_{RM}(n-1) \quad (7)$$

The sinusoidal estimation error of the input voltage, $\Delta e_S(n-1)$ can be obtained in (8).

$$\Delta e_S(n-1) = e(n-1) - e_{SM}(n-1) \quad (8)$$

According to $\Delta e_S(n-1)$, the amplitude $E_{SM}(n)$ and the phase angle $\theta_{SM}(n)$ can be estimated in (9) and (10)

$$E_{SM}(n) = E_{SM}(n-1) + K_{SE} \Delta e_S(n-1) \cos\theta_{SM} \quad (9)$$

$$\theta_{SM}(n) = \theta_{SM}(n-1) + \omega_S T_c - K_{S\theta} \Delta e_S(n-1) \sin\theta_{SM} \quad (10)$$

where K_{SE} and $K_{S\theta}$ are the gains for the estimation of the amplitude and the phase angle, respectively. ω_S is the angular frequency of the input voltage and T_c is the control period.

The rectangular estimation error of the input voltage, $\Delta e_R(n-1)$ can be obtained in (11).

$$\Delta e_R(n-1) = e(n-1) - e_{RM}(n-1) \quad (11)$$

According to $\Delta e_R(n-1)$, the amplitude $E_{RM}(n)$ and the phase angle $\theta_{RM}(n)$ can be estimated in (12) and (13)

$$E_{RM}(n) = E_{RM}(n-1) + K_{RE}\Delta e_R(n-1)\text{rec}\theta_{RM} \dots\dots\dots (12)$$

$$\theta_{RM}(n) = \theta_{RM}(n-1) + \omega_R T_c - K_{R\theta}\Delta e_R(n-1)\text{rec}(\theta_{RM} - \pi/2) \dots\dots\dots (13)$$

where K_{RE} and $K_{R\theta}$ are the gains for the estimation of the amplitude and the phase angle, respectively. ω_R is the angular frequency of the input voltage.

The average absolute errors for the sinusoidal and rectangular waveforms J_S and J_R are obtained in (14) and (15), respectively

$$J_S(n-1) = \frac{\omega_c T_c}{1 + \omega_c T_c} |\Delta e_S(n-1)| + \frac{1}{1 + \omega_c T_c} J_S(n-2) \dots\dots\dots (14)$$

$$J_R(n-1) = \frac{\omega_c T_c}{1 + \omega_c T_c} |\Delta e_R(n-1)| + \frac{1}{1 + \omega_c T_c} J_R(n-2) \dots\dots\dots (15)$$

where ω_c is the cutoff angular frequency. In the steady-state, J_S and J_R are compared and then the corresponding estimated voltage waveform of the smaller one is selected as the input voltage waveform. For example, in case $J_S < J_R$, the sinusoidal input voltage waveform is selected.

3. Stability and Estimation Gains

3.1 Stability of Sinusoidal Voltage Estimation Algorithm The sinusoidal estimation errors of the amplitude and the phase angle $\Delta e_S(n)$ and $\Delta \theta_S(n)$ are defined in (16) and (17), respectively.

$$\Delta e_S(n) = E_{amp} - E_{SM}(n) \dots\dots\dots (16)$$

$$\Delta \theta_S(n) = \theta - \theta_{SM}(n) \dots\dots\dots (17)$$

The estimation error vector $\mathbf{e}_S$ is defined in (18).

$$\mathbf{e}_S = \begin{bmatrix} \Delta e_S / E_{amp} \\ \Delta \theta_S \end{bmatrix} \dots\dots\dots (18)$$

The stability of the estimation algorithm can be obtained from the convergence to 0 of $|\mathbf{e}_S|$. From (1), (6), (8), (16) and (17), the estimation error of the input voltage, $\Delta e_S(n-1)$ is obtained by using $\sin \Delta \theta_S \approx \Delta \theta_S$, $\cos \Delta \theta_S \approx 1$ under $\Delta \theta_S \approx 0$ in (19).

$$\Delta e_S(n-1) \approx \Delta e_S(n-1) \cos \theta_{SM} - E_{amp} \Delta \theta_S(n-1) \sin \theta_{SM} \dots\dots\dots (19)$$

By using (9), (16) can be rearranged in (20).

