Identification Method of Current Conditions Suitable for Synchronous Machines Having Multiple Operating Points

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To perform finite element analysis (FEA) for estimating the characteristics of synchronous machines (SMs), a current condition corresponding to an operating point is necessary as an analysis input. Accurate identification for the current condition is a difficult problem because it is strongly susceptible to magnetic saturation, e.g., the cross-coupling saturation between d - and q -axes. Especially in a wound-field SM, its flexibility makes the identification more complicated; in other words, three independent variables, such as the amplitude and phase of armature current and amplitude of field current, have to be identified for SMs, whereas two variables are required for a permanent magnet SM or synchronous reluctance machine. Thus, numerous researchers have studied construction of the identification method. This paper proposes a novel identification method for the current condition of wound-field SMs using saturation functions defined by the flux maps. It is demonstrated that the proposed method can correctly identify a large number of current conditions in a short computation time. In addition, the influences of magnetic saturation on the characteristics of the SM were investigated by comparing the identification results obtained by the linear and nonlinear FEA. It is revealed that the magnetic saturation drastically affects the current condition and must be considered in the identification methodology.

Keywords: synchronous machine, wound-field, field current estimation, magnetic saturation, cross-coupling between axes

1. Introduction

The wound-field synchronous machine (SM) has long been widely used in MW-class large-capacity motors that drive rolling mills and compressors as well as main generators in various power plants. In this paper, SMs with three-phase armature windings are mainly focused on. The most important feature of SMs is that their field current can be freely changed. They have four electrical degrees of freedom, and for example, the amplitude of armature voltage V_a , output power P , reactive power Q , and mechanical angular velocity ω_m can each be changed independently. Henceforth, the combination of V_a , P , Q , and ω_m are called the operating point. This feature not only provides the desired mechanical output, but also minimizes the power capacity of the drive inverter by operating with a power factor of 1 ($Q = 0$), or adjusts Q according to the requirements of the power supply system.

Meanwhile, the following problems occur due to their high degree of freedom. Magnetic field analysis by finite element method (FEA) is performed to evaluate the detailed

characteristics of SM such as loss (copper loss, iron loss, eddy current loss, etc.), electromagnetic force, torque ripple, etc.^{(1)–(3)}. Generally, FEA requires the four variables as the input that consist of the amplitude I_a and phase β of the three-phase armature current, electrical angular velocity ω_e , and field current I_f ; the operating point can then be obtained from the output of the analysis. In this paper, I_a , β , ω_e , and I_f are called the current conditions. However, in actual analysis work, there are many cases where only the operating point is specified without giving the current condition. In these cases, as shown in Fig. 1, the current conditions need to be identified by solving the inverse problem before performing the FEA. The current conditions are also required in FEA for a permanent-magnet synchronous machine (PMSM) or a synchronous reluctance motor (SynRM). However, the degrees of freedom in them are three each, and it is relatively easy to identify the current conditions in these cases. Therefore, many identification methods^{(4)–(6)} have been proposed, but it is difficult to directly adapt these to SM. Although a method which treats SM and PMSM in an integrated manner⁽⁷⁾ has also been proposed, the identification method for the current conditions has not been investigated. As described above, there is the problem in SM that its high degree of freedom causes difficulty in identifying the current conditions.

Incidentally, in the design of SM, the field current under load operation needs to be estimated, and various formulas have long been developed⁽⁸⁾. This can be regarded as equivalent to identifying I_f , which is one of the current conditions. In the technical report of the Institute of Electrical Engineers

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Fig. 1. Basic process of characteristics calculation using FEA.

of Japan⁽⁹⁾, the coefficient obtained from the no-load saturation curve was incorporated into the estimated formula to consider magnetic saturation in the calculation of I_f . However, the direction of the magnetic flux changes due to the armature reaction under load, so the magnetic flux density distribution is significantly different from that under no load. Particularly in salient-pole SMs, a large magnetic saturation occurs locally at one side of the pole head⁽¹⁰⁾, and it is difficult to obtain an accurate I_f with the estimated formula.

As far as the authors are aware, the method in Ref. (11) most accurately considers the magnetic saturation. This method comprises the repetition of three processes: (a) calculation of the magnetic permeability distribution using non-linear FEA, (b) identification of inductance using FEA with the frozen permeability distribution, and (c) calculation of current conditions through solving d - and q -axis equations. Although the objective of this reference was the estimation of the field current, the method can simultaneously estimate the armature current. Cases where these currents were used as a current condition to be input to FEA have also been reported⁽¹¹⁾⁽¹²⁾. This method is very effective when identifying a small number of current conditions, but in principle, FEAs are performed repeatedly to identify each current condition, so if multiple current conditions are to be identified, the computation time will increase. In recent years, FEA may be required to be performed on a large number of operating points distributed in a two-dimensional plane ($\omega_m - P$ plane), e.g., in the development of PMSMs for electric vehicles⁽¹³⁾. In the case of SMs as well, a new method should be developed that can be applied for the identification of a large number of current conditions.

The objective of this study is to propose a new method for identifying current conditions in SMs. In particular, the following two points will be considered:

- The method can consider complex magnetic saturation characteristics such as the cross-coupling between axes.
- The method can calculate multiple current conditions easily.

First, in Chapter 2, the reasons for examining the identification method that does not involve inductance, as well as its applicability and expandability, are described. In Chapter 3, a suitable calculation method for the equations that must be solved to identify the current conditions is proposed. In Chapter 4, a calculation example for the salient-pole SM is shown, and the accuracy of the identified current conditions is verified (i.e., whether the calculation result of the FEA where the identified current conditions were input agrees with the

Fig. 2. Typical equivalent circuits of the SM.

specified operating point). Finally, in Chapter 5, the importance of considering magnetic saturation in identifying current conditions is discussed.

