Optimal Control of DC-DC Converter using Mixed Logical Dynamical System Model

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Keywords: dc-dc converter, hybrid dynamical systems, powerelectronics, model predictive control and multi-parametric mixed integer quadratic programming

Recently, control systems with switching of control modes and the discrete input, which is called hybrid dynamical systems, are actively researched. The hybrid dynamical systems consist of different types of dynamical systems, that is continuous time systems and the discrete event systems. Generally, the continuous time system is expressed by differential or difference equations. On the other hand, the discrete event system is described by the logic or state machines such as automata. If a system can be regarded as the hybrid dynamical system, both continuous behaviors and discrete events can be dealt with concurrently. Therefore, the hybrid dynamical system can model many systems as a single model without dividing into two systems.

Systems containing power electronic circuits include both the continuous and discontinuous behaviors. The continuous time behavior like current or voltage is yielded by the electric circuit by passive elements whereas the discontinuous one is on-off signal given to switching devices like MOSFET and/or IGBT. Thus, such a system is typical example of the hybrid dynamical system.

For design problem of the hybrid dynamical system, a variety of approaches have been proposed. Especially, modeling and synthesis based on mixed logical dynamical (MLD) systems by Bemporad et al. has much potential in the synthesis since the formulation is similar to the linear discrete time state space representation. The solution of the design is obtained by solving the optimization problem with help of model predictive control (MPC). Specifically, the problem is solved as a mixed-integer quadratic programming (MIQP) problem. The method is expected that it can generate better control performance than that by the conventional method if applied to the output control of the power converter.

However, it is difficult to solve the optimal problem online due to computation burden as the control period is considerably short in power converters compared to mechanical systems.

This paper proposes an output control method using the MPC for the output control problem of the dc-dc converter. The considered system is described as a MLD system. The control should achieve

2 quick tracking to the reference in transient state while the minimum switching frequency in steady state. The control input is obtained by calculation of the optimization problem which is solved by means of MPC.

To verify the effectiveness of the proposed method, some numerical simulations compared online with offline are illustrated.

Fig. 1 depicts the state space partition. Fig. 1 is obtained off-line. Fig. 2 is the simulation result using the state space partition. It can be seen that the output is tracked the reference.

Fig. 1. State partition closeup (active mode)

Fig. 2. Output voltage v_o

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This paper presents an optimal control method of dc-dc converters. The method is based on hybrid dynamical system theory. Specifically, the input of the considered system is confined to discrete value whereas the output to be controlled is continuous value. The system is modeled as a mixed logical dynamical system. Then, the optimal control law is obtained by means of model predictive control. The effectiveness of the proposed approach is illustrated through some numerical simulations.

Keywords: dc-dc converter, hybrid dynamical systems, powerelectronics, model predictive control and multi-parametric mixed integer quadratic programming

1. Introduction

Recently, control systems with switching of control modes and the discrete input, which is called hybrid dynamical systems, are actively researched⁽¹⁾⁻⁽³⁾. The hybrid dynamical systems consist of different types of dynamical systems, that is continuous time systems and the discrete event systems. Generally, the continuous time system is expressed by differential or difference equations while the discrete event system is described by logics or state machines such as automata. If a system can be regarded as the hybrid dynamical system, both continuous behaviors and discrete events can be dealt with concurrently. Therefore, the hybrid dynamical system can model many systems as a single model without dividing into two systems.

Systems containing power electronic circuits include both the continuous and discontinuous behaviors. The continuous behavior like current or voltage is yielded by the electric circuit by passive elements whereas the discontinuous one is on-off signal given to switching devices like MOSFET and/or IGBT. Thus, such a system is typical example of the hybrid dynamical system. In addition, recent developments of DSP and FPGA are expected to propel the digital control in power electronic circuits.