$$\Delta e_S(n) = \Delta e_S(n-1) - K_{SE} \Delta e_S(n-1) \cos \theta_{SM}(n-1) \dots\dots\dots (20)$$

By using (10), (17) can be rearranged in (21).

$$\Delta \theta_S(n) = \Delta \theta_S(n-1) + K_{S\theta} \Delta e_S(n-1) \sin \theta_{SM}(n-1) \dots\dots\dots (21)$$

From (19), (20) and (21), the error equation can be obtained in (22)

$$\mathbf{e}_S(n) = \mathbf{A}_S(\theta_{SM}) \mathbf{e}_S(n-1) \dots\dots\dots (22)$$

where

$$\mathbf{A}_S(\theta_{SM}) = \begin{bmatrix} 1 - K_{SE} \cos^2 \theta_{SM} & K_{SE} \sin \theta_{SM} \cos \theta_{SM} \\ K'_{S\theta} \sin \theta_{SM} \cos \theta_{SM} & 1 - K'_{S\theta} \sin^2 \theta_{SM} \end{bmatrix} \dots\dots\dots (23)$$

$$K'_{S\theta} = E_{amp} K_{S\theta} \dots\dots\dots (24)$$

$\mathbf{A}_S(\theta_{SM})$ includes the nonlinear functions. By using the following transformation matrix $\mathbf{T}_S^{-1}$

$$\mathbf{T}_S^{-1} = \begin{bmatrix} \sqrt{\frac{K'_{S\theta}}{K_{SE}}} \sin \theta_{SM} & \sqrt{\frac{K_{SE}}{K'_{S\theta}}} \cos \theta_{SM} \\ \cos \theta_{SM} & -\sin \theta_{SM} \end{bmatrix}, \dots\dots\dots (25)$$

(22) can be diagonalized in (26)

$$\mathbf{e}_{ST}(n) = \lambda_S(\theta_{SM}) \mathbf{e}_{ST}(n-1) \dots\dots\dots (26)$$

where

$$\mathbf{e}_{ST} = \mathbf{T}_S^{-1} \mathbf{e}_S \dots\dots\dots (27)$$

$$\lambda_S(\theta_{SM}) = \mathbf{T}_S^{-1} \mathbf{A}_S(\theta_{SM}) \mathbf{T}_S = \begin{bmatrix} \lambda_{S1} & 0 \\ 0 & \lambda_{S2} \end{bmatrix} \dots\dots\dots (28)$$

$$\begin{cases} \lambda_{S1} = 1 \\ \lambda_{S2} = 1 - K_{SE} \cos^2 \theta_{SM} - K'_{S\theta} \sin^2 \theta_{SM} \end{cases} \dots\dots\dots (29)$$

λ_{S1} and λ_{S1} are the eigenvalues of $\mathbf{A}_S(\theta_{SM})$.

(1) $K_S = K_{SE} = K'_{S\theta}$

The eigenvalues in (29) are $\lambda_{S1} = 1$, $\lambda_{S2} = 1 - K_S$ and $\lambda_S(\theta_{SM})$ in (26) can be linear.

$\mathbf{T}_S^{-1}$ in (25) is the transformation matrix to the $\alpha - \beta$ coordinate system that rotates synchronizing with θ_{SM} . Since the coordinate system rotates synchronizing with θ_{SM} , $|\mathbf{e}_S|$ converges to 0 under $\lambda_{S1} = 1$ if the absolute value of λ_{S2} is less than 1. Therefore the stability condition of the gain K_S is obtained in (30).

$$0 < K_S < 2 \quad (K_S = K_{SE} = K'_{S\theta}) \dots\dots\dots (30)$$

(2) $K_{SE} \neq K'_{S\theta}$

The eigenvalues λ_{S1} and λ_{S2} in (29) change with θ_{SM} and $\lambda_S(\theta_{SM})$ in (26) is nonlinear. Therefore $|\mathbf{e}_S|$ may not converge to 0 even if the absolute value of λ_{S2} is less than 1.

From the cases of (1) and (2), the stability of the estimation algorithm can be obtained in (30).