2. Introduction of Saturation Function in the Identification of Current Conditions

2.1 Problems of the Identification Method Using Inductance

In the discussion below, a general SM with a three-phase armature winding on the stator and a field winding on the rotor is considered. The direction of the magnetomotive force of the field winding is defined as the d -axis, and the direction that is electrically orthogonal to it is defined as the q -axis. The equivalent circuit after dq conversion is as shown in Fig. 2. R_a is the armature resistance; L_{ad} and L_{aq} are the armature reaction inductances of the d - and q -axes; L_{ld} and L_{lq} are the armature leakage inductances of the d - and q -axes; and R_{fd} and L_{fd} are the field resistance and leakage inductance converted to the armature side. Additionally, v_d and v_q are the armature voltages of the d - and q -axes; i_d and i_q are the armature currents of the d - and q -axes; Φ_d and Φ_q are the magnetic fluxes linking the armature windings in the d - and q -axes; and v_{fd} and i_{fd} are the field voltage and current converted to the armature side. In this paper, the effects of the iron loss or stray loss are ignored, with only copper loss considered. The steady state (i.e., operation at synchronous speed) is also assumed, and the damper current is ignored.

In the steady state, the mechanical angular velocity ω_m and the electrical angular velocity ω_e satisfy the following relationship:

$$\omega_e = \frac{p}{2} \omega_m \quad (1)$$

where p is the pole number. This means that ω_m and ω_e have a one-to-one relationship and can be easily converted. Therefore, the substantial unknown variables in the identification of current conditions are I_a , β , and I_f .

From the equivalent circuit in Fig. 2, I_a , β , and I_f corresponding to the specified operating point V_a , P , Q , and ω_m can be calculated as follows. First, V_a , P , and Q are given by the current, magnetic flux linkage and resistance of the circuit through Eq. (2).

$$\begin{cases} V_a = \sqrt{\left(R_a i_d - \frac{p}{2} \omega_m \Phi_q\right)^2 + \left(R_a i_q + \frac{p}{2} \omega_m \Phi_d\right)^2} \\ P = \frac{p}{2} \omega_m (\Phi_d i_q - \Phi_q i_d) \\ Q = -\frac{p}{2} \omega_m (\Phi_d i_d + \Phi_q i_q) \end{cases} \dots\dots\dots (2)$$

The derivation process is shown in Appendix 1. The relationship between the current and the magnetic flux linkage is expressed as follows.

$$\begin{cases} \Phi_d = L_{ld} i_d + L_{ad} (i_d + i_{fd}) \\ \Phi_q = L_{lq} i_q + L_{aq} i_q \end{cases} \dots\dots\dots (3)$$

i_d , i_q , and i_{fd} can be converted to I_a , β , and I_f by the following equations:

$$\begin{cases} I_a = \sqrt{(i_d^2 + i_q^2)} / 3 \\ \beta = \tan^{-1}(i_q / i_d) \\ I_f = \sqrt{2/3} i_{fd} / k \end{cases} \dots\dots\dots (4)$$

Here, k is the turn ratio of the field windings reference to the armature winding. In this paper, k was calculated from the number of turns N_a of the armature winding and the number of turns N_f of the field winding using the following equation:

$$k = \frac{2pC_a N_f}{N_s N_a} \dots\dots\dots (5)$$

where C_a is the number of parallel circuits of the armature winding, and N_s is the number of slots of the stator. The current conditions can be calculated if Eq. (3) is substituted into Eq. (2) to solve the simultaneous equations, and the obtained i_d , i_q , and i_{fd} are converted into I_a , β , and I_f by Eq. (4). For example, the specific calculation of I_f is as follows:

$$I_f = \sqrt{\frac{2}{3}} \frac{2V_a}{p\omega_m k L_{ad}} \frac{1 + \frac{L_d}{L_q} \left(\frac{p\omega_m P L_q}{2V_a^2}\right)^2}{\sqrt{1 + \left(\frac{p\omega_m P L_q}{2V_a^2}\right)^2}} \dots\dots\dots (6)$$

Here, the power factor is set as 1 ($Q = 0$), R_a was ignored, and $L_d = L_{ld} + L_{ad}$, $L_q = L_{lq} + L_{aq}$.

It is assumed in the above scenario that all of the equivalent circuit constants are known. The resistance can be regarded as a constant value, but the inductance changes greatly depending on the operating point because of the effect by the magnetic saturation of the iron core. Expressing this characteristic correctly requires the four inductances (L_{ad} , L_{aq} , L_{ld} , and L_{lq}) to be treated as a three-variable function for the current i_d , i_q , and i_{fd} . However, it is not easy to accurately determine these functions due to the essential problem of the arbitrariness in the turn ratio and the mutual leakage inductance⁽¹²⁾⁽¹⁴⁾. Furthermore, cases of variable inductance depending on the current cannot be solved explicitly as Eq. (6), and complicated iterative calculations are required. Additionally, magnetic saturation causing the cross-coupling between the d - and q -axes and preventing correct expression with the equivalent circuit in Fig. 2 has also been indicated as a problem⁽¹¹⁾.