For design problem of the hybrid dynamical system, a variety of approaches have been proposed. Especially, modeling and synthesis based on mixed logical dynamical (MLD) systems by Bemporad et al. has much potential in the synthesis since the formulation is similar to the linear discrete time state space representation⁽⁷⁾. The solution of the design is obtained by solving the optimization problem with help of model predictive control (MPC)⁽⁴⁾⁽⁵⁾. Specifically, the problem is solved as a mixed-integer linear or quadratic programming (MILP or MIQP) problem. It is expected that the

method can achieve better control performance than that by conventional methods if applied to the output control of the power converter. However, it is difficult to solve the optimal problem online due to computation burden since the control period is considerably short in power converters compared to mechanical systems.

This paper proposes a control method using the MPC for the output control problem of the dc-dc converter⁽⁸⁾⁽⁹⁾. The considered system is described as an MLD system. In the work⁽⁹⁾, one control period is divided into N submodels. Thus, further auxiliary variables are needed. In addition, the state variable among the submodels is handled as some kind of averaged one. On the contrary, this paper requires no averaging method. The explicit switching law is given as direct gate signal for the switching devices. Moreover, it is emphasized that the quadratic objective function not addressed in the work⁽⁹⁾ is addressed in this paper so that not only the tracking error but also the switching loss can be taken into account. The control should achieve quick tracking to the reference in transient state while the minimum switching frequency in steady state. The control input is obtained by calculation of the optimization problem which is solved by means of MPC. To verify the effectiveness of the proposed method, some numerical simulations compared online with offline are illustrated.

This paper is organized as follows. In Section 2, a step-down dc-dc converter and MLD are introduced. After that the optimization problems are described. In Section 3, an optimal control method is proposed. In the method, formulation of constraints and transformation to mp-MIQP are described. The effectiveness of the proposed method is shown through some simulations in Section 4. Finally, Section 5 concludes this paper.

2. Preliminaries

In this section, a step-down dc-dc converter is considered as the plant to be controlled. After the formulation,

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Fig. 1. Topology of the step-down dc-dc converter

MLD system, MPC, MIQP and multi-parametric MIQP (mp-MIQP) are reviewed.

2.1 Step-down Dc-Dc Converter A circuit topology of the step-down dc-dc converter is shown in Fig. 1. The dc-dc converter controls the load voltage v_o by on or off of the switch. The resistance r_o expresses the application driven by the dc-dc converter equivalently. The Equivalent Series Resistance (ESR) of the capacitor and the internal resistance of the inductor are denoted by r_c and r_l , respectively while x_l and x_c represent the inductance and the capacitance of the low-pass filtering stage, respectively. The switches S_1 and S_2 can not be conducted simultaneously. Together with the diode D , the switch S_2 provides a path for the inductor current i_l regardless whether it is positive or negative. In this paper, the dc-dc converter shown in Fig. 1 is the intended system. Continuous time state-space representation of the system is given as follows,

$$\dot{x}(t) = A_c x(t) + B_c u(t) \dots \dots \dots (1)$$

$$y(t) = C_c x(t) \dots \dots \dots (2)$$

where $x(t) = [i_l(t) \ v_c(t)]^T$. Denoted by $i_l(t)$ and $v_c(t)$ are the inductor current and the capacitor voltage, respectively. The matrices A_c , B_c and C_c are given by

$$A_c = \begin{bmatrix} -\frac{1}{x_l} \left(r_l + \frac{r_o r_c}{r_o + r_c} \right) & -\frac{1}{x_l} \left(\frac{r_o}{r_o + r_c} \right) \\ \frac{1}{x_c} \left(\frac{r_o}{r_o + r_c} \right) & -\frac{1}{x_c} \left(\frac{1}{r_o + r_c} \right) \end{bmatrix},$$

$$B_c = \begin{bmatrix} \frac{1}{x_l} \\ 0 \end{bmatrix}$$

and $C_c = \left[\frac{r_o r_c}{r_o + r_c} \ \frac{r_o}{r_o + r_c} \right]$, respectively. Eqs. (1) and (2) are sampled by T_s . The discrete time is described anew as t . Thus, the considered system is recast as follows,

