Modeling and Experimental Validation of Air-Fuel Ratio under Individual Cylinder Fuel Injection in Gasoline Engines

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This paper presents a modeling approach for describing the dynamics from the fuel injection command of individual cylinders to the oxygen sensor output in spark-ignition multicylinder engines. In the presented approach, it is shown that the dynamics can be modeled as a single-input, single-output, periodic time-varying system, in which the periodicity is attributable to the cyclically physical phenomenon of the engine system. Furthermore, as compared to the conventional air-fuel ratio model, the proposed model provides the characteristics of the air-fuel ratio in a small-scale sampling rate under individual cylinder fuel injection. To demonstrate the effectiveness, numerical simulation results are presented, and finally, experimental validation is provided on the basis of the results of the identification tests conducted on a six-cylinder gasoline engine control test bench.

Keywords: air-fuel ratio, individual fuel injection, linear periodic time-varying model, injector disturbance

1. Introduction

In internal combustion (IC) engines, the mass ratio of air to fuel is an important index of engine performances\(^{(1)}\). For example, in a cylinder, high or low air-fuel ratio (lean or rich fuel) will increase pollutants in exhaust gas such as carbon monoxide and nitric oxide, and even lead to misfire\(^{(7)-(9)}\). The air-fuel ratio of gas is required to be close to the stoichiometric value in order for the catalytic converter to work at maximum efficiency\(^{(3)-(6)}\). According to the aspirated air mass, the corresponding fuel mass must be injected to guarantee the stoichiometric air-fuel ratio. However, in practice this is not achieved easily for multicylinder engines, due to disturbances of fuel injectors, cylinder-to-cylinder imbalance, and the gas mixing phenomenon in the exhaust manifold.

There has been a great deal of research which focuses on the air-fuel ratio control. Usually, a single sensor is placed in the exhaust manifold and the sensed air-fuel ratio is used as the feedback to modify the fuel injection in the closed-loop control system. Some new technologies are applied in the feedback control. For example, the neural network technology is applied in air-fuel ratio modeling and control\(^{(10)-(12)}\). To improve this method, the adaptive control is combined with the learning algorithm for air-fuel ratio control in papers\(^{(10)-(13)}\). In above control methods, same fuel injection command is issued for all cylinders. However, there exists difference among fuel injectors and cylinders. That means same fuel injection command may lead to different air-fuel ratios of individual cylinders and further the air-fuel ratio perturbations in the exhaust manifold\(^{(12)-(13)}\). To solve this problem, the individual air-fuel ratio control is proposed\(^{(14)-(15)}\). The key of the control strategies is estimation of air-fuel ratios of individual cylinders from the sensed mixed air-fuel ratio. Usually, the estimation is obtained from a model that describes the behavior of gas mixing. A model is proposed in literature\(^{(16)}\), which treats air-fuel ratios of individual cylinders and the sensed air-fuel ratio in manifold as the inputs and the output, respectively. Based on the model, a linear quadratic Gaussian (LQG) controller is presented and experimentally validated\(^{(17)}\). From a similar consideration, an inverse model with individual air-fuel ratios as outputs is presented\(^{(18)-(19)}\). Moreover, some papers focused on the diesel engines and proposed the approaches to estimate the individual air-fuel ratio by introducing an extended Kalman filter and nonlinear observer with the estimation of the exhaust manifold pressure\(^{(20)-(22)}\).

From the view of control, the individual air-fuel ratio control problem is a multiple-input and multiple-output (MIMO) system, i.e. the air-fuel ratio of the gas burnt in each cylinder is controlled via the fuel injection command of each cylinder, respectively, and as mentioned above, the key of this problem is to estimate the air-fuel ratio of each cylinder without extra sensors for each cylinder. Most of the literatures focused on the estimation of air-fuel ratio in each cylinder with single sensor installed at the gas mixing point. However, control of the air-fuel ratio in each cylinder is not crucial to control of the air-fuel ratio in exhaust gas at the gas mixing point, but, adjusting the fuel injection for each cylinder is necessary due to the cylinder-to-cylinder imbalance in the fuel paths. It makes the system multiple-input and single-output and causes complexity in system analysis and design.