2.2 Advantages of the Identification Method Using Saturation Function

The above-mentioned problems are caused by the use of Eq. (3), which involves inductance, in the relationship between the current and the magnetic flux linkage. Therefore, this paper proposes the introduction of the saturation functions ϕ_d and ϕ_q with the variables of the armature current and field current⁽¹⁵⁾. Cases where the saturation function (also named as the magnetic flux map) was adopted in PMSM and SynRM⁽¹³⁾⁽¹⁶⁾ have been reported, but there are no examples where this was applied to SMs. Eq. (3) can be rewritten as follows when ϕ_d and ϕ_q are introduced:

$$\begin{cases} \Phi_d = \phi_d(i_d^\dagger, i_q, i_{fd}) \\ \Phi_q = \phi_q(i_d^\dagger, i_q, i_{fd}) \end{cases} \dots\dots\dots (7)$$

Here, the d -axis current sum $i_d + i_{fd}$ is defined as the combined d -axis excitation current $i_d^\dagger$ ⁽¹⁷⁾. The reason why $i_d^\dagger$ was introduced will be explained in Chapter 3. Eq. (7) is substituted into Eq. (2) to obtain the following simultaneous equations:

$$\begin{cases} f = \left\{ R_a (i_d^\dagger - i_{fd}) - \frac{p}{2} \omega_m \phi_q \right\}^2 + \left\{ R_a i_q + \frac{p}{2} \omega_m \phi_d \right\}^2 - V_a^2 = 0 \\ g = \frac{p}{2} \omega_m \{ \phi_d i_q - \phi_q (i_d^\dagger - i_{fd}) \} - P = 0 \\ h = -\frac{p}{2} \omega_m \{ \phi_d (i_d^\dagger - i_{fd}) + \phi_q i_q \} - Q = 0 \end{cases} \dots\dots\dots (8)$$

Eq. (8) means that the problem of identifying the current conditions corresponding to the specified operating point can be reduced to the problem of solving the nonlinear system of equations with three unknowns $f(i_d^\dagger, i_q, i_{fd}) = g(i_d^\dagger, i_q, i_{fd}) = h(i_d^\dagger, i_q, i_{fd}) = 0$.

The flowchart of the proposed method is shown in Fig. 3(a). Figure 3(b) is the flowchart of the conventional method in Ref. (11). The proposed method has the following features.

- (a) The saturation function can be obtained by just applying a dq -conversion to the magnetic flux linkage of each

Fig. 3. Flowcharts of identification method for current condition. Proposed method in this paper can save computation time even if numerous current conditions are required.

winding obtained by FEA. All magnetic saturation phenomena (e.g., cross-coupling saturation between axes) are taken into consideration, and there is also no need for any complicated calculations for determining the inductance.

- (b) FEA is performed only in the determination of the saturation function. The time required for identifying the current conditions (computation time for solving simultaneous equations (8)) is negligibly shorter than that for the FEA, and the computation time does not depend on the number of current conditions to be identified.
- (c) Each analysis point for determining the saturation function is independent of each other. Therefore, parallel processing can be easily introduced into the FEA, and the computation time is greatly reduced.
- (d) As shown in Appendix 2, the saturation function can be reused if the cross-section of the iron core is the same, even if the number of turns of each winding or the core length are changed, i.e., FEA needs not to be performed in these cases. This enables the method to be applied for the characteristic evaluation of SMs with the different designs as well.
- (e) As shown in Appendix 3, changing the f , g , and h in Eq. (8) can be expanded to identify current conditions that correspond to various states and characteristics, such as the maximum output power (pull-out torque) under voltage limits.

The method using the saturation function is suitable for the objectives of this research because of the features of (a) and (b). Although the computation time for determining the saturation function is predominant, the features of (c) can easily shorten this time. Additionally, as described in (d) and (c), the method has high applicability and expandability, thereby it is expected to be used not only for the identification of current conditions but also for various situations in the evaluation of SM characteristics.

3. Proposal of Calculation Method Suitable for Solving Simultaneous Equations Including Saturation Function

3.1 Introduction of Combined d -axis Excitation Current to Reduce Computation Time Figure 4(a) shows the trajectory of the current conditions when P is gradually increased under the conditions of $V_a = \text{constant}$, $Q = 0$ var (power factor of 1), and $\omega_m = \text{constant}$ (three conditions; 42, 84, 126 rad/s) for the 2 kW SM studied in this paper. The specific values for $\omega_m = 42$ rad/s are $i_d = 0$ A, $i_{fd} = 14$ A at $P = 0$ kW, and $i_d = -31$ A, $i_{fd} = 48$ A at $P = 5$ kW. In general, $i_{fd} (> 0)$ is increased and $i_d (< 0)$ is decreased with increases in P because the weakening field control is performed so that V_a keeps constant according to the specifications of the inverter or power system. Therefore, in the above-mentioned specific example, at the very least, the saturation function must be defined in the domain of $-31 \leq i_d \leq 0$ (A) and $14 \leq i_{fd} \leq 48$ (A), and the domain is further expanded as the P increases.

If the combined d -axis excitation current $i_d^\dagger = i_d + i_{fd}$ is introduced, $i_d^\dagger = 0 + 14 = 14$ A at $P = 0$ kW, and $i_d^\dagger = -31 + 48 = 17$ A also at $P = 5$ kW. This means that the domain of the saturation function becomes smaller in the

Fig. 4. Trajectories of current condition which represents load conditions of various rotating speeds.

Fig. 5. Analysis points on $i_d^\dagger - i_q - i_{fd}$ coordinate for calculations of saturation functions.

$i_d^\dagger$ direction as shown in Fig. 4(b), and the computation time required to calculate the saturation function can be reduced. For these reasons, i_d was replaced with $i_d^\dagger$.