$$x(t+1) = Ax(t) + Bu(t), \dots \dots \dots (3)$$

$$y(t) = Cx(t), \dots \dots \dots (4)$$

where $A = e^{A_c T_s}$, $B = \int_0^{T_s} e^{A_c \tau} d\tau B_c$ and $C = C_c$. Note that the value of input is limited to either 0 or v_s , which can be rewritten as follows,

$$(\forall t) u(t) \in \{0, v_s\}. \dots \dots \dots (5)$$

2.2 MLD System⁽⁷⁾ The MLD systems are described by linear dynamic equations subject to linear mixed-integer inequalities. Discrete events included in process are represented using logic variables. One of the advantages is in that

Fig. 2. Outline of MPC

the logical formula can be described linear inequalities and MPC can be applied to the MLD system.

The general MLD form is given as follows,

$$x(t+1) = Ax(t) + B_1 u(t) + B_2 \delta(t) + B_3 z(t) \dots \dots \dots (6)$$

$$y(t) = Cx(t) + D_1 u(t) + D_2 \delta(t) + D_3 z(t) \dots \dots \dots (7)$$

$$E_2 \delta(t) + E_3 z(t) \leq E_1 u(t) + E_4 x(t) + E_5 \dots \dots \dots (8)$$

where $x(t)$, $u(t)$ and $y(t)$ are states, inputs and outputs as is the usual linear systems, respectively. Additionally, $\delta(t)$ and $z(t)$ are introduced as binary and auxiliary continuous variables, respectively. A , B_i , C , D_i and E_i are appropriate matrices. As a result, Eqs. (6), (7) and (8) consist of the usual linear discrete time state space representation with the auxiliary variables of δ and z and the set of linear inequalities which represents the constraints on the system. Eq. (8) is also used to determine δ or z uniquely. Thus, the hybrid dynamical systems can be represented a familiar formulation and inequalities by the MLD system form.

2.3 MPC Fig. 2 shows the outline of the MPC. Suppose that output $y(t)$ is observed at t . Then, $y(k|t)$ is calculated by the use of the model, where $y(k|t)$ denotes the predicted output at $t+k$. The input sequence $u(0|t), u(1|t), \dots, u(N_m-1|t)$ over control input horizon $[t, t+N_m]$ is determined so that the predicted trajectory approaches the reference for predictive horizon $[t+1, t+N_p]$. N_m satisfies $N_m \leq N_p$. Only $u(t) = u(0|t)$, the first term of the input sequence, is actually input to the plant and hold until $t+1$. When $y(t+1)$ is observed at $t+1$, the input is similarly determined by sliding the predicted horizon to one step ahead. Then, the procedure is repeated. In this paper, N_m is simply set $N_p = N_m$.

2.4 MIQP and Mp-MIQP⁽⁶⁾ The MIQP derives the values which minimize an estimation of a given objective function under constraints given by inequalities and/or equalities concerning integer variables. The general MIQP form is given as follows,

$$\min_{v_t} v_t' S_1 v_t + 2(S_2 + x(t)' S_3) v_t, \dots \dots \dots (9)$$

$$\text{subject to } F_1 v_t \leq F_2 + F_3 x(t), \dots \dots \dots (10)$$

where v_t is

$$\nu_t = [\Omega'_t \quad \Delta'_t \quad \Xi'_t]', \dots\dots\dots (11)$$

$$\Omega_t = [u(0|t) \quad \dots \quad u(N_p - 1|t)]', \dots\dots\dots (12)$$

$$\Delta_t = [\delta(0|t) \quad \dots \quad \delta(N_p - 1|t)]', \dots\dots\dots (13)$$

$$\Xi_t = [z(0|t) \quad \dots \quad z(N_p - 1|t)]' \dots\dots\dots (14)$$

The mp-MIQP is a kind of MIQP parameterized by multiple parameters. The MIQP in the MPC often appears as the mp-MIQP. In such a case, the mp-MIQP parameterized by the state x of the model is described as follows,