Therefore, a single input signal, so-called unified injection command, is introduced in this paper to formulate the system as a single-input and single-output system (SISO). Another feature of the proposed model is periodic time-varying. Indeed, a periodic time-varying model has been used to represent the engine dynamics. For example, in the paper\(^{(23)}\), full-scaled engine model including intake manifolds, fuel injectors and rotational dynamics is formulated as periodic time-varying MIMO system. It should be noted that
the periodicity in the model results from the cylinder-to-cylinder combustion-event shifting. In contrast, the periodicity in the proposed model is caused by the restored parameter cyclically under the assumption that the exhaust gas remains in runners less than one cycle.

In the following, a periodic time-varying discrete model is derived with BDC (bottom dead center) sampling rate, in which the single input and the single output are unified injection command providing the individual fuel injection mass and the air-fuel ratio measured at the gas mixing point, respectively. The imbalance between the fuel path of cylinders are represented as offset in the input of the injector of each cylinder. It is noted that the fuel-air ratio is used instead of air-fuel ratio for linear representation. The simulation result is provided to demonstrate the model in principle, and finally the model is validated on a V6 engine.

2. System Description

Consider the ignition-combustion engines with multicylinders. The main purpose of this paper is to present a model that describes dynamical behavior from the fuel injection command of each cylinder to the output of fuel-air ratio sensor at gas mixing point in the exhaust manifold.

To illustrate the dynamics of the process from fuel injection to fuel-air ratio measurement, the combustion and the exhaust system in an engine with three cylinders is sketched as an example in Fig. 1, which is also motivated from the convenience in Section 5 where a V6 gasoline engine, with three cylinders sharing a common exhaust manifold, will be used to conduct the experimental validation. In this case, the exhaust phase of the three cylinders will occur sequentially with the interval $\frac{4\pi}{3}$[rad] in crankshaft angle (the engine cycle is $4\pi$ for four-stroke engines). In each cylinder, injected fuel and aspirated air are sparked and the combusted gas is exhausted to the corresponding runner when the exhaust valve opens. The gas flow will pass through the runner and mix at the exhaust manifold with the gases exhausted from other cylinders. Then, the mixed gas will pass through the head of oxygen sensor and the three-way catalyst, and finally be exhausted outside.

As sketched in the top plot of Fig. 2, the exhausted gas flow ($m_{oi}(t), i = 1, 2, 3$) of each cylinder occurs during the exhaust phase of the cycle where $\theta_{oi}$ and $\theta_{ci}$ ($i = 1, 2, 3$) are exhaust valve opening and closing angles of No. $i$ cylinder, respectively. However, due to the geometrical difference between the exhaust runners, the exhaust gas passing through different runners reaches the mixing point with different time. The bottom plot of Fig. 2 shows the gas flow ($m_{si}(t), i = 1, 2, 3$) from each cylinder at the mixing point and the mixed gas flow ($m_{si}(t)$). The fuel-air ratio of each cylinder that is propagated with the exhaust gas flow will be mixed also in the manifold and be sensed by the oxygen sensor. For the seek of simplicity, assume that the exhausted gas consists of fresh air and fuel. The fuel-air ratio sensor is installed at the gas mixing point. Fig. 3 shows a response of the fuel-air ratio sensor under varied fuel injection command, which is conducted on a V6 engine test bench (for details, see Section 5).

Note that in the case of direct injection, the fuel mass and the air charge will be determined in the intake phase of the same cycle. Moreover, the delay property of the sensor dynamics must be taken into account in the dynamics of the whole system. Therefore, from the view of dynamical system, the exhaust system of multicylinder engines can be represented with the block diagram shown in Fig. 4 where the
notations are defined in Section 3. In the following, we will present an approach to describe this system with linear periodic time-varying model.