3.2 Stability of Rectangular Voltage Estimation Algorithm The rectangular estimation errors of the amplitude and the phase angle $\Delta e_R(n)$ and $\Delta \theta_R(n)$ are defined in (31) and (32), respectively.

$$\Delta e_R(n) = E_{amp} - E_{RM}(n) \dots\dots\dots (31)$$

$$\Delta \theta_R(n) = \theta - \theta_{RM}(n) \dots\dots\dots (32)$$

The estimation error vector $\mathbf{e}_R$ is defined in (33).

$$\mathbf{e}_R = \begin{bmatrix} \Delta e_R / E_{amp} \\ \Delta \theta_R \end{bmatrix} \dots\dots\dots (33)$$

Fig. 4. Input voltage waveforms with amplitude error

Fig. 5. Input voltage waveforms with phase angle error

The stability of the estimation algorithm can be obtained from the convergence to 0 of $|\epsilon_R|$. Figure 4 shows the voltage waveforms with the estimation error of the amplitude ΔE_R between the input voltage e and the estimated voltage e_{RM} . The estimation error waveform $\Delta e_E (= e - e_{RM})$ is expressed in (34).

$$\Delta e_E = \Delta E_R \text{rec} \theta_{RM} \dots \dots \dots (34)$$

Figure 5 shows the voltage waveforms with the estimation error of the phase angle $\Delta \theta_R$ between the input voltage e and the estimated voltage e_{RM} . The estimation error waveform $\Delta e_\theta (= e - e_{RM})$ is approximated in (35)

$$\Delta e_\theta \approx -\frac{3}{\pi} E_{amp} \Delta \theta_R \text{rec} \left(\theta_{RM} - \frac{\pi}{2} \right) \dots \dots \dots (35)$$

where Δe_θ is approximated to obtain the equivalent area of $e - e_{RM}$. The estimation error of the input voltage, $\Delta e_R(n-1)$ is the summation of Δe_E in (34) and Δe_θ in (35), and is obtained in (36).

$$\begin{aligned} \Delta e_R(n-1) &= \Delta e_E(n-1) + \Delta e_\theta(n-1) \\ &\approx \Delta E_R(n-1) \text{rec} \theta_{RM} \\ &\quad - \frac{3}{\pi} E_{amp} \Delta \theta_R(n-1) \text{rec} \left(\theta_{RM} - \frac{\pi}{2} \right) \end{aligned} \dots \dots \dots (36)$$

By using (12), (31) can be rearranged in (37).

$$\begin{aligned} \Delta E_R(n) &= \Delta E_R(n-1) \\ &\quad - K_{RE} \Delta e_R(n-1) \text{rec} \theta_{RM}(n-1) \dots \dots \dots (37) \end{aligned}$$

By using (13), (32) can be rearranged in (38).

$$\begin{aligned} \Delta \theta_R(n) &= \Delta \theta_R(n-1) \\ &\quad + K_{R\theta} \Delta e_R(n-1) \text{rec} \left\{ \theta_{RM}(n-1) - \frac{\pi}{2} \right\} \end{aligned} \dots \dots \dots (38)$$

Table 1. $A_R(\theta_{RM})$ for phase θ_{RM}

phase θ_{RM}	$A_R(\theta_{RM})$	mode
$0 \leq \theta_{RM} < \frac{\pi}{6}$	$\begin{bmatrix} 1 - K_{RE} & 0 \\ 0 & 1 \end{bmatrix}$	1
$\frac{\pi}{6} \leq \theta_{RM} < \frac{\pi}{3}$	$\begin{bmatrix} 1 - K_{RE} & \frac{3}{\pi} K_{RE} \\ K'_{R\theta} & 1 - \frac{3}{\pi} K'_{R\theta} \end{bmatrix}$	2
$\frac{\pi}{3} \leq \theta_{RM} < \frac{2\pi}{3}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 - \frac{3}{\pi} K'_{R\theta} \end{bmatrix}$	3
$\frac{2\pi}{3} \leq \theta_{RM} < \frac{5\pi}{6}$	$\begin{bmatrix} 1 - K_{RE} & -\frac{3}{\pi} K_{RE} \\ -K'_{R\theta} & 1 - \frac{3}{\pi} K'_{R\theta} \end{bmatrix}$	4
$\frac{5\pi}{6} \leq \theta_{RM} < \frac{7\pi}{6}$	$\begin{bmatrix} 1 - K_{RE} & 0 \\ 0 & 1 \end{bmatrix}$	1
$\frac{7\pi}{6} \leq \theta_{RM} < \frac{4\pi}{3}$	$\begin{bmatrix} 1 - K_{RE} & \frac{3}{\pi} K_{RE} \\ K'_{R\theta} & 1 - \frac{3}{\pi} K'_{R\theta} \end{bmatrix}$	2
$\frac{4\pi}{3} \leq \theta_{RM} < \frac{5\pi}{3}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 - \frac{3}{\pi} K'_{R\theta} \end{bmatrix}$	3
$\frac{5\pi}{3} \leq \theta_{RM} < \frac{11\pi}{6}$	$\begin{bmatrix} 1 - K_{RE} & -\frac{3}{\pi} K_{RE} \\ -K'_{R\theta} & 1 - \frac{3}{\pi} K'_{R\theta} \end{bmatrix}$	4
$\frac{11\pi}{6} \leq \theta_{RM} < 2\pi$	$\begin{bmatrix} 1 - K_{RE} & 0 \\ 0 & 1 \end{bmatrix}$	1

