Online Frequency Estimation using Power Series Type Wavelet Transform

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Keywords: wavelet transform, time-frequency analysis, online frequency estimation

1. Introduction
The wavelet analysis is able to calculate a time-frequency analysis and acquire the frequency information on a signal. However, the wavelet analysis has problems, which are various amounts of calculation, accuracy and so on to apply it online to a power system. The Gabor function or several functions are widely used for the wavelet analysis, accuracy and so on to apply it online to a power system. The wavelet analysis has problems, which are various amounts of calculation decreases. The effectiveness of the proposed function is verified to compare with the Gabor function by simulation. Moreover, applying it to the experiment system using a DSP, the online frequency estimation is successfully performed and the good results are obtained.

2. Power Series Type Wavelet Transform
The proposed wavelet function, which is consisted of the power series expression, is described.

\[ f(t) = \left( \frac{\sigma^2}{240} + \frac{\sigma^8}{960} - \frac{\sigma^9}{8640} \right) \exp(-\sqrt{\omega}t + j\omega t) u(t) \]

where \( \omega_0 \) and \( \sigma \) are constant. \( u(t) \) is the unit step function.

The proposed function can easily obtain the discrete time function by the z-transform and shorten the calculation time in practice. The filter function of the proposed wavelet function, which has the reversed time at \( t = 0 \).

The proposed wavelet function can be transformed to a discrete time system by using the convolution integration and z-transform. The calculation time can be shortened by the transformation. The z-transform of the proposed wavelet transform is expressed as

\[ W_d(a,k) = a \frac{1}{2} T \left( \sum_{m=1}^{9} \delta_m s(k-m) \right) - \left( \sum_{m=1}^{10} \lambda_m W_d(a,k-m) \right) \]

where \( \delta_m \) and \( \lambda_m \) are coefficients related to \( \omega, \sigma, \) and \( a \) and sampling period \( T \). (2) enables fast, recursive computation for the convolution integral.

The real part \( W_r(\tau) \) and the imaginary part \( W_i(\tau) \) of \( W_d(a,\tau) \) can be utilized to estimate the instantaneous phase and frequency.

\[ \theta(\tau) = \tan^{-1} (W_i(\tau)/W_r(\tau)) \]

The estimated frequency is calculated by the following equation using the instantaneous phase \( \theta(k) \) and \( \theta(k-1) \) at the sampling time \( kT \) and \( (k-1)T \).

\[ f(k) = \frac{1}{2\pi} \frac{\theta(k) - \theta(k-1)}{T} \]

3. Experimental Results
In order to check whether the online frequency presumption is possible with the proposal system using a DSP (TMS320C32), the examined signal is computed by the DSP as the virtual sinusoidal current. The frequency estimation is performed by carrying out a step change of the frequency from 40 Hz to 50 Hz as shown in Fig. 1.

The frequency estimation can be confirmed within 0.04 s. The frequencies of the signal are quickly estimated on line by the proposed method.

Further the online frequency estimation for the power system (the IM current and the rectifier input current) is achieved by the experiment.

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**Fig. 1.** Experimental results of frequency estimation \( f_0 = 40 \) [Hz], \( \omega_0 = \sigma = 2\pi \)
Online Frequency Estimation using Power Series Type Wavelet Transform

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In this paper, a new approach to estimate the power system frequency using wavelet transform is proposed. The wavelet function is expressed by the power series and is able to be performed by the z-transform, which can shorten the estimating time of the signal frequency compared with the Gabor function. Simulation results, in which the fundamental frequency (60 Hz) or the harmonics frequency (300 Hz) of the signals are quickly estimated by the proposed method, are presented and the online estimation of the frequencies is successfully achieved by the proposed method in the experimental system using DSP.

Keywords: wavelet transform, time-frequency analysis, online frequency estimation

1. Introduction

Recently it is reported that the developments of power electronics make the harmful influence of harmonics or a noise. Because the electric power systems increasingly complicate, wavelet analysis attracts attention as the new method (1). Those are required the long calculation time for the convolution integral. Thus it is difficult to estimate frequencies on line.