3. Modeling

According to the physics introduced in Section 2, a dynamic model will be constructed that describes the behavior from the fuel injection command to the fuel-air ratio measured at the gas mixing point.

It will be shown that the system can be represented as a periodic time-varying linear system, if the sampling is conducted at BDC of each cylinder, i.e., the sampling period \( T_s = 4\pi/N \) (\( N \) denotes the number of cylinders), and use a unified signal \( u(j) \) (the fuel injection command at the sampling index \( j \)) as the control input which is issued to each cylinder according to the intake phase. We will start with continuous-time domain model, and then discretize the model with the sampled data at BDC, since in-cylinder burnt gas is exhausted outside when the exhaust valve opens, usually near to BDC, and the control algorithms in ECU is also triggered by BDC time.

3.1 Continuous-time Model

Essentially, the system is a hybrid system consisting of continuous time phenomenon such as the exhaust gas flows in runners and manifold, and discrete-time event such as the mass of air charge and injected fuel per cycle of each cylinder. For the latter, the sampling-and-holding signal will be used. For the seek of simplicity, in this subsection continuous-time signal will be used for all variables.

Let \( m_{ai}(t) \) (\( i = 1,2,\ldots,N \)) denote the gas mass flow exhausted from \( No. \ i \) cylinder, and \( m_{af}(t) \) and \( m_{fi}(t) \) represent the masses of fresh air and fuel included in \( m_{ai}(t) \), respectively. Then under the assumption that the components in exhaust gas are the fresh air and the fuel only, we have \( m_{ai}(t) = m_{af}(t) + m_{fi}(t) \). Furthermore, suppose that the behavior of the exhaust gas passing through the runner and reaching the gas mixing point in the manifold, where the oxygen sensor is installed, can be represented as the first-order linear system with time constant \( T_i \), Then, the gas flow observed at the gas mixing point is as follows:

\[
m_{ai}(t) = \int_0^t \frac{1}{T_i} m_{ai}(\tau) e^{-\frac{t-\tau}{T_i}} d\tau \quad \text{................. (1)}
\]

or, equivalently, the air mass flow and the fuel mass flow at the same position are given by

\[
m_{af}(t) = \int_0^t \frac{1}{T_i} m_{af}(\tau) e^{-\frac{t-\tau}{T_i}} d\tau \quad \text{................. (2)}
\]

\[
m_{fi}(t) = \int_0^t \frac{1}{T_i} m_{fi}(\tau) e^{-\frac{t-\tau}{T_i}} d\tau \quad \text{................. (3)}
\]

Therefore, the total mass flow of the fresh air and the fuel at the mixing point, \( m_{af}(t) \) and \( m_{fi}(t) \), can be calculated as the following equations, respectively,

\[
m_{af}(t) = \sum_{i=1}^N m_{af}(t) = \sum_{i=1}^N \int_0^t \frac{1}{T_i} m_{af}(\tau) e^{-\frac{t-\tau}{T_i}} d\tau \quad \text{................. (4)}
\]

\[
m_{fi}(t) = \sum_{i=1}^N m_{fi}(t) = \sum_{i=1}^N \int_0^t \frac{1}{T_i} m_{fi}(\tau) e^{-\frac{t-\tau}{T_i}} d\tau \quad \text{................. (5)}
\]

This means that the fuel-air ratio measured at the gas mixing point is determined by

\[
\eta(t) = \frac{m_{fi}(t)}{m_{af}(t)} \quad \text{................. (6)}
\]

Furthermore, let \( \eta_i(t) \) be the fuel-air ratio of the gas flow from \( No. \ i \) cylinder, i.e,

\[
\eta_i(t) = \frac{m_{fi}(t)}{m_{af}(t)} \quad \text{................. (7)}
\]

Combining equations (4)–(7), the mixed fuel-air ratio is expressed as

\[
\eta(t) = \sum_{i=1}^N \gamma_i(t) \eta_i(t) \quad \text{................. (8)}
\]

where

\[
\gamma_i(t) = \frac{m_{af}(t)}{\sum_{i=1}^N m_{af}(t)}
\]

