Stability Study of a Permanent Magnet Synchronous Motor Sensorless Vector Control System Based on Extended EMF Model

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A simplified sensorless vector control system is derived by using the extended electromotive force (EMF) model. By using a steady-state voltage equation approximately, the rotor speed and position are computed from the output voltage of a γ-axis proportional plus integral (PI) current controller with decoupling control. A linear model is proposed for the small perturbation around a steady-state operating point. The system stability is discussed in terms of the trajectories of the system matrix eigenvalues for speed estimation and control parameters. The comparison between the simulation results obtained using a nonlinear model and the experimental results validates the derived system.

Keywords: permanent magnet synchronous motor (PMSM), sensorless vector control, extended EMF, stability analysis

1. Introduction

Because of high efficiency characteristics, permanent magnet synchronous motors (PMSM) are used in many applications. Sensorless vector control that does not use any position sensors is studied and various methods have been proposed. Sensorless control method using the extended electromotive force (EMF) model in rotating reference frame is attractive for its functional structure and can be applied to interior magnet-type motor. In these papers, a disturbance observer is used to compute the extended EMF. On the other hand, a simplified sensorless vector control for electrical household appliances has been proposed by using the extended EMF model. Control structure is simplified by eliminating the speed and current proportional plus integral (PI) controllers and the extended EMF is computed by neglecting derivative term without using the disturbance observer. Another simplified method which does not use any motor constants is proposed. In this method, the output voltage of d-axis PI current controller is used to compute the rotor speed and position. A similar method which uses the output voltage of d-axis PI current controller is reported, but it is limited to non-salient PMSM.

In this paper, a simplified sensorless vector control system is derived by using the extended EMF model and a steady-state voltage equation approximately. The output voltage of γ-axis PI current controller with decoupling control is used to compute the rotor speed and position. A linear model is proposed for small perturbation around a steady-state operating point. The system stability is discussed in terms of the trajectories of the system matrix eigenvalues for speed estimation and control parameters. Experimental system is constructed by a digital signal processor (DSP) based PWM inverter drive. The comparison between simulation results obtained using a nonlinear model and the experimental results shows validity and usefulness of the proposed models and the derived system.

2. Sensorless System

The definition of d-q axis and γ-δ axis studied in this paper is shown in Fig. 1. The γ-δ axis rotates synchronously with an estimated magnetic pole’s angle.

The γ-δ axis equation of interior permanent magnet synchronous motor (IPMSM) is described by using extended EMF as follows:

\[
\begin{bmatrix}
\dot{e}_γ \\
\dot{e}_δ
\end{bmatrix} = \begin{bmatrix}
R_e + pL_δ & -\omega_e L_q \\
\omega_e L_q & R_e + pL_δ
\end{bmatrix} \begin{bmatrix}
i_γ \\
i_δ
\end{bmatrix} + \begin{bmatrix}
\dot{\theta}_γ \\
\dot{\theta}_δ
\end{bmatrix}
\]

(1)

where,

\[
E_{ex} = \omega_e \left( (L_d - L_q) i_q + \psi \right) - (L_d - L_q) p i_q
\]

(2)

\[
\begin{bmatrix}
\dot{\theta}_γ \\
\dot{\theta}_δ
\end{bmatrix} = E_{ex} \begin{bmatrix}
\sin \theta_e \\
\cos \theta_e 
\end{bmatrix} + (\dot{\omega}_e - \omega_e) L_d \begin{bmatrix}
i_γ \\
i_δ
\end{bmatrix}
\]

(3)

Fig. 1. Definition of d-q axis and γ-δ axis

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rotor angle is expressed as

\[
\hat{\gamma} = \frac{\hat{\theta}}{r} - \omega_t \tag{4}
\]

Conventional sensorless vector control method estimates \( \gamma-\delta \) extended EMF \( \hat{\epsilon}_r, \hat{\epsilon}_\theta \) by using a disturbance observer from (1) and computes approximate error angle \( \hat{\theta}_e \) of (4) as follows:

\[
\hat{\theta}_e = \tan^{-1} \left( \frac{\hat{\epsilon}_r}{\hat{\epsilon}_\theta} \right) \tag{5}
\]

The rotor speed is computed by PI controlling \( \hat{\theta}_e \) so as to converge to zero \((2,3)\).