$$\min_{\nu} \nu' H \nu + 2x' F \nu + x' Y x + 2C_f \nu + 2C_x x, \dots\dots (15)$$

$$\text{subject to } G \nu \leq W + E x, \dots\dots\dots (16)$$

where ν is

$$\nu = [\Omega' \quad \Delta' \quad \Xi']', \dots\dots\dots (17)$$

$$\Omega = [u_0 \quad \dots \quad u_{N_p-1}]', \dots\dots\dots (18)$$

$$\Delta = [\delta_0 \quad \dots \quad \delta_{N_p-1}]', \dots\dots\dots (19)$$

$$\Xi = [z_0 \quad \dots \quad z_{N_p-1}]' \dots\dots\dots (20)$$

Note that there exists a clear difference between notations of ν_t and ν . The former is utilized for MIQP while the latter is for mp-MIQP.

If solved, the optimal solution of the mp-MIQP is given as the piece-wise affine state feedback form. Namely, the control law of explicit form parameterized by the state x is obtained as follows,

$$\nu = K_i x + h_i \text{ if } x \in X_i, \dots\dots\dots (21)$$

where X_i ($i = 1, 2, \dots$) are regions partitioned in the state space, K_i and h_i are the corresponding constant matrices and vectors. As Eq. (21) is available offline, the optimal input is determined online according to measured state at each sampling.

3. Optimal Output Control Method of Dc-Dc Converter

In this section, a problem that the carrier frequency can not be set so high in Pulse Width Modulation (PWM) is described. The system to be controlled is modeled as the hybrid dynamical system. For the system, a model predictive control (MPC) based control method is proposed.

3.1 Conventional Method The basic method which is currently used for the control of dc-dc converters is the PWM with the triangular wave. The PWM, which determines on-off switching timing, controls the average output voltage. It employs relatively high carrier frequency so that the output voltage tracks to the voltage reference. Under such a condition, the voltage reference at the rising edge and that at the half period of the triangular wave can be assumed to be almost equal. Thus, the average voltage in the time can approximate the voltage reference. However, the reference may vary in the half period of triangular wave carrier if the carrier frequency is lowered to decrease switching loss for the energy saving. Therefore, the average voltage can not approximate the voltage reference any longer. One of possible reasons is in that the control frequency is determined by

the carrier frequency only. Other reason may be in that the PWM focuses on only the average output characteristic and excludes switching behaviors.

In this section, the method based on the MPC for the output control is proposed, regarding the power converter as the hybrid dynamical system. It derives the optimal input by predicting controlled variables using the mathematical plant model and minimizing a given objective function.

3.2 Representation by MLD System The dc-dc converter is rewritten to the MLD system representation to obtain the plant model. As shown by Eqs. (3), (4) and (5), the output which is a continuous variable is controlled by the discrete input for the dc-dc converter.

The auxiliary δ of 0-1 variable is introduced as a new input variable to describe the discrete variable. The variable is associated with as follows,

$$[\delta(t) = 1] \rightarrow [z(t) = v_s], \dots\dots\dots (22)$$

$$[\delta(t) = 0] \rightarrow [z(t) = 0], \dots\dots\dots (23)$$

where $z(t)$ is,

$$0 \leq z(t) \leq v_s, \dots\dots\dots (24)$$

Note that $z(t)$ is a seeming continuous auxiliary variable. Eqs. (22) and (23) indicate that $z(t) = v_s$ if $\delta(t) = 1$ whereas $z(t) = 0$, otherwise. By replacing Eqs. (22) and (23) by their equivalent linear inequalities, the MLD system representation can be described as follows,

$$x(t+1) = Ax(t) + Bz(t), \dots\dots\dots (25)$$

$$y(t) = Cx(t), \dots\dots\dots (26)$$

$$\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} z(t) \leq \begin{bmatrix} 0 \\ -v_s \\ v_s \\ 0 \end{bmatrix} \delta(t) + \begin{bmatrix} v_s \\ 0 \\ 0 \\ 0 \end{bmatrix}, \dots\dots\dots (27)$$

Inequality (27) reflects that $z(t) = v_s$ if $\delta(t) = 1$ whereas $z(t) = 0$ if $\delta(t) = 0$. Namely, $\delta(t)$ can be considered as the state of the switch: $\delta(t) = 1$ if the switch is on, $\delta(t) = 0$ if it is off.