3.2 Introduction of Linear Interpolation to Produce Continuous Saturation Function First, I_{dm} , I_{qm} , and I_{fdm} , which are the maximum values of $i_d^\dagger$, i_q , and i_{fd} , respectively, are determined for the domain of saturation function, and $(2M_d + 1) \times (2M_q + 1) \times (M_{fd} + 1)$ analysis points are set in the rectangular parallelepiped region as shown in Fig. 5.

$$\begin{cases} i_d^\dagger = \left(\frac{m_d}{M_d}\right) I_{dm} & (-M_d \leq m_d \leq M_d) \\ i_q = \left(\frac{m_q}{M_q}\right) I_{qm} & (-M_q \leq m_q \leq M_q) \dots\dots\dots (9) \\ i_{fd} = \left(\frac{m_{fd}}{M_{fd}}\right) I_{fdm} & (0 \leq m_{fd} \leq M_{fd}) \end{cases}$$

$m_d, m_q, m_{fd}, M_d, M_q,$ and M_{fd} are the integers that define the analysis points. FEAs are conducted at each point with the input of $I_a, \beta,$ and $I_f,$ which are obtained by converting Eq. (9) through Eq. (4). If the dq -conversion is applied to the magnetic flux linkage of the armature winding obtained from the FEAs, then discretized saturation functions $\phi_{d,m_d,m_q,m_{fd}}$ and $\phi_{q,m_d,m_q,m_{fd}}$ defined on the analysis points are obtained. Due to the influence of the slot ripple and the winding distribution, the magnetic flux pulsates according to the difference in relative positions of the stator and rotor even under the same current conditions. In the case of a three-phase armature winding, the pulsation has a periodicity with an electrical angle of $\pi/3$. Therefore, FEAs in which the rotor position θ_m was changed as follows were performed, and the average value was used as the saturation function.

$$\theta_m = \left(\frac{m_\theta}{M_\theta} \right) \frac{2\pi}{3p} \quad (0 \leq m_\theta < M_\theta) \dots \dots \dots (10)$$

where m_θ and M_θ are integers. In other words, FEA is conducted (total number of analysis points) $\times M_\theta$ times to determine the saturation function. All analyses are independent of each other; thus, parallel processing can be conducted in an arbitrary manner.

Since the saturation function must be continuous to solve Eq. (8), the following local coordinates $x, y,$ and z were introduced inside the rectangular parallelepiped surrounded by eight adjacent analysis points, and the inside was linearly interpolated.

$$\begin{cases} x = i_d^\dagger M_d / I_{dm} - m_d \\ y = i_q^\dagger M_q / I_{qm} - m_q \\ z = i_{fd}^\dagger M_{fd} / I_{fdm} - m_{fd} \end{cases} \dots \dots \dots (11)$$

$$\phi_d = \frac{1}{8} \begin{pmatrix} (1-x)(1-y)(1-z) \\ (1+x)(1-y)(1-z) \\ (1-x)(1+y)(1-z) \\ (1-x)(1-y)(1+z) \\ (1+x)(1+y)(1-z) \\ (1+x)(1-y)(1+z) \\ (1-x)(1+y)(1+z) \\ (1+x)(1+y)(1+z) \end{pmatrix} \begin{pmatrix} \phi_{d,m_d,m_q,m_{fd}} \\ \phi_{d,m_d+1,m_q,m_{fd}} \\ \phi_{d,m_d,m_q+1,m_{fd}} \\ \phi_{d,m_d,m_q,m_{fd}+1} \\ \phi_{d,m_d+1,m_q+1,m_{fd}} \\ \phi_{d,m_d+1,m_q,m_{fd}+1} \\ \phi_{d,m_d,m_q+1,m_{fd}+1} \\ \phi_{d,m_d+1,m_q+1,m_{fd}+1} \end{pmatrix} \dots \dots \dots (12)$$

The same applies for ϕ_q as well. From the above, a

Fig. 6. Different relative positions between stator and rotor.

continuous saturation function can easily be determined by the results of FEA.

3.3 Introduction of Newton-Raphson Method for Nonlinear Equation Nonlinearity of the saturation function requires iterative computations to solve Eq. (8). In this paper, the Newton-Raphson method, which is a typical method for solving nonlinear problems was applied⁽¹³⁾⁽¹⁵⁾. The Jacobian J is given by the partial derivatives of $f, g,$ and h as follows:

$$J = \begin{pmatrix} \partial f / \partial i_d^\dagger & \partial f / \partial i_q & \partial f / \partial i_{fd} \\ \partial g / \partial i_d^\dagger & \partial g / \partial i_q & \partial g / \partial i_{fd} \\ \partial h / \partial i_d^\dagger & \partial h / \partial i_q & \partial h / \partial i_{fd} \end{pmatrix} \dots \dots \dots (13)$$

The partial derivative of the saturation function, which is involved in $f, g,$ and $h,$ can be obtained by partially differentiating Eqs. (11) and (12). The solution vector at the $n + 1^{\text{th}}$ iterative computation $\mathbf{i}^{(n+1)}$ can be calculated using the n^{th} solution $\mathbf{i}^{(n)}$ as follows:

$$\mathbf{i}^{(n+1)} = \mathbf{i}^{(n)} - J^{-1}(f(\mathbf{i}^{(n)}) \quad g(\mathbf{i}^{(n)}) \quad h(\mathbf{i}^{(n)}))^t \dots \dots \dots (14)$$

The solutions $i_d^\dagger, i_q,$ and i_{fd} are obtained by repeating the calculation of Eq. (14). These are substituted into Eq. (4) in order to obtain the current conditions.

4. Discussion on the Identification Accuracy and Computation Time of Current Conditions

4.1 Specifications of the SM to be Examined

A six-pole SM with a rating of 145 V-9A-2 kW, base speed of 400 min^{-1} , and top speed of 1200 min^{-1} was examined. As shown in Fig. 7, a salient-pole rotor is employed in the SM. The armature winding is a three-phase distributed winding and the field winding is a concentrated winding, with $k = 1.8$. The FEA model was created using the JMAG-Designer ver. 18.1 from JSOL Co. Ltd., and the magnetic field analysis was conducted.