Meanwhile, if we focus on the in-cylinder fuel-air ratio which is determined by the mass of air charge and injected fuel per cycle, the fuel-air ratio of each cylinder can be represented as follows. Note that the mass of air charge and the fuel per cycle, the fuel-air ratio of each cylinder can be calculated as

\[
\gamma_i(t) = \frac{m_{fi}(t)}{m_{af}(t)} \quad \text{................. (9)}
\]

Instituting this in-cylinder fuel-air ratio into the gas mixing model (8) with rearranged coefficients

\[
r_i(t) = \frac{\gamma_i(t)}{m_{af}(t)} \quad \text{................. (10)}
\]

Fig. 5. Sampling-and-holding description of injected fuel mass
we have
\[ \eta(t) = \sum_{i=1}^{N} r_i(t) m_{fi}^{hold}(t) \quad \cdots \cdots \cdots \cdots \quad (11) \]

Note that the fuel injection command of No. \( i \) cylinder is issued at the beginning of the corresponding intake phase. For the sake of simplicity, we use the sampling-and-holding signal \( m_{fi}^{hold} \) [mm/cycle] which is synchronized with \( m_{fi} \) to represent the fuel injection command. Furthermore, the actual injected fuel, in the case of direct injection, is given by
\[ m_{fi}^{hold}(t) = m_{fi}(t) + d_i \quad \cdots \cdots \cdots \cdots \quad (12) \]
where the unknown offset \( d_i \) is to introduce cylinder-to-cylinder imbalance caused by the perturbation in gains and external disturbance, etc. Therefore, the model (11) is rewritten as
\[ \eta(t) = \sum_{i=1}^{N} r_i(t) (m_{fi}^{hold}(t) + d_i) \quad \cdots \cdots \cdots \cdots \quad (13) \]

Let the dynamics of the fuel-air ratio sensor be represented as the first-order linear system, i.e., the dynamics of sensed fuel-air-ratio \( \eta(t) \) is written as
\[ \dot{\eta}_s(t) = \frac{1}{\tau} [-\eta_s(t) + \eta(t)] \quad \cdots \cdots \cdots \cdots \quad (14) \]
where \( \tau \) is the time constant and \( \eta_s(t) \) is the sensor output.

As is well-known, the transient behavior of air-fuel sensor causes delay in measurement, the shape of exhaust manifold, and the location of the sensor result in waste time to obtain the air-fuel-ratio value. The model proposed deduced above taken these effects into account using the differential equations (2), (3), and (14) to represent the transient behavior.

### 3.2 Discrete-time Model

Let \( k \) and \( j \) denote the index of cycle and BDC, and initialize the BDC of No. 1 cylinder in the first cycle (\( k = 0 \)) as \( j = 0 \). Then, at No. \( i \) cylinder’s BDC in \( k - th \) cycle, the sampled-value of a continuous-time signal \( x(t) \) is
\[ x(j) = x(jT_s) = x(kT + (i - 1)T_s) \quad \cdots \cdots \cdots \cdots \quad (15) \]
where \( T_s \) is the interval between two adjacent BDCs, i.e., \( T_s = T/N \). In other words, the \( j - th \) sampling BDC corresponds to the BDC of No. \( i \) \((i=1, 2, \ldots, N)\) cylinder in the \( k - th \) cycle. Moreover, if we focus on the sampling-holding signal such as the fuel injection command of No. \( i \) cylinder \( u_{fi}^{hold}(t) \), its sampled-data is,
\[ u_{fi}^{hold}(kN + i - 1) = u_{fi}^{hold}(kN + i) \]
\[ = \cdots = u_{fi}^{hold}((k + 1)N + i - 1) \quad \cdots \cdots \cdots \cdots \quad (16) \]

With this fact in mind, by calculating the sampled-data at each sampling time \( t = jT_s \), the discrete-time model corresponding to the gas mixing model (13) is obtained as follows:

When \( Mod(j, N) = p - 1 \) (at No. 1 cylinder’s BDC),
\[ \eta(j) = r_i(0) [u_{fi}^{hold}(j) + d_i] + \sum_{i=1}^{N} r_i(0) [u_{fi}^{hold}(j - N + i - 1) + d_i] \quad \cdots \cdots \cdots \cdots \quad (17) \]
\[ \cdots \]

When \( Mod(j, N) = N - 1 \) (at No. \( N \) cylinder’s BDC),
\[ \eta(j) = \sum_{i=1}^{N} r_i(0) [u_{fi}^{hold}(j - N + i) + d_i] \]
\[ \cdots \]

where the sampled value \( u_{fi}^{hold}(j) \) is replaced according to
\[ u_{fi}^{hold}(j) = \begin{cases} 
  u_{fi}^{hold}((k - 1)N + i - 1) & j < kN + i - 1 \\
  u_{fi}^{hold}(j) & j = kN + i - 1 \\
  u_{fi}^{hold}(kN + i - 1) & j > kN + i - 1 
\end{cases} \quad \cdots \cdots \cdots \cdots \quad (20) \]

As mentioned in Subsection 3.1, the exhaust gas flow of each cylinder stays in the exhaust runner and manifold not longer than one cycle. Then, if the engine is operated in static model with constant speed, the aspirated air mass per cycle is fixed and the profile of air flow in exhaust gas is periodic with period \( T \), i.e., for No. \( i \) cylinder \((i=1, 2, \ldots, N)\), we have
\[ m_{sai}(t) = m_{sai}(t + T) \]
\[ m_{sai}^{hold}(t) = C_i \quad \cdots \cdots \cdots \cdots \quad (21) \]
where \( C_i \) is a constant. This leads to the coefficients \( r_i(t) \) of the model (13) vary periodically, i.e., \( r_i(t) = r_i(t + T) \). Thus, it is deduced as
\[ r_j = \begin{cases} 
  r_0(0) & Mod(j, N) = 0 \\
  r_1(0) & Mod(j, N) = 1 \\
  \vdots & \vdots \\
  r_{(N-1)}(N-1) & Mod(j, N) = N - 1 
\end{cases} \quad \cdots \cdots \cdots \cdots \quad (22) \]

where \( i = 1, 2, \ldots, N \).

Moreover, introduce a unified control signal \( u_{fi}(j) \) to denote the fuel injection command for all cylinders which will be issued to corresponding cylinder as follows,
\[ u_{fi}^{hold}(j) = u_{fi}(j), \quad \text{where } Mod(j, N) = i - 1 \cdots \cdots \cdots \cdots \quad (23) \]

Then, (17)–(19) become a periodic model with the unified input signal \( u_{fi}(j) \):

When \( Mod(j, N) = 0 \),
\[ \eta(j) = r_0(0) [u_{fi}(j) + d_i] + \sum_{i=2}^{N} r_i(0) [u_{fi}(j - N + i - 1) + d_i] \quad \cdots \cdots \cdots \cdots \quad (24) \]
\[ \cdots \]

When \( Mod(j, N) = p - 1 \),
equation (14) is written as

$$
\eta(j) = \sum_{i=1}^{p} r_i(j-p+1)[u_f(j-p+i) + d_i] + \sum_{i=p+1}^{N} r_i(j-N-p+i) + d_i]
$$

\ldots
\ldots
\ldots (25)

When $Mod(j,N) = N - 1$,

$$
\eta(j) = \sum_{i=1}^{N} r_i(N-1)[u_f(j-N+i) + d_i]\ldots (26)
$$

With sampling time $T_s$, the discrete-time representation of equation (14) is written as

$$
\eta_s(j+1) = g \cdot \eta_s(j) + (1-g) \cdot \eta(j) \ldots \ldots (27)
$$

where $g = 1 - T_s / \tau$.