3.3 Output Control Method by MIQP To derive the input sequence by applying the MPC for the plant model given by the MLD system form, the objective function is set as follows,

$$J(x(t), \Delta_t, \Xi_t) = \sum_{k=1}^{N_p} \|y(k|t) - v_{\text{ref}}\|_2^2 + \Delta'_t \tilde{H} \Delta_t + 2L \Delta_t, \dots\dots\dots (28)$$

where v_{ref} denotes the voltage reference. Eq. (28) is rewritten as the general MIQP form like Eqs. (9) and (10) to solve the minimization problem. By using Eqs. (3) and (4), $y(k|t)$ is described as follows,

$$\begin{aligned} y(k|t) &= C(A^k x(t) + \sum_{j=0}^{k-1} A^{k-j-1} B z(j)) \\ &= C(A^k x(t) + G_k \Xi'_k), \dots\dots\dots (29) \end{aligned}$$

where $G_k = [A^{k-1} B \quad A^{k-2} B \quad \dots \quad B]$. By substituting Eq. (29) into Eq. (28), Eq. (28) is given as follows,

$$\begin{aligned}
& J(x(t), \Delta_t, \Xi_t) \\
&= \sum_{k=1}^{N_p} \Xi_t' G_k' C' C G_k \Xi_t - 2 \sum_{k=1}^{N_p} v_{\text{ref}}' C G_k \Xi_t \\
&+ 2 \sum_{k=1}^{N_p} x(t)' A^k C' C G_k \Xi_t + \Delta_t' \tilde{H} \Delta_t + 2L \Delta_t.
\end{aligned} \tag{30}$$

Connected with Eq. (9), the optimization problem of Eq. (30) is

$$\min_{\Delta_t, \Xi_t} \begin{bmatrix} \Delta_t \\ \Xi_t \end{bmatrix}' S_1 \begin{bmatrix} \Delta_t \\ \Xi_t \end{bmatrix} + 2(S_2 + x(t)' S_3) \begin{bmatrix} \Delta_t \\ \Xi_t \end{bmatrix}, \dots \tag{31}$$

where S_1 , S_2 and S_3 are, respectively,

$$S_1 = \begin{bmatrix} \tilde{H} & O \\ O & \sum_{k=1}^{N_p} G_k' C' C G_k \end{bmatrix} \in \mathbb{R}^{2N_p \times 2N_p}, \dots \tag{32}$$

$$S_2 = \begin{bmatrix} L & -\sum_{k=1}^{N_p} v_{\text{ref}}' C G_k \end{bmatrix} \in \mathbb{R}^{1 \times 2N_p}, \dots \tag{33}$$

$$S_3 = \begin{bmatrix} O & \sum_{k=1}^{N_p} A^k C' C G_k \end{bmatrix} \in \mathbb{R}^{2 \times 2N_p}. \dots \tag{34}$$

In this paper, $\tilde{H}$ and L are zeros.

Let us rewrite the constraints as the general form like inequality (10). Recall that only two discrete input is permitted in the considered system. Such a constraint can be represented as follows, associating Eq. (27) with Eq. (10),

$$\tilde{F}_1 \begin{bmatrix} \Delta_t \\ \Xi_t \end{bmatrix} \leq \tilde{F}_2 + \tilde{F}_3 x(t), \dots \tag{35}$$