A rated current of 9 A in the target machine equivalents to 15.6 A on the $d-q$ plane, so $I_{dm} = I_{qm} = 46.8 \text{ A}$ was set with considering a 300% margin against the rated value. The same applied to I_{fdm} , and a maximum value of 59.5 A was obtained from a field current of 300% to the rated load.

Figure 8 shows the analysis results of the no-load saturation curve. The saturation coefficient σ at 1.2 times the rated voltage is 1.8 for the base speed, and this means that it is classified as a remarkably saturated condition⁽¹⁸⁾. In contrast to the base speed, the SM can be regarded as operated under the unsaturated condition at the top speed. Studying this SM

Fig. 7. FEA model of the target SM.

Fig. 8. No-load saturation characteristics of the target SM.

Fig. 9. Pulsation of the magnetic flux depending on the relative position.

enables the evaluation of a wide range of saturation states.

Figure 9 shows the changes in Φ_d relative to θ_m under no-load conditions for the rated voltage and base speed. There is a maximum pulsation of 0.55% for an average value of 1.1602 Wb in one cycle, and this is considered to be a guideline for the error when the saturation function is calculated with $M_\theta = 1$. The average value of the five points plotted in Fig. 9 is 1.1601 Wb, and the error for $M_\theta = 5$ was sufficiently small at 0.008%. The magnitude of the error caused by the pulsation of the magnetic flux will change depending on the operating points. However, as described later, the error caused by the number of divisions of the current (M_d, M_q, M_{fd}) shows a sufficiently higher order of magnitude. For these reasons, M_θ was fixed at 5 in this paper. In addition, the condition of $M_d = M_q = M_{fd}$ is imposed, and the computational accuracy of the current conditions was investigated when these were changed to 3, 5, 10, and 20.

4.2 Influence of the Division Number of Analysis Points on Identification Accuracy and Computation Time

The current conditions were identified by the method with the saturation function when P was changed from 0 kW (no-load) to 5 kW (2.5 pu) at the rated voltage ($V_a = 145.5$ V), power factor 1 ($Q = 0$ var), and base speed ($\omega_m = 42$ rad/s). Figures 10(a), (b), and (c) show I_a , β , and I_f , respectively. I_a is the same value regardless of the number of divisions. As explained in Appendix 4, this is because I_a is uniquely determined independent to the saturation function. Meanwhile, β and I_f , which depend on the saturation function, are affected

(a) Characteristics of I_a

(b) Characteristics of β

(c) Characteristics of I_f

Fig. 10. Identified current conditions of the target SM.

Fig. 11. Difference in the saturation function due to division number.

by the number of divisions.

Figures 11(a) and (b) show the ϕ_d for the different division numbers of the analysis points, which are 3 and 20 divisions, respectively. Although the saturation function is highly nonlinear and the curve is not smoothly expressed with three

Table 1. Identification error of current conditions for the rated voltage, zero reactive power, and base speed with various division numbers.

(a) Calculated value and error of V_a								
P (pu)	$M_d M_q M_{fd}=3$		$M_d M_q M_{fd}=5$		$M_d M_q M_{fd}=10$		$M_d M_q M_{fd}=20$	
	Calc. (pu)	Error (%)	Calc. (pu)	Error (%)	Calc. (pu)	Error (%)	Calc. (pu)	Error (%)
N.L.	1.021	2.05	1.025	2.51	1.001	0.09	1.001	0.10
0.5	1.038	3.83	1.053	5.26	1.006	0.58	1.001	0.11
1	1.016	1.63	1.046	4.56	1.009	0.92	1.003	0.34
1.5	1.036	3.59	1.028	2.85	1.012	1.19	1.003	0.32
2	1.045	4.50	1.019	1.93	1.010	0.99	1.003	0.33
2.5	1.045	4.52	1.013	1.28	1.008	0.82	1.005	0.53

(b) Calculated value and error of P								
P (pu)	$M_d M_q M_{fd}=3$		$M_d M_q M_{fd}=5$		$M_d M_q M_{fd}=10$		$M_d M_q M_{fd}=20$	
	Calc. (pu)	Error (%)	Calc. (pu)	Error (%)	Calc. (pu)	Error (%)	Calc. (pu)	Error (%)
0.5	0.520	3.93	0.527	5.41	0.503	0.60	0.501	0.11
1	1.017	1.74	1.048	4.84	1.010	0.99	1.004	0.37
1.5	1.559	3.96	1.547	3.15	1.520	1.32	1.505	0.36
2	2.102	5.09	2.045	2.24	2.023	1.15	2.008	0.38
2.5	2.632	5.30	2.539	1.57	2.525	1.00	2.516	0.65

(c) Error of power factor angle				
P (pu)	$M_d M_q M_{fd}=3$	$M_d M_q M_{fd}=5$	$M_d M_q M_{fd}=10$	$M_d M_q M_{fd}=20$
Error (deg.)	Error (deg.)	Error (deg.)	Error (deg.)	Error (deg.)
0.5	1.132	1.095	0.150	-0.016
1	0.519	1.480	0.261	0.017
1.5	1.586	0.990	0.423	-0.004
2	2.843	0.652	0.317	-0.051
2.5	3.631	0.010	0.108	0.017

divisions, increasing the number of divisions to 20 can generally capture the curve. It is obvious that the error of the linear interpolation caused by insufficient division number affects the identification results of the current condition.