3.3 Linear Periodic Time-varying Model

In order to get a unified expression for equations (24)–(26), two matrices are introduced:

$$
\Gamma = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 \\
0 & 0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 1 & 0 \\
1 & 0 & \cdots & \cdots & 0
\end{bmatrix}_{N \times N}
$$

$$
A = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}_{N \times N}
$$

Then, the model (24)–(26) can be represented as the unified expression

$$
\eta(j) = \sum_{i=1}^{N} (a^i R_i) a^j M(j) \ldots \ldots (28)
$$

where

$$
M(j) = \begin{bmatrix}
u_f(j) + m(j)d \\
u_f(j-1) + m(j-1)d \\
\vdots \\
u_f(j-N+1) + m(j-N+1)d
\end{bmatrix}
$$

$$
d = \begin{bmatrix}
d_1 \\
d_2 \\
\vdots \\
d_N
\end{bmatrix}, \quad m(j) = a^j, \quad R_i = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}_{1 \times 1}
$$

Define the state variables as

$$
\begin{cases}
x_1(j) = u_f(j-1) + m(j-1)d \\
x_2(j) = u_f(j-2) + m(j-2)d \\
\vdots \\
x_{N-1}(j) = u_f(j-N+1) + m(j-N+1)d \\
x_N(j) = \eta_s(j)
\end{cases}
$$

then

$$
\begin{cases}
x_1(j) = u_f(j) + m(j)d \\
x_2(j) = x_1(j) \\
\vdots \\
x_{N-1}(j) = x_N(j-1) \\
x_N(j) = \eta_s(j+1)
\end{cases}
$$

From (28), we have

$$
\eta(j) = \sum_{i=1}^{N-1} l_i(j)x_i(j) + l_N(j)[u(j) + m(j)d] \ldots \ldots (29)
$$

where $l_i(j)$ ($i = 1, 2, \ldots, N$) represent the periodic transfer from fuel mass to fuel-air ratio.

Combining the state definition, the model (29) and the sensor dynamics (27), the state system with sensor output $\eta_s(j)$ as system output is represented as

$$
\begin{bmatrix}
x_1(j) \\
x_2(j) \\
\vdots \\
x_N(j)
\end{bmatrix}
= A(j) \begin{bmatrix}
x_1(j) \\
x_2(j) \\
\vdots \\
x_N(j)
\end{bmatrix}
+ B(j)[u(k) + m(j)d]
$$

$$
y(j) = C \begin{bmatrix}
x_1(j) \\
x_2(j) \\
\vdots \\
x_N(j)
\end{bmatrix}
$$

where

$$
A(j) = \begin{bmatrix}
0 & 0 & \cdots & 0 \\
1 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix}_{N \times N}
$$

$$
B(j) = \begin{bmatrix}
1 \\
0 \\
\vdots \\
0
\end{bmatrix}
$$

$$
C = (0, \ldots, 0, 1)
$$

Obviously, this is a linear periodic time-varying model, since $l_i(j), \ldots, l_N(j)$ and $m(j)$ are periodic with period $N$.

4. Simulation

An exhaust system with three cylinders is considered in the numerical simulation. For three-cylinder exhaust system, the linear periodic time-varying model (expressed by (30) and (31)) is rewritten as

$$
\begin{bmatrix}
x_1(j+1) \\
x_2(j+1) \\
x_3(j+1)
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
(1-g)l_1(j) & (1-g)l_2(j) & g
\end{bmatrix}
\begin{bmatrix}
x_1(j) \\
x_2(j) \\
x_3(j)
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
(1-g)l_3(j)
\end{bmatrix}
[ua(j) + m(j)d]
$$

\ldots \ldots (32)
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Fig. 6. Data for model identification

Table 1. Identified parameters ($\times 10^{-3}$)

<table>
<thead>
<tr>
<th>Model $(j, 3)$</th>
<th>$r = 0$</th>
<th>$r = 1$</th>
<th>$r = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - g)l_1(j)$</td>
<td>1.4124</td>
<td>2.3698</td>
<td>1.4438</td>
</tr>
<tr>
<td>$(1 - g)l_2(j)$</td>
<td>0.4890</td>
<td>0.2893</td>
<td>0.4068</td>
</tr>
<tr>
<td>$(1 - g)l_3(j)$</td>
<td>8.2323</td>
<td>7.0710</td>
<td>7.6733</td>
</tr>
</tbody>
</table>

$y(j) = (0, 0, 1) \begin{bmatrix} x_1(j) \\ x_2(j) \\ x_3(j) \end{bmatrix}$  \hspace{1cm} (33)

where the model parameters, $(1 - g)l_1(j), (1 - g)l_2(j), (1 - g)l_3(j)$ and $g$, will be identified and the model will be validated by simulation in this section.