where $\tilde{F}_1$, $\tilde{F}_2$ and $\tilde{F}_3$ are, respectively,

$$\begin{aligned}
\tilde{F}_1 &= \begin{bmatrix} 0 & 1 & & & \\ v_s & -1 & & & \\ -v_s & 1 & & & \\ 0 & -1 & & & \\ & & \ddots & & \\ & & & 0 & 1 \\ O & & & v_s & -1 \\ & & & -v_s & 1 \\ & & & 0 & -1 \end{bmatrix} \in \mathbb{R}^{4N_p \times 2N_p}, \\
\tilde{F}_2 &= \begin{bmatrix} v_s \\ 0 \\ 0 \\ 0 \\ 0 \\ v_s \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^{4N_p}, \quad \tilde{F}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4N_p \times 2}.
\end{aligned} \tag{36}$$

Constraints of the inductor current limitation are added to avoid the switching device being broken due to the overcurrent to the inductance. More specifically, if the predictive inductor current at $t+1$, i.e. $i_l(1|t)$, exceeds the limited value,

$i_{l,\max}$, then the switch is forced to be off. Such an additional condition can be described as follows,

$$[i_l(1|t) > i_{l,\max}] \rightarrow [\delta(0) = 0]. \dots \tag{37}$$

Transformed into the inequality, Eq. (37) is described as,

$$i_l(1|t) - i_{l,\max} \leq M(1 - \delta(0)), \dots \tag{38}$$

where M is the upper limit of i_l . As $x = [i_l \quad v_c]'$, replaced the first row of A and the first element of B with A_1 and b_1 , respectively, $i_l(1|t)$ is recast in,

$$i_l(1|t) = A_1 x(t) + b_1 z(0). \dots \tag{39}$$

Consequently, by Eq. (39), inequality (38) has the following form,

$$M\delta(0) + b_1 z(0) \leq (M + i_{l,\max}) - A_1 x(t). \dots \tag{40}$$

Add Eq. (40) as new constraints to the last row of Eq. (36), then Eq. (36) is modified as follows,

$$\begin{aligned}
F_1 &= \begin{bmatrix} & & & \tilde{F}_1 & & & \\ M & 0 & \dots & 0 & b_1 & 0 & \dots & 0 \end{bmatrix}, \\
F_2 &= \begin{bmatrix} \tilde{F}_2 \\ M + i_{l,\max} \end{bmatrix}, \quad F_3 = \begin{bmatrix} \tilde{F}_3 \\ A_1 \end{bmatrix}.
\end{aligned} \tag{41}$$

If the objective function is described the general MIQP form as above, the optimal input can be derived. However, the optimization problem of the objective function of which its computation amount is so much should be solved every control period. Therefore, it is difficult to apply to the system with short control frequency like electrical circuits, especially, dc-dc converter. Consequently, the method is desired which solves the optimization problem by not online but offline and utilizes the result for the control. Such a method is realized by solving the optimization problem as mp-MIQP.

3.4 Output Control Method by Mp-MIQP Eq. (28) is adopted as the objective function. Multiplied out, Eq. (28) is described as follows,

$$\begin{aligned}
& J(x, \Delta, \Xi) \\
&= \sum_{k=1}^{N_p} \Xi' G_k' C' C G_k \Xi + 2 \sum_{k=1}^{N_p} x' A^k C' C G_k \Xi \\
&+ \sum_{k=1}^{N_p} x' A^k C' C A^k x - 2 \sum_{k=1}^{N_p} v_{\text{ref}}' C G_k \Xi \\
&- 2 \sum_{k=1}^{N_p} v_{\text{ref}}' C A^k x + \Delta' \tilde{H} \Delta + 2L \Delta. \dots \tag{42}
\end{aligned}$$