Detailed FEA, which simulates for half an electric angle cycle, was conducted with the input of the current conditions in Fig. 10 to verify the identification accuracy. Specifically, I_a , β , and ω_e shown in Fig. 10 were converted into three-phase alternating current and given to the armature winding of the FEA model as an input condition. I_f was also given to the field winding, and the rotation speed was set as $2\omega_e/p$.

Table 1(a) shows the V_a calculated from the fundamental component of the armature voltage waveform obtained from the detailed FEA. The per-unit value based on the rated voltage of 145.5 V and the rated output of 2 kW was used in the table. When the division number of the analysis points was set as $M_d = M_q = M_{fd} = 3$, the V_a obtained from the detailed FEA had a maximum error of approximately 5% against the specified operating point of $V_a = 1$ pu. The error decreased with increases in the division number, with the error dropping to less than 1% at $M_d = M_q = M_{fd} = 20$. Similarly for P , the error was less than 1% at $M_d = M_q = M_{fd} = 20$, as shown in Table 1(b). The error of Q was evaluated equivalently as the difference in the power factor angle. The error was confirmed to be less than 0.1 deg and sufficiently reduced at $M_d = M_q = M_{fd} = 20$, as shown in Table 1(c). The accuracy needed in the current conditions depends on the objective of the detailed FEA, but setting each division number to around 20 is reasonable in this case if less than 1% is assumed as a guide for the error of V or P as an example.

Table 2. Identification error of current conditions for the operating points with the top speed or leading power factor.

(a) Rated voltage, no reactive power, and top speed					
P (pu)	V_a		P (pu)	Power Factor angle	
	Calc. (pu)	Error (%)		Error (deg.)	
N.L.	1.005	0.47	0.000	-	-
0.5	1.013	1.33	0.507	1.37	0.052
1	1.008	0.75	1.008	0.80	0.171
1.5	1.009	0.88	1.515	0.98	0.200
2	1.007	0.68	2.016	0.79	0.177
2.5	1.007	0.73	2.522	0.90	-0.122

(b) Rated voltage, leading power factor 0.9, and base speed					
P (pu)	V_a		P (pu)	Power Factor angle	
	Calc. (pu)	Error (%)		Error (deg.)	
N.L.	1.001	0.10	0.000	-	-
0.5	1.003	0.35	0.502	0.32	0.039
1	1.005	0.48	1.005	0.46	0.073
1.5	1.006	0.56	1.508	0.53	0.108
2	1.004	0.36	2.010	0.49	-0.046
2.5	i_{fd} was exceeding the domain of saturation functions I_{fdm}				

As another condition, the accuracy of the current conditions under $M_d = M_q = M_{fd} = 20$ for the operating point with rated voltage and power factor of 1 at the top speed ($\omega_m = 126$ rad/s) is shown in Table 2(a). The error is the same level as the base speed. In addition, the accuracy of the current conditions for a leading power factor of 0.9 at the base speed is shown in Table 2(b). A similar degree of accuracy as the case for power factor of 1 was obtained even when driving at the leading power factor. It was demonstrated from the above results that sufficient accuracy can be provided in a wide operating range regardless of the saturation state and power factor.

The computation time required for the FEA to determine the saturation function was 5, 10, 40, and 228 min., in ascending order of division number. The CPU of the computer was an Intel® Xeon® Gold 6154 @ 3 GHz, and the FEA was executed in four parallel. On the other hand, the computation time required for the detailed FEA was approximately 0.5 min. per condition. In Ref. (13), an optimal algorithm that was specialized for the identification of β was proposed, and this requires about three times of iterations to reach a solution. Three variables need to be identified in SMs and thus the total number of iterations can be assumed to be $3^3 = 27$, then the computation time of each condition is estimated to be 13.5 min. Performing four parallel computations, the time required to identify 100 points of current conditions is 338 min. In other words, at the situation as identifying approximately 100 points of current conditions, the method with the saturation function will be more advantageous than the conventional method where the detail FEA is executed repeatedly in terms of computation time.

4.3 Error of Linear Interpolation and Determination Method for Appropriate Division Number The error was evaluated for the SM in Fig. 7 in Section 4.2. However, the shape of the saturation function changes if the SM design is different, so the number of divisions required to achieve an error of less than 1% may also change. Here, let us consider a simple method for finding the appropriate number of divi-

 Fig. 12. Derivatives of d -axis saturation function.

sions.

When the function $y = s(x)$ is linearly interpolated in the interval $[a, b]$, the maximum error value E_m is given by the following⁽¹⁹⁾.

$$E_m = \frac{(b-a)^2}{8} \max_{a \leq x \leq b} \left| \frac{d^2 s}{dx^2}(x) \right| \dots \dots \dots (15)$$

Applying this to the saturation function, $b - a$ corresponds to the interval between adjacent analysis points, and $d^2 s/dx^2$ corresponds to its second derivative.

Figure 12(a) is the partial derivative of ϕ_d in Fig. 11(b) with $i_d^\dagger$. This represents the differential inductance, which decreases as the current increases due to the magnetic saturation. Figure 12(b) shows further differentiated values with $i_d^\dagger$. There are a maximum and minimum points near the origin, where the change rate in differential inductance becomes the largest. According to Eq. (15), the error of linear interpolation is the largest at these two points. When the current becomes sufficiently large, the iron core is completely saturated and the change in the differential inductance becomes gradual, which reduces the error.