In this paper, in order to perform online frequency estimation of a signal, a new wavelet function is proposed. It can be transformed to discrete time system by the \( z \)-transform, which has various amounts of calculation, accuracy and so on to apply it online to a power system. The Gabor function or several functions are widely used for the wavelet transform. It presents the band pass frequency property of \( \varphi(t) \), where \( \psi(\omega) \) is the Fourier transform of \( \varphi(t) \).

The wavelet analysis is able to calculate a time-frequency analysis and acquire the frequency information on a signal (2). In general, the wavelet analysis has problems, which are various amounts of calculation, accuracy and so on to apply it online to a power system. The Gabor function or several functions are widely used for the wavelet transform. Those are required the long calculation time for the convolution integral. Thus it is difficult to estimate frequencies on line.

In this paper, in order to perform online frequency estimation of a signal, a new wavelet function is proposed. It can be transformed to discrete time system by the \( z \)-transform, by which the amount of calculation decreases. The effectiveness of the proposed function is verified to compare with the Gabor function by simulation. Moreover, applying it to the experiment system using a DSP, the online frequency estimation is successfully performed and the good results are obtained.

2. Wavelet Analysis

2.1 Wavelet Transform

In order to acquire the local information on the observed signal \( s(t) \), a core function, called a mother wavelet function, is defined. It is expanded, reduced or carried out its parallel time shifting. The wavelet transform can be derived from a mother wavelet function by introducing scaling factor \( a \) and time shifting factor \( \tau \).

\[
\varphi_{a\tau}(t) = \frac{1}{\sqrt{a}} \varphi\left(\frac{t - \tau}{a}\right) \quad \ldots \quad (1)
\]

Mother wavelet function \( \varphi(t) \) is defined as an analytical function which satisfies the admissibility condition expressed in (2).

\[
C_\varphi = \int_{-\infty}^{\infty} \frac{(|\psi(\omega)|^2)}{\omega} d\omega < \infty \quad \ldots \quad (2)
\]

i.e. \( \psi(\omega)|_{\omega = 0} = \int_{-\infty}^{\infty} \varphi(t) dt = 0 \quad \ldots \quad (3)

It presents the band pass frequency property of \( \varphi(t) \), where \( \psi(\omega) \) is the Fourier transform of \( \varphi(t) \).

The wavelet transform of \( s(t) \) by the wavelet function is defined as (4).

\[
W_\varphi(a, \tau) = \int_{-\infty}^{\infty} s(t) \overline{\varphi_{a\tau}(t)} dt \quad \ldots \quad (4)
\]

Replacing \( \overline{\varphi_{a\tau}(t)} = \eta_a(t - \tau) \), we have

\[
W_\psi(a, \tau) = \int_{-\infty}^{\infty} s(t) \eta_a(t - \tau) dt \quad \ldots \quad (5)
\]

\( \overline{\varphi(t)} \) is the complex conjugate of \( \varphi(t) \). If the scale factor \( a \) increases, a frequency band of the wavelet function spread and the band becomes narrow if \( a \) decrease.

2.2 Wavelet Transform by the Gabor Function

Generally, the Gabor function is used as a wavelet transform method. The Gabor function \( \gamma(t) \) is expressed as (6).

\[
\gamma(t) = \frac{1}{2 \sqrt{\pi a}} \exp\left( -\frac{t^2}{4a} + j\omega_0 t \right) \quad \ldots \quad (6)
\]

By integrating with the Gabor function and an observed signal \( s(t) \) the wavelet transform can be expressed by (7).

\[
W_\gamma(a, \tau) = \int_{-\infty}^{\infty} s(t) \gamma_a(t - \tau) dt \quad \ldots \quad (7)
\]

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The proposed function includes the power series function and can easily obtain the discrete time function by the z-transform and shorten the calculation time in practice. When the z-transform is applied in (10), ten of sampling data are required. The coefficients of the power series are determined so as to satisfy the admissible condition in (2).