According to the physical models discussed in Subsection 3.1, the simulator is constructed on MATLAB/SIMULINK. In the simulator, the injected fuel mass of each cylinder with individual offset (12), the dynamics of mass flows (2) and (3), and the mixing process (4), (5) and (8) are involved. The physical parameters like offsets in fuel injection path, time constants of gas passing through runners and the time constant of the sensor are set as $d_1 = -0.068 [\text{mg}], d_2 = 0.068 [\text{mg}], d_3 = 0.136 [\text{mg}], T_1 = 15 [\text{ms}], T_2 = 10 [\text{ms}] T_3 = 5 [\text{ms}]$ and $\tau = 50 [\text{ms}]$, respectively. Moreover, white noise is added on injected fuel mass per cycle.

Suppose the engine is operated in a static mode with speed 1600 [rpm], and the air charge mass is 99 [mg/cycle], 101 [mg/cycle] and 102 [mg/cycle] for No. 1, No. 2 and No. 3 cylinders, respectively. Then, the BDC-scaled sampling time is $T_s = 25 [\text{ms}]$. Same fuel injection command is issued to three cylinders and the response of the sensor output is gotten as shown in the bottom plot of Fig. 6. Suppose the injector disturbance is known, then the injected fuel mass into each cylinder is shown in top three plots in Fig. 6. With the input-output data of the simulator in Fig. 6, after being sampled at BDCs, parameters are identified based on least square (LS) estimation and their values list in Table 1.

Validation of the identified model is demonstrated in Fig. 7, in which the unified fuel injection command $u(j)$ is issued to each cylinder sequentially with initial assignment $u(0) = u_{f1}(0)$, i.e., $u(0)$ is issued to the No. 1 cylinder. The fuel mass entering three cylinders and their unified fuel mass are shown in Fig. 7 (a) and (b), respectively. Under the actuation of the fuel injection, the responses of the simulator and the model with the identified parameters are shown in Fig. 7[c], and a view of zoom-in is shown in Fig. 8 in detail. Fig. 7[d] provides the model error and it shows less than 0.001.

5. Experimental Validation

The experiment is conducted on the test bench with a V6 gasoline engine (displacement: 3.5L) in which six cylinders are installed on both banks and cylinders on same bank share an exhaust system. If cylinders are numbered from No. 1 to
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Fig. 9. Control structure of test bench

Table 2. Identified parameters ($\times 10^{-3}$)

<table>
<thead>
<tr>
<th>Mod($j$, 3)</th>
<th>r = 0</th>
<th>r = 1</th>
<th>r = 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - g)l_i(j)$</td>
<td>0.0767</td>
<td>0.0695</td>
<td>0.0545</td>
</tr>
<tr>
<td>$g$</td>
<td>0.0428</td>
<td>0.0427</td>
<td>0.0498</td>
</tr>
<tr>
<td>$(1 - g)l_i(j)$</td>
<td>0.1020</td>
<td>0.1534</td>
<td>0.1581</td>
</tr>
</tbody>
</table>

Fig. 10. Experimental validation of the model by comparing sensor output and model output

No. 6 according to spark sequence, then No. 1, No. 3 and No. 5 cylinders are on a bank and other three cylinders are on the other one. In this experiment, the cylinders on one side bank are considered and they are renumbered from No. 1 to No. 3 according to the spark sequence (also exhaust BDC sequence). The dynamometer provides torque on the engine to simulate the load and the friction in practice. Fig. 9 provides the system configuration of the test bench. The experiment is conducted on dSPACE which delivers control signals to and receives real-time sampling data from ECU via CANBUS. In addition, dSPACE can also get analogue signal from sensors, such as oxygen sensors, intake manifold pressure sensor, etc. Dynamometer controller can be operated via its user interface.

