Force Control by Flexible Manipulator Based on Resonance Ratio Control using Position Sensitive Detector

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Keywords: motion control, disturbance observer, flexible manipulator, two-mass resonant system, force control, vibration suppression

This paper presents a novel force control method to suppress torsional vibration of two-mass resonant system based on the resonance ratio control. The reaction torque is estimated by using position sensitive detector (PSD). Since the method does not need the parameter identification, the torsion information is obtained with accuracy.

The total force controller is shown in Figure 1. Here, the disturbance torque \( \tau_{\text{dist}} \) is compensated by the disturbance observer, and the arm disturbance is rejected by the arm disturbance observer. \( K_v \) denotes the stiffness of the environment. The transfer function from the force command \( F_{\text{cmd}} \) to the force response \( F_{\text{res}} \) is represented as follows,

\[
\frac{F_{\text{res}}}{F_{\text{cmd}}} = \frac{K_v K_a \omega_a^2}{s^2 + K_v s^3 + \omega_a^2 s^3 + K_a \omega_a^3 s + K_v K_a \omega_a^4}, \quad (1)
\]

The resonance ratio is uniquely determined as \( \sqrt{6} \) when force control is implemented to two-mass resonant systems. Each gain is given as follows,

\[
K_v = \frac{\omega_a^2}{K_v}, \quad (2)
\]

\[
K_a = 4\omega_a, \quad (3)
\]

\[
K_v = \frac{5}{J_v}. \quad (4)
\]

The arm disturbance is compensated by the feedback of the estimated arm disturbance torque through the inverse system of the motor portion

\[
\frac{F_{\text{cmd}}}{\tau_{\text{dist}}} = \frac{s^2 + K_v s + K_v K_f}{K_p K_f}, \quad (5)
\]

The experimental results without reaction torque feedback are shown in Figure 2. From Figure 2, it turns out that the force and position responses are vibrated. This is because the system does not distinguish the torsional reaction torque from the arm disturbance. The system compensates the torsional reaction torque and the force response becomes worse because of canceling out the antiresonance zero by the disturbance observer.

The experimental results by the proposed force control based on the resonance ratio control are shown in Figure 3. The proposed system can distinguish the torsional reaction torque from the arm disturbance. Since the torsional reaction torque is compensated by the resonance ratio control, and the vibrations are well suppressed. As a result, the proposed force control system can realize both the suppression of the inner torsional reaction torque and the adaptation to unknown outer force responses.

The numerical and experimental results showed viability of the proposed method. The total system has a very simple structure and the vibration of the arm portion was well suppressed by the proposed method.
Force Control by Flexible Manipulator Based on Resonance Ratio Control using Position Sensitive Detector

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This paper presents a novel force controller to suppress torsional vibration of two-mass resonant system. The resonance ratio control is one of the effective control methods of two-mass resonant system. In this method, the ratio between the resonant frequency of motor and arm is determined arbitrary according to the feedback of estimated reaction torque. The reaction torque is estimated by using position sensitive detector (PSD). Since the estimation method does not need the parameter identification, the torsion information is obtained with accuracy.

To attain the affinity and adaptability to environment, motion systems should control the reaction force from the environment. In the force control system, the force response is regarded as a disturbance of the arm portion. The arm disturbance is observed by the arm disturbance observer. The proposed force control system is based on both the conventional PD control and the resonance ratio control and the determination method of pole placement is discussed. The proposed force control system can realize both the suppression of the inner torsional reaction torque and the adaptation to outer force inputs. The numerical and experimental results show viability of the proposed method.

Keywords: motion control, disturbance observer, flexible manipulator, two-mass resonant system, force control, vibration suppression

1. Introduction

With the progress of robust control techniques, high performance in the motion control field has been realized. In the recent industrial application, faster motion is required. In this case, the motion response is affected by flexibility in driven system. In addition, in the field of space robot manipulators, it is required to reduce the total weight of manipulators from a view point of the cost of transport. However, reduction of the weight leads to a fall of stiffness. As a result, the tip of manipulators tends to vibrate and the motion responses deteriorate.

To address this issue, several control methods to suppress the vibration have been developed [1][2][3][4][5][6]. The resonance ratio control [6] is one of the effective control methods of two-mass resonant system. In this method, the ratio between the resonant frequency of motor and arm is determined arbitrary according to the feedback of estimated reaction torque.