The filter function of the proposed wavelet function, which has the reversed time at \( t = 0 \),

\[
\eta(t) = \frac{\sqrt{a}}{\pi} \exp(-\sigma t + j\omega_0 t) u(t)
\]

\[
\eta_s(t) = \frac{1}{\sqrt{a}} \eta \left( \frac{t}{a} \right)
\]

\[
a = \frac{\omega_0}{2\pi f_0}
\]

where \( f_0 \) is the filter frequency. Fig. 3 shows an example of the real and imaginary parts of the filter function \( \eta(t) \) at \( f_0 = 40 \text{ Hz} \).

Spectrum of the function \( \eta(t) \) is calculated by (14).

\[
F[\eta(t)] = \int_{-\infty}^{\infty} \eta(t) e^{-j\omega t} dt
\]

\[
= \int_{0}^{\infty} \left[ -\frac{\sigma^7}{240} + \frac{\sigma^8}{960} - \frac{\sigma^9}{8640} \right] \exp(-\sigma t + j(\omega_0 - \omega)t) dt
\]

Replacing \( s = j(\omega - \omega_0) \) in (14), we have

\[
F[\eta(t)] = \int_{0}^{\infty} \left[ -\frac{\sigma^7}{240} + \frac{\sigma^8}{960} - \frac{\sigma^9}{8640} \right] e^{-st} dt \cdots (15)
\]

Applying (16) to (15), a simple equation is obtained as shown in (17).

\[
\int_{-\infty}^{\infty} t e^{-st} dt = \frac{k !}{s^{k+1}} \cdots (16)
\]

\[
F[\eta(t)] = -\frac{7! \cdot \sigma^7 s^7 / 240 + 8! \cdot \sigma^8 s / 960 - 9! \cdot \sigma^9 / 8640}{s^{10}}
\]

Replacing \( s = j(\omega - \omega_0) \) in (17), the spectrum \( F(j\omega) \) can be expressed in (18).

\[
F(j\omega) = \frac{-21\sigma^7 \left[ s^2 - 2\sigma s + 2\sigma^2 \right]}{[\sigma + j(\omega - \omega_0)]^{10}} \cdots (18)
\]
From (18), the admissible condition in (3) is expressed by

\[ \omega_0 = \sigma \cdot \frac{2\pi}{\sqrt{3}} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (19) \]

Fig. 5 and Fig. 7 illustrate the frequency spectrums of the proposed filter functions \( \eta_\omega(t) \) shown in Fig. 4 and Fig. 6 respectively. In these cases, the filter frequency \( f_0 \) is set to 300 Hz, \( \omega_0 \) is set to 2\( \pi \) and \( \sigma \) is varied. In Figs. 6 and 7, the condition of (19) is not satisfied. However the admissible condition is approximately satisfied.

Characteristics of the filter function by changing \( \sigma \) are the following two points.
- Variation of convergence time of the filter function
  Convergence time of the filter function shortens as \( \sigma \) becomes large. The function at \( \sigma = 2\pi \) shown in Fig. 4 converges at 0.013 s and the function at \( \sigma = \pi/3 \) shown in Fig. 6 converges at 0.02 s.
- Variation of frequency range of spectrum
  Frequency range of the spectrum spreads as \( \sigma \) becomes large. The range of the spectrum at \( \sigma = 2\pi \) shown in Fig. 5 becomes from 100 Hz to 500 Hz and the range of the spectrum at \( \sigma = \pi/3 \) becomes from 200 Hz to 400 Hz.

When the frequency of the complex signal which includes the fundamental frequency and the harmonics frequencies is estimated, it is required to squeeze the frequency range for estimating the specified frequency. Thus the frequency range of the filter function must be adjusted by changing \( \sigma \) corresponding to the estimated frequency.