Assuming that the rated voltage is V_r and the rated frequency is f_r , the index e (%) expressing the error can be calculated as follows.

$$e = 100 \frac{2\pi f_r E_m}{V_r} \dots \dots \dots (16)$$

In the SM targeted in this paper, the maximum value of $|\partial^2 \phi_d / \partial i_d^{\dagger 2}|$ is 0.0097 H/A and the division interval of $i_d^\dagger$ is 2.3 A. From these values, e can be derived 0.56% and 1.67% for the base and top speed, respectively. Strictly speaking, the partial derivatives in the i_q and i_{fd} directions, as well as that of ϕ_q , need to be considered. However, the d -axis is more likely to be affected by the magnetic saturation than the q -axis in a salient-pole type, i.e., $\partial^2 \phi_d / \partial i_d^{\dagger 2}$ will be dominant in terms of effects on the error. Actually, as shown in Table 1, the error of V_a , which is proportional to the saturation function, remains lower level than e at either speed and this is a reasonable result. In the case of a cylindrical SM, ϕ_q has nearly the same characteristics as ϕ_d , and thus, appropriate division number can be determined by considering only $\partial^2 \phi_d / \partial i_d^{\dagger 2}$ as well as the case of the salient-pole SM.

Incidentally, the no-load saturation curve is the relationship between the field current i_{fd} and the armature voltage $v_q (= \omega \Phi_d)$, and is equivalent to the saturation function ϕ_d for $i_d^\dagger = i_{fd}$ and $i_q = 0$. Furthermore, as seen in Fig. 12, a maximum or minimum point of $\partial^2 \phi_d / \partial i_d^{\dagger 2}$ places on the $i_d^\dagger$ axis

Fig. 13. Derivatives calculated from no-load saturation characteristics.

($i_q = 0$). These allow that the maximum value of $|\partial^2 \phi_d / \partial i_d^{\dagger 2}|$ can also be calculated by differentiating the no-load saturation curve with i_{fd} . As shown in Fig. 13, the minimum value obtained from the no-load saturation curve is -0.0093 H/A, which is almost the same as the value calculated from the saturation function. There are small differences depending on whether the magnetomotive force source is i_d or i_{fd} , but this is acceptable when determining the guideline for the number of divisions. In this way, the appropriate number of divisions can be determined even from the no-load saturation curve that can be obtained relatively easily.

5. Discussion of the Magnetic Saturation Effect on the Identification Results

5.1 Effect on Saturation Function To discuss the effect of magnetic saturation, a linear FEA where the relative permeability of the stator core and rotor core was fixed at 5000 was conducted and its result was compared with the nonlinear FEA. Figures 14(a) and (b) are ϕ_d for $i_{fd} = 0$ A given by the nonlinear and linear FEA, respectively. Note that a different scale is used in the ϕ_d -axis direction in order to make the changes easier to understand. The nonlinear FEA produces a curved surface along the $i_d^\dagger$ -axis as Fig. 14(a), but the linear FEA produces a plane with a constant inclination as Fig. 14(b). The curve along the $i_d^\dagger$ -axis exhibits direct saturation characteristics where the magnetic path in the d -axis is saturated by the d -axis magnetomotive force. Meanwhile, a curved surface can also be seen along the i_q -axis. A q -axis magnetic flux is generated by i_q , and a region in which this flux intersects the d -axis magnetic flux is created. Since this locally increases the magnetic flux density and advances magnetic saturation at the region, the ϕ_d indirectly changes. This interference between the d - and q -axes is usually called cross-coupling saturation, which is a component expressed as $-L_{dq}i_q$ in Ref. (11), and causes indirect saturation characteristics. These direct and indirect saturation characteristics are fully taken into account by the method with the saturation function.

Figures 14(c) and (d) are ϕ_d for $i_{fd} = 59.5$ A (300% of the field current at rated load). The ϕ_d of (c) is smaller than (a) overall even if $i_d^\dagger$ or i_q were the same. This is similar for the linear FEA case as well, with the ϕ_d in (d) is smaller than in (b). Figure 15 shows the values on the $i_d^\dagger$ -axis each taken out from Figs. 14(a)–(d). Decreases in the ϕ_d can be confirmed in

Fig. 14. Saturation functions of the target SM.

Fig. 15. Decrease in the saturation function due to leakage magnetic flux of the armature winding.

both the linear and nonlinear cases. The reasons for this are as follows. The armature winding has a leakage inductance (L_{ld} in Fig. 2), and a leakage magnetic flux is generated by i_d . In contrast to this, i_{fd} does not affect L_{ld} and does not generate a leakage magnetic flux in the armature winding. Therefore, even if i_d^+ were the same, the leakage magnetic flux would change due to the balance between i_d and i_{fd} , whereby changes in ϕ_d appear.

Figures 14(e) and (f) are ϕ_q for $i_{fd} = 59.5$ A. In the case of a salient-pole SM, since the q -axis magnetic path has a longer air-gap than the d -axis magnetic path, the effect of the magnetic saturation is relatively small. From this tendency, the magnetic saturation in the q -axis sometimes seems to be ignored in each calculation⁽¹⁷⁾. However, in reality, the direct

Fig. 16. Current conditions obtained by non-linear and linear FEAs.

saturation is also generated in ϕ_q , as seen in Fig. 14(e). Furthermore, ϕ_q changes depending on i_d^+ due to the indirect saturation as well as ϕ_d . From the above, ϕ_q also should be treated as a function of i_d^+ , i_q , and i_{fd} .

5.2 Effect on Identification Results of Current Conditions The current conditions of the base and top speed at the rated voltage and power factor of 1 were identified using the saturation functions which were produced by the nonlinear FEA and linear FEA (relative permeability of 5000), respectively. Figures 16(a) and (b) show the characteristics of β and I_f , respectively. The division number of the saturation function is 20. Different identification results were obtained depending on whether the magnetic saturation was considered or not, and these reasons are discussed below.