In this paper, we derive a simpler method without using the disturbance observer. We compute the \( \gamma \)-axis extended EMF by neglecting derivative term in (1) as follows:

\[
e^*_r = v^*_r - R^*_i i^*_d + \omega_t L^*_q i^*_q \tag{6}
\]

From (5) and (6), we estimate the rotor angular speed by

\[
\hat{\omega}_0 = -\left( K_{pe} + \frac{K_{ie}}{s} \right) e^*_r \tag{7}
\]

In order to reduce high frequency ripple, a low pass filter is used to compute the rotor speed as follows:

\[
\hat{\omega}_r = \frac{\omega_t}{s + \omega_t} \hat{\omega}_0 \tag{8}
\]

The sensorless vector control system derived in this paper is shown in Fig. 2. Fig. 3 shows an approximate block diagram of speed estimation and rotor angle estimation. The transfer function of rotor angle is expressed as

\[
\frac{\hat{\theta}}{\hat{\theta}_r} = \frac{E^*_c K_{pe} s + E^*_e K_{ie}}{s^2 + E^*_c K_{pe} s + E^*_e K_{ie}} \tag{9}
\]

where,

\[
E^*_c = \hat{\omega}_r \left( L_d - L_q \right) i^*_q + \omega_t \]

By using damping coefficient \( \zeta \) and natural angular frequency \( \omega_n \), the PI control gains are expressed as

\[
K_{pe} = \frac{2\zeta \omega_n}{E^*_c}, \quad K_{ie} = \frac{\omega_n^2}{E^*_e} \tag{10}
\]

3. Analysis of Sensorless System

3.1 Nonlinear Model  In order to analyze the system shown in Fig. 2, we choose the \( d-q \) axis and \( \gamma-\delta \) axis of Fig. 1. By assuming ideal voltage control of PWM inverter, we have

\[
\begin{align*}
\gamma_a &= v^*_a, \quad \gamma_b = v^*_b, \quad \gamma_c = v^*_c \tag{11}
\end{align*}
\]

Since \( \gamma-\delta \) transformation of controller is the same as the analysis, the following relation is obtained.

\[
\begin{align*}
\gamma_a &= v^*_a, \quad \gamma_b = v^*_b \tag{12}
\end{align*}
\]

By using the actual error angle \( \hat{\theta}_e \), the \( d-q \) variables are expressed by the following co-ordinate transformation.

\[
\begin{bmatrix}
\hat{v}_d \\
\hat{v}_q
\end{bmatrix} =
\begin{bmatrix}
\cos \hat{\theta}_e & -\sin \hat{\theta}_e \\
\sin \hat{\theta}_e & \cos \hat{\theta}_e
\end{bmatrix}
\begin{bmatrix}
\hat{v}_a \\
\hat{v}_b
\end{bmatrix} \tag{13}
\]

\[
\begin{bmatrix}
\hat{i}_d \\
\hat{i}_q
\end{bmatrix} =
\begin{bmatrix}
\cos \hat{\theta}_e & -\sin \hat{\theta}_e \\
\sin \hat{\theta}_e & \cos \hat{\theta}_e
\end{bmatrix}
\begin{bmatrix}
\hat{i}_a \\
\hat{i}_b
\end{bmatrix} \tag{14}
\]

The \( d-q \) state equations of PMSM are obtained by Park’s equation as follows:

\[
\begin{align*}
p i_d &= \frac{1}{L_d} (v_d - R_i i_d + \omega_t L_q i_q) \tag{15}
p i_q &= \frac{1}{L_q} (v_q - R_i i_q - \omega_t L_d i_d - \omega_t \psi) \tag{16}
p \omega_t &= \frac{P}{2J} \left[ \psi i_q + (L_d - L_q) i_d i_q \right] - \frac{P}{2J} T_L \tag{17}
\end{align*}
\]

where, \( P \) is number of poles, \( J \) is moment of inertia, and \( T_L \) is load torque.

The controller is expressed by following equations.

PI speed controller:

\[
\begin{align*}
p w_1 &= \omega^*_r - \hat{\omega}_r \tag{18}
p w_2 &= \hat{i}_q - i_q \tag{19}
p w_3 &= K_{pe} (\omega^*_r - \hat{\omega}_r) + K_{ie} w_1 \tag{20}
p w_4 &= K_{pe} i_q + K_{ie} w_2 \tag{21}
p w_5 &= K_{pe} (\hat{i}_q - i_q) + K_{ie} w_3 \tag{22}
p w_6 &= \hat{\omega}_r - \omega_t \tag{23}
\end{align*}
\]

By taking derivative of (4), we have

\[
p \theta_e = \hat{\omega}_t - \omega_t \tag{27}
\]

A nonlinear model of the sensorless system shown in Fig. 2 is obtained by (6) and (13)–(27). By using this model, we can compute transient responses.
3.2 Linear Model  

The steady-state solutions are obtained by setting derivative operator $\dot{p}$ to zero in the nonlinear model. When the stator resistance and $q$-axis inductance of (6) are correct, the steady-state actual error angle $\theta_{e0}$ becomes zero.