In this paper, simulations and experiments are carried out at \( \omega_0 = 2\pi \) (constant), \( \sigma = 2\pi \) and \( \sigma = \pi/3 \) respectively for comparing the estimation performance.

### 3.2 Wavelet Transform using z-transform

The proposed wavelet function can be transformed to a discrete time system by using the convolution integration and z-transform \(^{22}\). The calculation time can be shortened by the transformation. The z-transform of (9) is expressed as (20).

\[ W_\eta(z) = a^{-1/2} T s(z) nT/a \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (20) \]

where \( s(z) \) and \( \eta(z) \) are the z-transform of the discrete signal \( s(nT/a) \) and the discrete function \( \eta(nT/a) \) respectively. Furthermore, \( \eta(z) \) can be expressed as the following polynomial form.

\[ \eta(z) = \sum_{n=0}^{\infty} \eta(nT/a) z^{-n} = \frac{\sum_{m=1}^{9} \delta_m z^{-m}}{1 + \sum_{m=1}^{10} \lambda_m z^{-m}} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (21) \]

\[ \lambda_m = k_m \left[ \exp \left( - \frac{T(\sigma - j\omega_0)}{a} \right) \right]^{m} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (22) \]

\[ \delta_m = \left( p_m \left( \frac{\sigma T}{a} \right)^{7} + q_m \left( \frac{\sigma T}{a} \right)^{8} + r_m \left( \frac{\sigma T}{a} \right)^{9} \right) \times \left[ \exp \left( - \frac{T(\sigma - j\omega_0)}{a} \right) \right]^{m} \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (23) \]

\( \delta_m \) and \( \lambda_m \) are coefficients related to \( \omega, \sigma, a \) and sampling period \( T, k_m, p_m, q_m, r_m \) are constant coefficients.

(20) and (21) lead to

\[ W_\eta(z) = a^{-1/2} T \left( \sum_{m=1}^{9} \delta_m z^{-m} s(z) \right) - \left( \sum_{m=1}^{10} \lambda_m z^{-m} W_\eta(z) \right) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (24) \]

By digitizing (24), the following equation is obtained.

\[ W_\eta(a, k) = a^{-1/2} T \left( \sum_{m=1}^{9} \delta_m s(k-m) \right) - \left( \sum_{m=1}^{10} \lambda_m W_\eta(a, k-m) \right) \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (25) \]
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(25) enables fast, recursive computation for the convolution integral.

3.3 Estimation of Frequency

The real part $W_R(\tau)$ and the imaginary part $W_I(\tau)$ of $W_\phi(a, \tau)$ can be utilized to estimate the instantaneous phase and frequency.

$$\theta(\tau) = \tan^{-1} \left( \frac{W_I(\tau)}{W_R(\tau)} \right) \quad \text{-------------(26)}$$

where $W_R(\tau) \equiv \text{Re} \left( W_\phi(a, \tau) \right)$, $W_I(\tau) \equiv \text{Im} \left( W_\phi(a, \tau) \right)$.

The estimated frequency is calculated by the following equation using the instantaneous phase $\theta(k)$ and $\theta(k - 1)$ at the sampling time $kT$ and $(k - 1)T$.

$$f(k) = \frac{1}{2\pi} \frac{\theta(k) - \theta(k - 1)}{T} \quad \text{-------------(27)}$$

4. Simulation Results

Fig. 8 shows the simulation result of the transient response of the proposed algorithm compared with the Gabor method. The input signal is a single sinusoidal wave that changes in step from 40 Hz to 50 Hz and from 40 Hz to 30 Hz at 0.5 s and 1.5 s, respectively. The sampling period is 4 ms, which must be selected enough smaller than the period of the estimated signal. Parameters of Gabor function and the proposed function are selected so as to become almost the same window width as shown in Fig. 1 and Fig. 3.

The frequency estimation can be confirmed within 0.05 s respectively and the response of the proposed wavelet transform is almost the same as that of the Gabor wavelet transform.