Recently, the force control of flexible manipulators has been done. Chiou [7] studied stability of constrained flexible manipulators. Matsuno [8] and Yoshikawa [9] proposed position/force controllers for flexible manipulators. However, since the control stiffness of the ideal force control should be 0, the feedback of position responses is undesirable from this point of view. In other words, force control systems should control the reaction force despite their position responses. The amount of research addressing the force control with low control stiffness of flexible manipulators is limited.

This paper presents a novel force control method to suppress torsional vibration of two-mass resonant system based on the resonance ratio control. The reaction torque is estimated by using position sensitive detector (PSD) [6]. PSD is kind of position sensor which consists of a camera and a light emitting diode (LED) target. Since the estimation method does not need the parameter identification, the torsion information is obtained with accuracy.

In the force control system, the force response is regarded as a disturbance of the arm portion. The arm disturbance is observed by the arm disturbance observer [7]. The proposed force control system is based on both the conventional PD control and the resonance ratio control, and the determination method of pole placement is discussed. The proposed method is applied to a one-degree-of-freedom flexible manipulator. The flexible manipulator is driven by a direct-drive motor. As a result, high affinity and adaptability to environment with suppression of vibration is achieved.

This paper is organized as follows. In the following section, a modeling of the flexible manipulator is explained. In Section 3, vibration suppression by a resonance ratio control is shown. In Section 4, force controller with low control stiffness is proposed to suppress the vibration. The numerical simulation is conducted in Section 5. Then, the proposed method is applied to a flexible manipulator and the experimental results are shown in Section 6. Finally, this paper is summarized.

2. Modeling of Flexible Manipulator

In this paper, we introduce a model of a two-mass resonant system which consists of the motor portion and the arm portion. The flexible manipulator is driven by a direct-drive motor. To attain the robustness of the motor portion, acceleration

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High-performance motion controller is a requisite for more skillful and dexterous motion. To attain such high performance, motion systems should be robust against the load change and parameter variation. Sabanovic mentioned that acceleration controller makes a motion system robust by analyzing sliding mode control. A disturbance observer which was proposed by Ohishi et al. is a good candidate for attainment of robust acceleration control. A disturbance observer identifies the total mechanical load torque and parameter change. In other words, the identification of disturbance torque is essential for motion control robustness to realize various applications.

This subsection introduces acceleration controller by a disturbance observer. A disturbance observer is designed to cancel the disturbance torque as quickly as possible. The estimated disturbance torque is obtained from the velocity response \( \dot{\theta}_m \) and the current reference \( P_{ref} \) as shown in Fig. 3.

The disturbance torque \( \tau_{dism} \) is represented as

\[
\tau_{dism} = (J_m - J_m^{nom})\dot{\theta}_m + (K_m - K_m^{nom})P_{ref} + F_{cm} + D_{m}\dot{\theta}_m + \tau_{react} \tag{4}
\]

where

\[
F_{cm} \quad \text{: Coulomb friction}
\]
\[
D_m \quad \text{: Viscous friction.}
\]

In (4), the first term is the torque due to the self-inertia variation. The second term is torque pulsations due to the variation of the torque coefficient. The third term and the fourth term denote the coulomb and the viscous friction. The last term is
the reaction torque caused by the shaft torsion.

Equation (5) shows that the disturbance torque is estimated through the first-order low-pass filter

\[ \hat{\tau}_{\text{dis}} = \frac{g_{\text{dis}}}{s + g_{\text{dis}}} \tau_{\text{dis}}, \]  

(5)

where \( g_{\text{dis}} \) denotes the cut-off frequency of the low-pass filter. The disturbance torque estimated by (5) is used for a realization of robust motion control. And robust motion controller makes a motion system to be an acceleration control system

The measured angle from the PSD is defined as follows,

\[ \theta_{\text{psd}} = \theta_{\text{ref}} - \theta_{\text{a}}. \]  

(8)

If the value of the spring coefficient \( K_{\text{fn}} \) is known \textit{a priori}, the reaction torque is estimated as follows,

\[ \hat{\tau}_{\text{react}} = K_{\text{fn}} \theta_{\text{psd}}. \]  

(9)

3.3 Feedback of Reaction Torque In this paper, a resonance ratio control which is one of the effective control methods for vibration suppression of two-mass resonant system is applied. In this method, the ratio between the resonant frequency and the anti-resonant frequency is determined arbitrarily according to the feedback of the estimated reaction torque.