The system is linearized about its equilibrium state to obtain a linear model for small signal performance evaluations, such as stability and transient response. The linearized equation of PMSM in matrix form is obtained from (15)–(17) as

$$p\Delta x = A_p\Delta x + B_p\Delta u_s + B_T\Delta T_L \quad \cdots \cdots \cdots \cdots \cdots (28)$$

where,

$$\Delta x = [\Delta i_d \Delta i_q \Delta \omega_r]^T$$

$$\Delta u_s = [\Delta \nu_d \Delta \nu_q]^T$$

The elements of the matrices are shown in the appendix.

By taking small perturbation of (13), we have

$$\Delta i_d = i_d - i_d^* \quad \cdots \cdots \cdots \cdots \cdots (29)$$

$$\Delta i_q = i_q - i_q^* \quad \cdots \cdots \cdots \cdots \cdots (30)$$

The linear state equation of controller is expressed from (18), (20), (22), (24), (26), and (27) as follows:

$$p\Delta w = A_w\Delta w + A_x\Delta x_s + B_x\Delta r \quad \cdots \cdots \cdots \cdots \cdots (31)$$

where,

$$\Delta w = [\Delta \omega_r \Delta \omega_1 \Delta \omega_2 \Delta \omega_3 \Delta \omega_4 \Delta \theta_p]^T$$

$$\Delta r = [\Delta \omega^* \Delta \omega^*]^T$$

The input vector is expressed by using state variables and references as

$$\Delta u_s = F_w\Delta w + F_x\Delta x_s + F_r\Delta r \quad \cdots \cdots \cdots \cdots \cdots (32)$$

For simplicity, the PI gains of speed estimator is assumed constant to derive (31) and (32). From (28), (31), and (32), we obtain a linear model of the complete system as

$$p\begin{bmatrix} \Delta x_s \\ \Delta w \end{bmatrix} = \begin{bmatrix} A_x & B_x F_w \\ A_w & B_w \end{bmatrix} \begin{bmatrix} \Delta x_s \\ \Delta w \end{bmatrix} + \begin{bmatrix} B_x F_r \\ B_r \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta T_L \end{bmatrix} \quad \cdots \cdots \cdots \cdots \cdots (33)$$

The stability and transient responses of the system shown in Fig. 2 can be studied by the linear model. We have confirmed that the transient responses obtained by the nonlinear model are in good agreement with those by (33) in the neighborhood of operating point.

3.3 Stability Analysis  

By using the parameters of tested PMSM, the stability is studied. For simplicity, the $d$-axis current command $i_d^*$ is set to 0. The band width of the PI current controller is designed as 1000 rad/s. Fig. 4 shows the trajectories of the poles (eigenvalues) in the $s$-plane calculated from (33), in which $\omega_s$ of (10) is changed from 10 to 200 and $\omega_{sc}$ is changed from 30 to 100. Where, $\omega_{sc}$ is the gain crossover angular frequency of PI speed control. When the pole is near the imaginary axis, the system becomes oscillating. Therefore, suitable value of $\omega_s$ must be chosen. If the $\omega_{sc}$ is small, the system is more stable, however the response of speed becomes slow. Fig. 5 shows the trajectories of the poles, in which $\zeta$ of (10) is changed from 0 to 10 and $\omega_{sc}$ is changed from 30 to 100. When $\zeta$ is small the system becomes unstable and large value of $\zeta$ leads the system to high frequency instability. Therefore, we choose as $\omega_s = \omega_{sc}$.
Sensorless Control of PMSM (Mineo Tsuji et al.)

Fig. 7. Transient responses for the step change of speed

Fig. 8. Transient responses for the step change of speed

50, \( \zeta = 1.5 \), and \( \omega_{nc} = 30 \) to ensure the system stability. By using these parameters, Fig. 6 shows the trajectories of the poles, in which speed command \( N'_r \) is varied from 50 min\(^{-1}\) to 3000 min\(^{-1}\) and the load torque \( T_L \) is varied from 0 to 2.5 Nm. For the change of speed and load torque, the system is stable at all operating points.