This is a three-cylinder exhaust system and its model is represented by (32) and (33). The model parameters like $(1 - g)l_i(j) (i = 1, 2, 3)$ and $g$ will be identified by experiment. After the engine warms up, the opening angle of the throttle keeps at 8.6 degrees, the rotational speed is maintained at 1600 $rpm$ by the dynamometer, and the load torque is around 80 $Nm$. Fuel injection command of three cylinder and the corresponding sensor output are sampled at BDCs for identification. Using the least square method, model parameters are identified and listed in Table 2, in which the parameter $g$ is much bigger than the one in the simulation since the real sensor response is much slower.

The experiment is conducted for model validation by comparing the sensor output $\eta_s(j)$ and the model output $y(j)$ still under speed 1600 $rpm$. Let $e(j)$ denote the model error as $e(j) = y(j) - \eta_s(j)$. For quantitative analysis, mean value of error (abbreviated to MVoE), mean absolute error (MAE), and standard deviation (SD) are defined as follows,

$$MVoE = \frac{\sum_{j=1}^{M} e(j)}{M}$$

$$MAE = \frac{\sum_{j=1}^{M} | e(j) |}{M}$$

$$SD = \sqrt{\frac{\sum_{j=1}^{M} (e(j) - MVoE)^2}{M - 1}}$$

where $M$ is the number of data points.

In the following, two experiments are conducted under
open-loop and closed-loop control of air-fuel ratio, respectively. In experiment I, the fuel injection mass is adjusted in close-loop control, and Fig. 11 provides experimental results. Fig. 12 shows the detail of Fig. 11 from the 280–th to the 296–th BDC, in which the model input represents fuel injection command of an individual cylinder at its corresponding BDC time as presented in modeling process. In experiment II, the fuel injection mass is adjusted in open-loop control, and the comparison of the model output and the sensed fuel-air ratio is shown in Fig. 13. During the period 530–th to 580–th BDC, fuel injection command of No. 3 cylinder is reduced to 21.5 [mm]/[stroke] from 22.5 [mm]/[stroke], and the detail of fuel-air ratio is shown in Fig. 14. Analysis on model error for above experiment lists in Table 3.

6. Conclusion

This paper presented a modeling approach for fuel-air ratio following the physical process from fuel injection to fuel-air measurement. The discrete model is finally represented with a linear periodic time-varying representation. The model input is the unified fuel injection command for all cylinders and the model output is the BDC-sampled fuel-air ratio of exhaust gas in the exhaust manifold. Lastly, the modeling approach is validated by simulation and experiment. Experiments are conducted on a V6 gasoline engine in which three cylinders share an exhaust system. When the engines speed is 1600 rpm and the load is 80 Nm, the model output is compared with the sensed fuel-air ratio and the comparison shows that their mean absolute error is about 2.0 × 10^{-4}. The main contribution of this paper is to provide a modeling approach of the BDC-scaled SISO model for further air-fuel ratio control with smaller scale than usual air-fuel ratio control. However, we need much more experiments under different operation conditions for the model validation, and the case of speed changing is not considered in this paper.

Table 3. Model error (× 10^{-4})

<table>
<thead>
<tr>
<th>Exp.</th>
<th>MAE</th>
<th>MPE</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. I</td>
<td>0.67</td>
<td>1.59</td>
<td>0.07</td>
</tr>
<tr>
<td>Exp. II</td>
<td>1.33</td>
<td>1.76</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Fig. 14. Details of Fig. 13

References


(22) J. Chaunin, P. Moulin, G. Corde, N. Petit, and P. Rouchon: “Kalman filtering for real-time individual cylinder air fuel ratio observer on a diesel engine test
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