From (36), (37) and (38), the error equation can be obtained in (39)

$$\epsilon_R(n) = A_R(\theta_{RM}) \epsilon_R(n-1) \dots \dots \dots (39)$$

where

$$\begin{aligned} A_R(\theta_{RM}) &= \begin{bmatrix} 1 - K_{RE} \text{rec}^2 \theta_{RM} & \\ K'_{R\theta} \text{rec} \left(\theta_{RM} - \frac{\pi}{2} \right) \text{rec} \theta_{RM} & * \\ * & \frac{3}{\pi} K_{RE} \text{rec} \left(\theta_{RM} - \frac{\pi}{2} \right) \text{rec} \theta_{RM} \\ & 1 - \frac{3}{\pi} K'_{R\theta} \text{rec}^2 \left(\theta_{RM} - \frac{\pi}{2} \right) \end{bmatrix} \dots \dots \dots (40) \end{aligned}$$

$$K'_{R\theta} = E_{amp} K_{R\theta} \dots \dots \dots (41)$$

Table 1 shows $A_R(\theta_{RM})$ for the estimated phase angle θ_{RM} . $A_R(\theta_{RM})$ has 4 modes.

(1) Mode 1

The eigenvalues of $A_R(\theta_{RM})$ are obtained in (42).

$$\left. \begin{aligned} \lambda_{R11} &= 1 - K_{RE} \\ \lambda_{R12} &= 1 \end{aligned} \right\} \dots \dots \dots (42)$$

The condition that the absolute value of λ_{R11} is less than 1 is obtained in (43).

$$0 < K_{RE} < 2 \dots \dots \dots (43)$$

(2) Mode 2

The eigenvalues of $A_R(\theta_{RM})$ are obtained in (44).

$$\left. \begin{aligned} \lambda_{R21} &= 1 \\ \lambda_{R22} &= 1 - K_{RE} - \frac{3}{\pi} K'_{R\theta} \end{aligned} \right\} \dots \dots \dots (44)$$

The condition that the absolute value of λ_{R22} is less than 1 is obtained in (45).

$$-\frac{\pi}{3} K_{RE} < K'_{R\theta} < -\frac{\pi}{3} K_{RE} + \frac{2}{3} \pi \dots \dots \dots (45)$$

(3) Mode 3

The eigenvalues of $A_R(\theta_{RM})$ are obtained in (46).

$$\left. \begin{aligned} \lambda_{R31} &= 1 \\ \lambda_{R32} &= 1 - \frac{3}{\pi} K'_{R\theta} \end{aligned} \right\} \dots \dots \dots (46)$$

The condition that the absolute value of λ_{R32} is less than 1 is obtained in (47).

$$0 < K'_{R\theta} < \frac{2}{3}\pi \dots \dots \dots (47)$$

(4) Mode 4

The eigenvalues of $A_R(\theta_{RM})$ are obtained in (48).

$$\left. \begin{aligned} \lambda_{R41} &= 1 \\ \lambda_{R42} &= 1 - K_{RE} - \frac{3}{\pi} K'_{R\theta} \end{aligned} \right\} \dots \dots \dots (48)$$

The condition that the absolute value of λ_{R42} is less than 1 is obtained in (49).