Associated with Eq. (15), the optimization problem of Eq. (42) is transformed as follows,

$$\begin{aligned}
\min_{\Delta, \Xi} & \begin{bmatrix} \Delta \\ \Xi \end{bmatrix}' H \begin{bmatrix} \Delta \\ \Xi \end{bmatrix} + 2x' F \begin{bmatrix} \Delta \\ \Xi \end{bmatrix} + x' Y x \\
&+ 2C_f \begin{bmatrix} \Delta \\ \Xi \end{bmatrix} + 2C_x x, \dots \tag{43}
\end{aligned}$$

where H , F and C_f are $H = S_1$, $F = S_3$ and $C_f = S_2$, respectively. The others are,

$$Y = \sum_{k=1}^{N_p} A'^k C' C A^k, \dots\dots\dots (44)$$

$$C_x = - \sum_{k=1}^{N_p} v'_{\text{ref}} C A^k. \dots\dots\dots (45)$$

The constraints are given as

$$G \begin{bmatrix} \Delta \\ \Xi \end{bmatrix} \leq W + Ex, \dots\dots\dots (46)$$

where G , W and E are $G = F_1$, $W = F_2$ and $E = F_3$, respectively. Transformed as above, the optimization problem is solved as mp-MIQP offline. Then, the result is employed to control.

4. Numerical Simulation

In this section, the effectiveness of the method proposed in the previous section is shown by applying it to the output control of dc-dc converter shown in Fig. 2. The specification is to achieve quick tracking to the reference in transient state with minimal switching in steady state. The solution is available in two ways. One is solved online, the other, offline. The results are compared with each other.

4.1 Simulation Condition The circuit and control parameters in the simulations are listed in Tables 1 and 2, respectively.

The problem setup is as follows. The load is firstly in the sleep mode. Then, received by the request of some task, the operation mode changes from the sleep mode to the active mode. After completion of the task, the mode is switched back to the idling mode, again. In such a situation, the output voltage should be kept at the reference value. The operation mode is assumed to be switched to active from sleep at 0.3 [ms] and switched back to sleep from the active at 0.3 [ms] later after that.

4.2 Simulation Result The partition of the state obtained by offline calculation, that is, mp-MIQP and their closeups are shown in Figs. 3 and 4, respectively. In each region of Fig. 3, the optimal input sequence is given out. The figures of state partition as shown in Figs. 3 and 4 are generated by means of Multi-Parametric Toolbox⁽¹⁰⁾. In Fig. 3, the number of state partition is limited to at most 2^{N_p} . Each partition is constructed by linear inequalities. In each partition, the solution of mp-MIQP given by Eq. (43) is assigned. The state, $[i_l \ v_c]'$, is judged at each control sampling. It can be simply performed since the obtained state partition

is constructed by linear inequalities. Focus on the white region at the right bottom corner in Figs. 3 or 4. When the state $[i_l \ v_c]'$ enters the region, switch S_1 shown in Fig. 1 is turned off since the constraint about the inductor current given by Eq. (37) can be satisfied no longer.

The output voltage is shown in Fig. 5. From Fig. 5, it can be seen that the output voltage holds at the specified value in the steady state. To see the transition, the closeups of Fig. 5 and the corresponding switching signal around the mode changes are shown in Figs. 6 and 7. Fig. 6 corresponds to the mode

Fig. 3. State partition (top: sleep, bottom: active)

Fig. 4. State partition closeup (top: sleep, bottom: active)

Table 1. Circuit Parameters

source voltage v_s	5 [V]
inductance x_l	2.2 [μ H]
internal resistance r_l	20 [m Ω]
capacitance x_c	220 [μ F]
ESR r_c	20 [m Ω]

Table 2. Control Parameters

control period T_s	1 [μ s]
predictive horizon N_p	5
upper limit $i_{l,\text{max}}$	10 [A]
reference value v_{ref}	2 [V]
simulation period	0.9 [ms]

Fig. 5. Output voltage v_o (top: MIQP, bottom: mp-MIQP)

Fig. 6. Closeups of output voltage v_o (top) and switching signal (bottom), (active from sleep)

switching from sleep to active while Fig. 7, from active to sleep. Figs. 6 and 7 illustrate that the output voltage tracks to the voltage reference quickly in spite of mode changes.