The block diagram of the reaction torque feedback is shown in Fig. 6. Here \( K_r \) denotes the feedback gain of reaction torque. The estimated reaction torque is fed back to the motor which is an acceleration control system. When \( K_r = J_m^{-1} \), the system shown in Fig. 6 is equal to the uncontrolled system shown in Fig. 2.

In Fig. 6, the transfer functions from \( \dot{\theta}_{\text{ref}} \) to \( \theta_{\text{m}} \) and \( \dot{\theta}_{\text{a}} \) are represented as follows,

\[ \theta_{\text{m}} = \frac{J_m s^2 + K_f}{J_m s^2 + K_f (1 + K_r J_a)} \cdot \frac{1}{s} \dot{\theta}_{\text{m}}. \]  

(10)

\[ \dot{\theta}_{\text{a}} = \frac{K_f}{J_m s^2 + K_f} \theta_{\text{m}}. \]  

(11)

Here, the resonance frequency of the motor portion \( \omega_m \) and the resonance frequency of the arm portion \( \omega_a \) are defined as follows,

\[ \omega_m = \sqrt{\frac{K_f}{J_m} (1 + K_r J_a)} \]  

(12)

\[ \omega_a = \sqrt{\frac{K_f}{J_a}} \]  

(13)

The resonance ratio \( K \) is defined as

\[ K = \frac{\omega_m}{\omega_a}. \]  

(14)

\[ K = \sqrt{1 + K_r J_a}. \]  

(15)

\( \omega_a \) is determined according to the controlled plant because arbitrary parameters are not included. On the other hand, \( \omega_m \) is controlled by the feedback of the reaction torque.

4. Force Control of 2-Mass Resonant System

In this paper, to decompose the arm disturbance from the torsional reaction torque, the following four feedback loops are designed;

\begin{itemize}
  \item disturbance compensation by the disturbance observer;
  \item vibration suppression by the torsional reaction torque;
  \item force control by the estimated arm disturbance;
  \item nonlinear disturbance compensation of arm side.
\end{itemize}

The torsional reaction torque is necessary for stabilization of the system and the pole assignment is decided by the resonance ratio control. The arm disturbance is estimated by using the arm disturbance observer without force sensors\(^7\).

4.1 Arm Disturbance Estimation In this paper, we introduce the force control with vibration suppression. Since the arm disturbance should be estimated since it is the response of force control. The arm disturbance is observed by the arm disturbance observer.

The dynamical equations of two-mass resonant system lead to the following relation,

\[ \tau_{\text{dis}} = \dot{\tau}_{\text{react}} - J_m \ddot{\theta}_{\text{a}}. \]  

(16)

The arm disturbance is estimated from the estimated reaction torque and the arm acceleration. The block diagram of
the arm disturbance observer is shown in Fig. 7. To reduce the high frequency noise, the arm disturbance is observed through the first-order low-pass filter. \( g_o \) denotes the cut-off frequency of the low-pass filter.

### 4.2 Force Control Based on Resonance Ratio Control

In this paper, force control based on the resonance ratio control is proposed. The proposed force controller is shown in Fig. 8. The resonance ratio control considers that the control system is based on perfect acceleration control. Thus the cut-off frequency of the disturbance observer should be set as large as possible. Here, the disturbance torque \( \tau_{diss} \) is compensated by the disturbance observer, and the arm disturbance is rejected by the arm disturbance observer. \( K_s \) denotes the stiffness of the environment.

The transfer function from the force command \( F_{cmd} \) to the force response \( F_{res} \) is represented as follows,

\[
\frac{F_{res}}{F_{cmd}} = \frac{K_p K_s \omega_a^2}{s^4 + K_s s^3 + \omega_m^2 s^2 + K_s \omega_a^2 s + K_p K_s \omega_a^2} \tag{17}
\]

Here, we introduce the second-order transfer functions \( G_i(s) = \frac{\omega_i^2}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \) \((i = 1, 2)\) and (17) are transformed as follows,

\[
\frac{F_{res}}{F_{cmd}} = G_1(s)G_2(s). \tag{18}
\]

In case the damping coefficients of the transfer functions \( G_1(s) \) and \( G_2(s) \) which have the characteristics of the second-order systems are more than 1, the response is stable.