$$-\frac{\pi}{3} K_{RE} < K'_{R\theta} < -\frac{\pi}{3} K_{RE} + \frac{2}{3}\pi \dots \dots \dots (49)$$

Since the coordinate system changes with θ_{RM} , $|\epsilon_R|$ converges to 0 under $\lambda_{R12} = \lambda_{R21} = \lambda_{R31} = \lambda_{R41} = 1$ if the absolute values of λ_{R11} , λ_{R22} , λ_{R32} and λ_{R42} are less than 1. Therefore, from (43), (45), (47) and (49), the stability condition of the gain K_{RE} and $K'_{R\theta}$ is obtained in (50).

$$\left. \begin{aligned} 0 &< K_{RE} \\ 0 &< K'_{R\theta} \\ K'_{R\theta} &< -\frac{\pi}{3} K_{RE} + \frac{2}{3}\pi \end{aligned} \right\} \dots \dots \dots (50)$$

3.3 Design of Gains for Sinusoidal Voltage Estimation

$A_S(\theta_{SM})$ in (23) consists of the definite matrix A_{S0} and the nonlinear matrix A_{S1} as follows.

$$A_S(\theta_{SM}) = A_{S0} + A_{S1} \dots \dots \dots (51)$$

$$A_{S0} = \begin{bmatrix} 1 - \frac{K_S}{2} & 0 \\ 0 & 1 - \frac{K_S}{2} \end{bmatrix} \dots \dots \dots (52)$$

$$A_{S1} = \frac{K_S}{2} \begin{bmatrix} -\cos 2\theta_{SM} & \sin 2\theta_{SM} \\ \sin 2\theta_{SM} & \cos 2\theta_{SM} \end{bmatrix} \dots \dots \dots (53)$$

A_{S1} in (53) oscillates with twice the frequency of the input voltage and its average value is 0. Therefore $A_S(\theta_{SM})$ can be approximated in (54) if the convergence characteristic of the input voltage estimation is slow enough compared with twice the frequency of the input voltage

$$\begin{aligned} \epsilon_S(n) &= (A_{S0} + A_{S1})\epsilon_S(n-1) \\ &\simeq A_{S0}\epsilon_S(n-1) \\ &= \left(1 - \frac{K_S}{2}\right)^n \epsilon_S(0) \dots \dots \dots (54) \end{aligned}$$

where $\epsilon_S(0) = [\Delta E_S(0)/E_{amp} \ \Delta\theta_S(0)]^T$ is the initial value of the estimation error vector. From the equivalence of the response for the sampling point n in (54) and the time response for the first order system with time constant τ_S , the relationship between the eigenvalue $1 - K_S/2$ and time constant τ_S is given in (55).

$$1 - \frac{K_S}{2} = e^{-\frac{T_c}{\tau_S}} \dots \dots \dots (55)$$

From (24) and (55), the gains for input voltage estimation, K_{SE} and $K_{S\theta}$ can be determined in (56).

$$\left. \begin{aligned} K_{SE} &= 2\left(1 - e^{-\frac{T_c}{\tau_S}}\right) \\ K_{S\theta} &= \frac{2}{E_{amp}}\left(1 - e^{-\frac{T_c}{\tau_S}}\right) \end{aligned} \right\} \dots \dots \dots (56)$$

3.4 Design of Gains for Rectangular Voltage Estimation $A_R(\theta_{RM})$ in (40) consists of the definite matrix A_{R0} and the nonlinear matrix A_{R1} as follows.

$$A_R(\theta_{RM}) = A_{R0} + A_{R1} \dots \dots \dots (57)$$

$$A_{R0} = \begin{bmatrix} 1 - \frac{2}{3}K_{RE} & 0 \\ 0 & 1 - \frac{2}{\pi}K'_{R\theta} \end{bmatrix} \dots \dots \dots (58)$$

$$A_{R1} = \begin{bmatrix} -K_{RE}(\text{rec}^2\theta_{RM} - \frac{2}{3}) \\ K'_{R\theta}\text{rec}(\theta_{RM} - \frac{\pi}{2})\text{rec}\theta_{RM} \\ \frac{3}{\pi}K_{RE}\text{rec}(\theta_{RM} - \frac{\pi}{2})\text{rec}\theta_{RM} \\ -\frac{3}{\pi}K'_{R\theta}\{\text{rec}^2(\theta_{RM} - \frac{\pi}{2}) - \frac{2}{3}\} \end{bmatrix} \dots \dots \dots (59)$$