4. Experimental Results

The control system is implemented by a DSP (TMS320 C33) based PWM inverter. The sampling period of DSP is 100 \( \mu \)s and the switching frequency is 10 kHz. The tested PMSM has the following rated and nominal values: rated output 800 W, rated speed 3000 min\(^{-1}\), rated torque 2.55 Nm, \( P = 8 \), \( J = 0.0048 \) kgm\(^2\) (including DC generator), \( R_s = 0.4 \) \( \Omega \), \( L_d = 3.42 \) mH, \( L_q = 3.82 \) mH, and \( \psi = 0.1425 \) Wb which were also used in the analysis. Because the dead time and the non-ideal features of IGBT influence the output voltage of the inverter, a compensating algorithm is developed for the experimental system\(^9\).

Figs. 7–10 show the transient responses for step change of speed command. The simulation results are obtained by solving the nonlinear model described in section 3.1. In these figures, the speed command \( N'_r \) is stepped from 200 min\(^{-1}\) to 250 min\(^{-1}\) and then down to 200 min\(^{-1}\). When designed parameters (\( \omega_n = 50, \zeta = 1.5, \omega_{nc} = 30 \)) are used, desirable responses of actual speed \( N_r \) and estimated speed \( \hat{N}_r \) are obtained as shown in Fig. 7. The actual error angle \( \theta_e \) is defined as (4). When \( \omega_n \) is small, the \( \theta_e \) becomes large and the stability is not good as shown in Fig. 8 (\( \omega_n = 20 \)). When \( \omega_n \) is large, the PI speed estimation gains of (10) are large and high frequency ripples are observed in estimation speed and error angle as shown in Fig. 9 (\( \omega_n = 110 \)). When \( \zeta \) is small, the P gain of speed estimation is small and low frequency oscillation is observed as shown in Fig. 10 (\( \zeta = 0.5 \)). The poles corresponding to the equilibrium point of Fig. 10 are shown in Fig. 5. Since the poles are close to imaginary axis in this case, it seems that the PWM operation, machine parameter change and noise which are not considered in the analysis cause the difference between simulation results and experimental ones. It is confirmed that the simulation results can predict the experimental results in all cases, except high frequency ripples.

Fig. 11 shows the experimental results when the speed command is stepped from 1000 min\(^{-1}\) to 1050 min\(^{-1}\) and then down to 1000 min\(^{-1}\). Fig. 12 shows the experimental results when the speed command is stepped from 2500 min\(^{-1}\) to 2550 min\(^{-1}\) and then down to 2500 min\(^{-1}\). The designed parameters (\( \omega_n = 50, \zeta = 1.5, \omega_{nc} = 30 \)) are used and smooth
control of speed is obtained in both cases. By comparing the results of Figs. 11 and 12 with those of Fig. 7, it is observed that the high frequency ripples of $\hat{N}_r$ and $\hat{\theta}_e$ are decreased drastically. Since the extended EMF is large at high speed, we can estimate rotor position by small PI estimation gains. It seems that a little increase of overshoot in Fig. 12 comparing with that in Fig. 11 is caused by the poles shown in Fig. 6. The steady state experimental position error $\hat{\theta}_e$ becomes large as shown in Fig. 12 at high speed$^{10}$. The change of $q$-axis inductance $L_q$ and the PWM voltage error are considered as the reason. In order to reduce the error $\hat{\theta}_e$, an identification of $L_q$ is desired. In the simulation in Fig. 12, the actual $L_q$ is changed ($L_q^*$ in the controller is 3.82 mH). Large influence of $L_q$ on transient characteristics are not seen in the results. Therefore, the proposed linear model is useful for the basic estimation of stability.

Figs. 13 and 14 show the experimental results when the load torque is stepped from 0.15 Nm to 0.88 Nm and from 0.88 Nm to 0.15 Nm respectively. The load torque is changed by turning a switch of DC generator connected to PMSM between on and off. It is observed that the speed is disturbed by the change of load but it is recovered in a stable state.

5. Conclusions

We have derived a simplified sensorless vector control system by using the extended EMF model. This system does not need any observers which are used in the standard extended EMF based method. In order to analyze the stability of the system in detail, we have proposed a linear model for small perturbation around a steady-state operating point.
By the trajectories of system matrix eigenvalues, the system stability has been discussed for speed estimation and speed control parameters. Stability analysis of PMSM sensorless vector control system using linear model has not been reported. Since proposed stability analysis using the linear model can be executed in short computation time than commercial simulation tools, it is useful for the design of controller. Experimental system is constructed by DSP based PWM inverter drive. Comparison between simulation results using a nonlinear model and experimental results shows validity and usefulness of the derived system and the proposed analytical models.
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