The characteristic equation of the proposed system is given as

\[
s^4 + K_s s^3 + \omega_m^2 s^2 + K_s \omega_a^2 s + K_p K_s \omega_a^2 \tag{19}
\]

By using the second-order transfer functions \( G_1(s) \) and \( G_2(s) \), the characteristic equation of the transfer function is described as follows,

\[
(s^2 + 2\zeta_1 \omega_1 + \omega_1^2)(s^2 + 2\zeta_2 \omega_2 + \omega_2^2) = s^4 + 2(\zeta_1 \omega_1 + \zeta_2 \omega_2)s^3 + (\omega_1^2 + \omega_2^2 + 4\zeta_1 \zeta_2 \omega_1 \omega_2)s^2 + 2(\zeta_1 \omega_1^2 + \zeta_2 \omega_2^2)s + \omega_1^2 \omega_2^2 \cdots \cdots \cdots \cdots \cdot (20)
\]

Since (20) is equal to (19), the relation of coefficients leads to the following equations,

\[
K_s = 2(\zeta_1 \omega_1 + \zeta_2 \omega_2) \tag{21}
\]

\[
\omega_m^2 = \omega_1^2 + \omega_2^2 + 4\zeta_1 \zeta_2 \omega_1 \omega_2 \tag{22}
\]

\[
K_s \omega_a^2 = 2(\zeta_1 \omega_1 \omega_2 + \zeta_2 \omega_1 \omega_2) \tag{23}
\]

\[
K_p K_s \omega_a^2 = \omega_1^2 \omega_2^2 \tag{24}
\]

Setting \( \zeta_1 = \zeta_2 = 1 \) and \( \omega_1 \omega_2 = \omega_a^2 \), yields

\[
\omega_m^2 = \omega_1^2 + \omega_2^2 + 4\omega_1 \omega_2 \tag{25}
\]

Moreover, setting \( \omega_1 = \omega_2 = \omega_a \) the control system has the maximum bandwidth, and the characteristic equation has quadruple pole \( \omega_a \)

\[
\omega_m^2 = 6\omega_a^2 \tag{26}
\]

Therefore the resonance ratio is given as follows,

\[
K = \sqrt{6} = 2.449 \tag{27}
\]

These equations mean that the resonance ratio is uniquely determined as \( \sqrt{6} \) when force control is implemented to twomass resonant systems. Each control parameter is given as follows,

\[
K_p = \frac{\omega_m^2}{K_s} \tag{28}
\]

\[
K_s = 4\omega_a \tag{29}
\]

\[
K_r = \frac{5}{J_a} \tag{30}
\]

In order to attain the affinity and adaptability to environment, zero-stiffness motion controller is designed in this paper. The force response which is given as a disturbance of the arm portion is controlled by the proposed force control system. Since the proposed force control system does not have the position feedback loop, and compliant motion to the environment is realized.
4.3 Arm Disturbance Compensation

The arm disturbance is compensated by the feedback of the estimated arm disturbance torque through the inverse system of the motor portion. The transfer function from $F_{cmd}$ to $F_{res}$ is represented by \( (17) \). The transfer function from $\tau_{\text{disa}}$ to $F_{res}$ is represented as follows,

$$
\frac{F_{\text{res}}}{\tau_{\text{disa}}} = \frac{1}{s^3 + K_s s^2 + K_f K_r}.
$$

Thus, the inverse system from $\tau_{\text{disa}}$ to $F_{cmd}$ is given as follows,

$$
\frac{F_{\text{cmd}}}{\tau_{\text{disa}}} = \frac{s^3 + K_s s + K_f K_r}{K_p K_f}.
$$

The arm disturbance compensation is summarized as Fig. 9.

The total force controller is shown in Fig. 10. In Fig. 10, only the motor position $\theta_m$ and the torsional angle $\theta_{psd}$ are measured by the motor encoder and the PSD, respectively. The arm position $\theta_a$ is obtained by $\theta_m - \theta_{psd}$. $\theta_m$ and $\theta_a$ are estimated by the pseudo-derivative.

## 5. Simulations

To confirm the validity of the proposed method, numerical simulations are conducted with and without the reaction torque feedback. The force commands are input stepwise as 1 N. Each parameter of the experiments is shown in Table 1. The sampling time is 1 ms.