A_{R1} in (59) oscillates with twice the frequency of the input voltage and its average value is 0. Therefore, $A_R(\theta_{RM})$ can be approximated in (60) if the convergence characteristic of the input voltage estimation is slow enough compared with twice the frequency of the input voltage

$$\begin{aligned} \epsilon_R(n) &= (A_{R0} + A_{R1})\epsilon_R(n-1) \\ &\simeq A_{R0}\epsilon_R(n-1) \\ &= \begin{bmatrix} 1 - \frac{2}{3}K_{RE} & 0 \\ 0 & 1 - \frac{2}{\pi}K'_{R\theta} \end{bmatrix}^n \epsilon_R(0) \dots \dots \dots (60) \end{aligned}$$

where $\epsilon_R(0) = [\Delta E_R(0)/E_{amp} \ \Delta\theta_R(0)]^T$ is the initial value of the estimation error vector. From the equivalence of the response for the sampling point n in (60) and the time response for the first order system with time constant τ_R , the relationship between the eigenvalue $1 - 2K_{RE}/3$, $1 - 2K'_{R\theta}/\pi$ and time constant τ_R is given in (61).

$$\left. \begin{aligned} 1 - \frac{2}{3}K_{RE} &= e^{-\frac{T_c}{\tau_R}} \\ 1 - \frac{2}{\pi}K'_{R\theta} &= e^{-\frac{T_c}{\tau_R}} \end{aligned} \right\} \dots \dots \dots (61)$$

From (41) and (61), the gains for input voltage estimation, K_{RE} and $K_{R\theta}$ can be determined in (62).

$$\left. \begin{aligned} K_{RE} &= \frac{3}{2}\left(1 - e^{-\frac{T_c}{\tau_R}}\right) \\ K_{R\theta} &= \frac{\pi}{2E_{amp}}\left(1 - e^{-\frac{T_c}{\tau_R}}\right) \end{aligned} \right\} \dots \dots \dots (62)$$

4. Experimental Results

4.1 System Configuration Figure 6 shows a 600 W single-phase PFC converter for both sinusoidal and rectangular input voltage waveforms for experiments. The specifications are shown in Table 2. A DSP (TMS320C50) is used for the controller of the PFC converter.

Table 3 shows the gains for the input voltage estimation. The gains are designed as the time constant $\tau_S = \tau_R = 3.89$ ms in (56) and (62).

4.2 Sinusoidal Voltage Estimation Figure 7 shows the theoretical and experimental convergence characteristics for the amplitude disturbances of ± 30 V at $\theta = 0$ deg under the sinusoidal voltage estimation. Figure 8 shows the theoretical and experimental convergence characteristics for the phase angle disturbance of ± 30 deg at $\theta = 0$ deg. The dotted lines are the theoretical characteristics in (54), the solid lines are the experimental ones. From Figs. 7 and 8, the experimental convergence characteristics are close to the theoretical ones.

Fig. 6. Hardware configuration

Table 2. System specifications

Rated output Voltage source e	600 W AC200 V, 60 Hz, Sinusoidal or AC200 V, 400 Hz, Rectangular
Rated output voltage	DC380 V
Inductance L	1.72 mH
Smoothing capacitance C	1500 μ F
Switching frequency	58.8 kHz
Control period T_c	68 μ s

Table 3. Experimental conditions

Estimation of sinusoidal input voltage	Gain for estimation of amplitude $K_{SE} = 0.0347$ Gain for estimation of phase angle $K_{S\theta} = 0.000123 \text{ rad/V}$
Estimation of rectangular input voltage	Gain for estimation of amplitude $K_{RE} = 0.0260$ Gain for estimation of phase angle $K_{R\theta} = 0.000111 \text{ rad/V}$
Voltage control	Natural angular frequency $\omega_n = 2\pi \times 1 \text{ rad/s}$ Damping ratio $\zeta = 1$
Current control for sinusoidal input voltage	Natural angular frequency $\omega_n = 2\pi \times 600 \text{ rad/s}$ Damping ratio $\zeta = 1$
Current control for rectangular input voltage	Natural angular frequency $\omega_n = 2\pi \times 100 \text{ rad/s}$ Damping ratio $\zeta = 1$