Fig. 9 and Fig. 10 show the simulation results of the estimation of the harmonics frequency (300 Hz). The test signal has the fundamental component (60 Hz), 30% of the 3rd order, 15% of the 5th order, 10% of the 7th order and 5% of the 9th order harmonics as shown in Fig. 9 and the fundamental component (60 Hz), 30% of the 5th order and 10% of the 7th order harmonics in Fig. 10. Further only the 5th order component changes in step from 300 Hz to 310 Hz and the reverse at 0.5 s and 1.5 s respectively in Fig. 10. The filter frequency of the mother function is selected to 300 Hz to estimate the 5th order harmonics. The sampling period is 0.5 ms. The estimated frequency has a large error of which the maximum is 40 Hz at $\sigma = 2\pi$. Compared with it, the good estimation
is achieved at $\sigma = 2\pi/3$ since the frequency range of the filter function is squeezed and the other frequencies except the estimated frequency gives scarcely influences.

5. Online Frequency Estimation by DSP

In order to check whether the online frequency presumption is possible with the proposal system using a DSP (TMS320C32), the examined signal is computed by the DSP as the virtual sinusoidal current. The frequency estimation is performed by carrying out the same step change of the frequency as that of the simulation. The results are shown in Fig. 11. The online estimation is well realized by the proposed method as same as the simulation. In the online processing, the Gabor wavelet transform was unrealizable in both causes, because of processing time and the amount of calculation. Table 1 shows the processing time of the DSP per 1 period to estimate frequency by using the simple convolution integral wavelet transform of the function and the wavelet transform using z-transform. The processing time of the Wavelet transform using z-transform is much faster than that of the convolution integral wavelet transform.

6. Experimental Results

6.1 Induction Motor  In order to demonstrate the validity of the proposed method, the current supplied to an
Induction Motor (IM) from an inverter is detected and its frequency is estimated by the proposed wavelet transform. The system configuration is shown in Fig. 12. The control of IM is realized by the vector control. The speed of IM is controlled by giving the motor speed command.

The step change of the motor speed command is carried out for 1 s with 1200-1500-1200 r/min and 1200-900-1200 r/min. The sampling period is 4 ms and frequency estimation results are obtained at every sampling period. The results are shown in Figs. 13 and 14. As shown in Figs. 13 and 14, the frequency and the amplitude of the current change simultaneously on step change of the motor speed command. Although the estimated results during the transient period have ripples due to the complex current changes, the online frequency estimation is realized by this experimental system.

### 6.2 Diode Rectifier

To examine the harmonics frequency estimation, an input current of the three phase diode rectifier is detected and its frequency is estimated by the proposed method. Fig. 15 depicts the configuration of diode rectifier system and Table 2 shows the circuit parameters. The waveform of the input current $i_a$ is shown in Fig. 16 and an FFT analysis of the current $i_a$ is shown in Fig. 17. The current $i_a$ has the fundamental component (60 Hz), 10% of the 5th order and 5% of the 7th order harmonics component.

Fig. 18 shows the experimental result of the fundamental frequency estimation (60 Hz) and Fig. 19 shows the harmonics frequency estimation (300 Hz). Those have small ripples compared with the simulation. It is due to an influence of other frequency current, an accuracy of the current sensor and quantization of the signal. However the fundamental frequency and the harmonics frequency can be estimated within 3% errors and 2% errors respectively.

### 7. Conclusion

In this paper, the online frequency estimation using the proposed wavelet transform is verified. The proposed
wavelet function consists of the power series and can be transformed to the discrete time system by the $z$ transform. The fundamental frequency or the harmonics frequencies of the signals are quickly estimated by the proposed method. Compared with it with the Gabor function, it is verified that the proposed method can estimate the frequency the same speed as the Gabor method and it can shorten the calculation time for estimate. The online frequency estimation for the power system (the IM and the rectifier) is achieved by the experiment.

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References


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