Let us turn to the inductor current, i.e. i_l in Fig. 5, which is shown in Fig. 8. Focus on the startup in Fig. 8, then it can be seen that the inductor current is not over the limitation, 10 [A]. Let us focus on the tracking performance in the two figures in Fig. 5. It can be seen that the bottom figure in Fig. 5 is very similar to the top. The fact would hold unless modeling error and disturbance input exist although it has not been proved strictly so far. Recall that the bottom figure in Fig. 5 is given by mp-MIQP which is solved offline. Therefore, for practical point of view, the method by mp-MIQP with less computation burden is more useful than that by MIQP.

Now, there exist various parameter perturbation in electrical circuits. For the reason, in the subsequent, let us consider cases the actual values are different from those used in

Fig. 7. Closeups of output voltage v_o (top) and switching signal (bottom), (sleep from active)

Fig. 8. Inductor current i_l

Fig. 9. Output voltage v_o

Fig. 10. Output voltage v_o closeup

the model in the proposed method. Suppose that the value of capacitance or inductance is the nominal value plus 50%. Then, the output is shown in Fig. 9. Similarly, the closeups of Fig. 9 are shown in Figs. 10 and 11. From Figs. 9 to 11, the output voltage tracks to the voltage reference even if some

Fig. 11. Output voltage v_o closeup Fig. 12. Inductor current i_l Fig. 13. Output voltage v_o

parameters are different from those used in the model. Differently from the results above, control performance is somewhat deteriorated. The reason of performance degradation is in that the predicted state is different from the actual state. However, as shown in Fig. 12(b), the current oscillation is limited since the inductance x_l of which value is not so small smoothens it.

Next, let us consider how to reduce the switching loss. This can be achieved by adding the term about the number of switchings in the objective function of Eq. (28). In Eq. (28), $\tilde{H}$ is set as follows,

$$\tilde{H} = (\Pi_1 - \Pi_2)' Q (\Pi_1 - \Pi_2), \dots \quad (47)$$

where Π_1 , Π_2 and Q are, respectively,

$$\Pi_1 = \begin{bmatrix} 0 \\ \vdots \\ I_{N_p-1} \\ 0 \end{bmatrix} \in \mathbb{R}^{(N_p-1) \times N_p}, \dots \quad (48)$$

$$\Pi_2 = \begin{bmatrix} 0 \\ I_{N_p-1} \\ \vdots \\ 0 \end{bmatrix} \in \mathbb{R}^{(N_p-1) \times N_p}, \dots \quad (49)$$

$$Q = q I_{N_p-1}. \dots \quad (50)$$

Note that the value of the objective function of Eq. (28) increases in response to the number of switchings. Therefore,

Fig. 14. Output voltage closeup v_o Fig. 15. Output voltage closeup v_o Fig. 16. Inductor current i_l

the switching loss can be reduced dependent on $q > 0$ in Eq. (50). In Eq. (50), $q = 10^{-3}$ is determined by trial and error for simulation. Fig. 13 shows the output result for $\tilde{H}$ of Eq. (47). Similarly, the closeups of Fig. 13 are shown in Figs. 14 and 15. From Figs. 13 to 15, the output voltage tracks to the voltage reference even if the term to reduce switching is added in objective function. Fig. 16 also shows that the inductor current is not over the limitation, 10[A].

5. Conclusions

In this paper, a control method for dc-dc converter has been proposed. The dc-dc converter has been modeled as a mixed logical dynamical logic system since it has both continuous and discrete variables. For the control, a model predictive control (MPC) based method has been introduced. The optimization problem has been solved as a multi-parametric programming problem. The result has been obtained as the state space partition so that online control can be applied. As a result, the computation time was shortened without being deteriorated control performance. In addition, even if that the values used for prediction are different from those of the actual ones, the output voltage has been tracked to the voltage reference. Finally, introduction of the term to reduce the switching in the objective function, the output voltage has been tracked to the reference at the expense of tracking performance.

Implementation of the proposed method is currently being developed. Further researches include the robustness analysis for implementation.

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