### Table 1. Parameters of simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_m$</td>
<td>0.02 kg m²</td>
</tr>
<tr>
<td>$K_s$</td>
<td>3.0 N/m/A</td>
</tr>
<tr>
<td>$J_s$</td>
<td>0.24 kg m²</td>
</tr>
<tr>
<td>$K_{psd}$</td>
<td>54.0 N/rad</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>15.0 rad/s</td>
</tr>
<tr>
<td>$K_r$</td>
<td>60.0</td>
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<tr>
<td>$K_p$</td>
<td>20.8</td>
</tr>
<tr>
<td>$\phi_s$</td>
<td>200 rad/s</td>
</tr>
<tr>
<td>$\phi_a$</td>
<td>80 rad/s</td>
</tr>
<tr>
<td>$\phi_{psd}$</td>
<td>600 rad/s</td>
</tr>
</tbody>
</table>

The simulation results of force control are shown in Fig. 11. The simulation results without reaction torque feedback are shown in Fig. 11(a). From Fig. 11(a), it turns out that the force responses are vibrated. The simulation results by the resonance ratio control are shown in Fig. 11(b). In the proposed method, the vibrations are well suppressed.

### 6. Experiments

#### 6.1 Experimental Setup

To confirm the validity of the proposed method, experiments of force control of a flexible manipulator are performed. The overview of the experimental system is shown in Fig. 5. In the experiments, the motor angle information is obtained by an encoder, and the tip of arm is measured by the PSD. The descriptions of sensors are listed in Table 2.

Each parameter of the experiments is shown in Table 3. The control software is written in C language under ART–Linux. The sampling time is 1 ms. In the experiments, the force commands are input stepwise as 2 N.

#### 6.2 Experimental Results

The experimental results without reaction torque feedback are shown in Fig. 12. From Fig. 12, it turns out that the force and position responses are vibrated. This is because the system does not distinguish the torsional reaction torque from the arm disturbance. The system compensates the torsional reaction torque and the force response becomes worse because of canceling out the anti-resonance zero by the disturbance observer.
realize both the suppression of the inner torsional reaction torque and the adaptation to unknown outer force responses.

The experimental results by the proposed force control based on the resonance ratio control are shown in Fig. 13. The proposed system can distinguish the torsional reaction torque from the arm disturbance. The sensing noise is observed in the experimental results since the resolution of the PSD is lower than one of the motor encoder. To reduce the sensing noise, the arm position should be measured directly by the arm position encoder.

Since the torsional reaction torque is compensated by the resonance ratio control, the vibrations are well suppressed. Furthermore, only the reaction torque from the environment is observed by the arm disturbance observer, and the system generates the acceleration reference momentarily.

Since the proposed control system does not have the position feedback loop, compliant motion to the environment is achieved. As a result, the proposed force control system can realize both the suppression of the inner torsional reaction torque and the adaptation to unknown outer force responses.

### Table 2. Descriptions of sensors

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yokogawa Electric Corporation</td>
<td>65536 pulse/rev</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Model number</th>
<th>Sampling frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamamatsu Photonics K.K.</td>
<td>C5949</td>
<td>300 Hz</td>
</tr>
</tbody>
</table>

### Table 3. Parameters of experiments

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_a$ Motor inertia</td>
<td>0.02 kg m²</td>
</tr>
<tr>
<td>$K_t$ Torque coefficient</td>
<td>3.0 Nm/A</td>
</tr>
<tr>
<td>$J_e$ Arm inertia</td>
<td>0.165 kg m²</td>
</tr>
<tr>
<td>$K_s$ Spring coefficient</td>
<td>30.0 Nm/rad</td>
</tr>
<tr>
<td>$\omega_n$ Arm resonance frequency</td>
<td>13.5 rad/s</td>
</tr>
</tbody>
</table>

| Kp Proportion gain | 1.818 |
| Kr Velocity gain | 53.9 |
| Kt Reaction torque gain | 30.3 |

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_{dis}$ Cut-off frequency of disturbance observer</td>
<td>400 rad/s</td>
</tr>
<tr>
<td>$\omega_{arm}$ Cut-off frequency of arm disturbance observer</td>
<td>10 rad/s</td>
</tr>
<tr>
<td>$\omega_{pd}$ Cut-off frequency of pseudo derivation</td>
<td>600 rad/s</td>
</tr>
</tbody>
</table>

### Fig. 12. Experimental results without reaction torque feedback ($K_r = 0$)

(a) Force response  (b) Position response

### Fig. 13. Experimental results by the proposed force control based on reaction torque feedback ($K_r = 30.3$)

(a) Force response  (b) Position response

#### References


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