Fig. 7. Theoretical and Experimental Convergence characteristics for amplitude disturbances under sinusoidal voltage estimation

Figure 9 shows the convergence characteristics for the amplitude disturbances of $-300 \sim +300 \text{ V}$ at $\theta = 0 \text{ deg}$. Figure 10 shows the convergence characteristics for the phase

Fig. 8. Theoretical and Experimental Convergence characteristics for phase angle disturbances under sinusoidal voltage estimation

Fig. 9. Convergence characteristics for amplitude disturbances under sinusoidal voltage estimation

Fig. 10. Convergence characteristics for phase angle disturbances under sinusoidal voltage estimation

angle disturbances of $-180 \sim +180 \text{ deg}$ at $\theta = 0 \text{ deg}$. From Fig. 9, the disturbances of the amplitude do not affect the estimation errors of the phase angle $\Delta\theta_S$. From Fig. 10, the estimated amplitude E_{SM} decreases transiently for the disturbances of the estimated phase angle. Then it converges to the amplitude E_{amp} as the estimation error of the phase angle $\Delta\theta_S$ converges to 0.

Fig. 11. Theoretical and Experimental Convergence characteristics for amplitude disturbances under rectangular voltage estimation

Fig. 12. Theoretical and Experimental Convergence characteristics for phase angle disturbances under rectangular voltage estimation

As above, the sinusoidal voltage estimation algorithm can converge for the disturbances of the amplitude of $-300 \sim +300$ V and the phase angle of $-180 \sim +180$ deg within 40 ms of the input voltage.

4.3 Rectangular Voltage Estimation Figure 11 shows the theoretical and experimental convergence characteristics for the amplitude disturbances of ± 30 V at $\theta = 0$ deg under the rectangular voltage estimation. Figure 12 shows the theoretical and experimental convergence characteristics for the phase angle disturbances of ± 30 deg at $\theta = 0$ deg. The dotted lines are the theoretical characteristics in (60), the solid lines are the experimental ones. From Fig. 11 and 12, however the estimation errors of the amplitude and phase angle are respectively about 5 V and 1 deg in the maximum, the average experimental convergence characteristics are close to the theoretical ones. The reason of the steady-state errors is that the input voltage waveform is not ideal and the estimation error is always caused.

Figure 13 shows the convergence characteristics for the amplitude disturbances of $-250 \sim +250$ V at $\theta = 0$ deg. Figure 14 shows the convergence characteristics for the phase angle disturbances of $-180 \sim +180$ deg at $\theta = 0$ deg. From Fig. 13 and 14, the rectangular voltage estimation algorithm can respectively converge ΔE_R and $\Delta \theta_R$ to $0 \sim +5$ V and $-1 \sim +1$ deg for the disturbances of the amplitude of $-250 \sim +250$ V and the phase angle of $-180 \sim +180$ deg within 40 ms of the input voltage.

4.4 Control Characteristics Figures 15 and 16 show the experimental results of the proposed control scheme using the sinusoidal and rectangular input voltages under the

Fig. 13. Convergence characteristics for amplitude disturbances under rectangular voltage estimation

Fig. 14. Convergence characteristics for phase angle disturbances under rectangular voltage estimation

Fig. 15. Experimental results for sinusoidal input voltage

rated load, respectively. The power factor for the sinusoidal and rectangular input voltages, measured by using the Digital Power Meter (WT1030, YOKOGAWA), are 0.995 and 0.949, respectively. Although there are some estimation errors of the rectangular waveform from Fig. 11 to Fig. 14 because of the unideal input voltage, the good characteristics of the PFC

Fig. 16. Experimental results for rectangular input voltage

converter system are obtained. In Fig. 16 there are some current overshoot when the rectangular input current increases and decreases from 0 because of the discontinuous conduction mode of the reactor current.

5. Conclusions

This paper presents the input voltage estimation of the single-phase PFC converter for both the sinusoidal and rectangular input voltages. The stability of the estimation algorithm and the design of gains for the estimation are shown.

Using the proposed input voltage estimation algorithm, the estimation errors converge within 40 ms of the input voltage under the amplitude and phase angle disturbance. The power factors are 0.995 and 0.949 when the sinusoidal and rectangular waveforms are used as the input voltages, respectively.

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