We first focus on β . Figures 17(a) and (b) show the magnetic flux density distributions at the base speed under no-load and overload (output power 5 kW, 2.5 pu) conditions. The magnetic flux density of the stator teeth and rotor magnetic poles at the base speed is high at about 1.6 T, and the effect of the magnetic saturation can be seen already even in no-load conditions. Therefore, remarkable differences in β can be seen regardless of load depending on whether magnetic saturation is considered or not. Figures 17(c) and (d) are the magnetic flux density distributions at the top speed. The magnetic flux density under no-load is sufficiently small, but under load, it locally increases and reaches the saturation level at one end of the magnetic pole⁽¹⁰⁾. This means that the influence of the magnetic saturation appears only with increased load and consequently the difference in β gradually

Fig. 17. Distributions of the magnetic flux density.

Fig. 18. Rotation in axes due to magnetic saturation of the pole-head.

increases.

Next, we consider the reasons why β becomes smaller when the magnetic saturation is considered. The magnetic permeability distribution is symmetric against the geometric center of the magnetic pole under no-load conditions. This results that the center agrees with the d -axis as shown in Fig. 18(a). However, under load conditions, the magnetic permeability of one end of the magnetic pole (forward in the direction of rotation) decreases as shown in Fig. 18(b) since the phase of the magnetic flux shifts to the positive direction of the q -axis. When only the d -axis magnetomotive force (i_d or i_{fd}) is applied to this kind of asymmetric magnetic circuit, a magnetic flux which deviates to the negative direction of the q -axis relative to the geometric center of the magnetic pole is generated⁽²⁰⁾. In other words, not only d -axis but also q -axis magnetic flux is generated by the d -axis component of the magnetomotive force, and this can be regarded as the cross-coupling as well. Here, note that Ref. (20) assumes generator operation, so the magnetic saturation occurrence site is reversed when compared to Fig. 18(b), where motor operation is assumed. Looking at this phenomenon from a different point of view, the d - q coordinate system can be considered to have rotated clockwise due to the asymmetry of the magnetic circuit. This means that the direction of the magnetic flux shown in Fig. 18(b) becomes the actual d -axis. The phase of

 Fig. 19. $i_q - \Phi_q$ characteristics according to change of the load.

the current at this time also rotates with the actual d - q coordinate system, so β seems to have become smaller due to magnetic saturation when considering the geometric d - q coordinate system as a reference.

We consider the change of I_f using Eq. (6). Since $p\omega_m PL_q/2 V_a^2 \rightarrow 0$ under no-load conditions ($P = 0$), I_f is inversely proportional to L_{ad} . The L_{ad} greatly decreases due to magnetic saturation at the base speed and the difference in I_f between considering the magnetic saturation or not is large. On the contrary, in cases of sufficiently large P , the 1 in the denominator and numerator of the equation can be ignored and I_f is nearly proportional to L_d/L_{ad} . Even if L_{ad} decreases due to magnetic saturation, L_d simultaneously decreases, and thus the effect of the magnetic saturation relatively decreases.

Finally, we focus on Φ_q characteristics with respect to i_q shown in Fig. 19 to investigate the effect of cross-coupling saturation. The inclination of the curve ($= L_{lq} + L_{aq}$) is the same and constant regardless of the rotating speed when magnetic saturation is not considered. However, as for the characteristics of the top speed where magnetic saturation is considered, the inclination rapidly changes at around $i_q = 7$ A, and Φ_q rather decreases against the increase in i_q . This indicates that the cross-coupling saturation is extremely strong, and the amount of decrease in inductance is greater than the amount of increase in the magnetomotive force. In addition, as for the characteristics of the base speed where magnetic saturation is considered, the inclination remains small even when i_q approaches zero. This is because the pole-heads are already saturated with only Φ_d , as also shown in Fig. 17(a). In this way, the cross-coupling saturation occurs at various operating points and can even induce extremely sudden changes in the q -axis inductance under certain circumstances.

As seen above, the magnetic saturation has a large effect on current conditions and it cannot be ignored for accurate identification. The method using the saturation function that can accurately express this effect is useful. In this paper, the discussions were conducted for the cases of the salient-pole SM, but the method is applicable for the cylindrical SM as well.

6. Conclusions

This paper proposed the introduction of the saturation function for the identification of the current conditions input into the FEA for the characteristics calculation of SMs. Using

the saturation function allows for avoiding various problems when inductance is used, and calculation processes can be simplified. Furthermore, the computation time needed for identifying the current conditions from the saturation function is extremely short, and this is an effective method when multiple current conditions are needed.

Current conditions of the salient-pole SM were identified at multiple operating points using the proposed method, and FEA where those identified values were used as inputs was performed. The operating points obtained from the analysis results had an overall error of less than 1% when compared to the specified operating points. The computation time needed for obtaining this accuracy was less than four hours with four parallel processes. Increasing the number of parallel processes can reduce the computation time in an inversely proportional manner.

The phenomena reported in previous research that not only direct saturation characteristics but also indirect saturation characteristics caused by the cross-coupling between the d - and q -axes is generated were reconfirmed by investigating for the characteristics of the saturation function. It is also found that magnetic saturation had a large effect on the current conditions even in actual operation conditions (i.e., voltage or power factor are limited) as a result of comparisons to a virtual SM on the linear FEA without the magnetic saturation. Accurate identification of the current conditions strongly requires consideration of the magnetic saturation, and the proposed method which accurately considers its effects using the saturation function is effective. The proposed method enables the easy identification of current conditions at multiple operating points, and thus, this realizes practical analyses that consider actual operating conditions and not just where the amplitude or phase of the current are changed.

This paper is the addition of new results and discussion to the content presented at the 2019 Technical Meeting of the Institute of Electrical Engineers of Japan⁽¹⁵⁾.

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Appendix

1. Derivation Process of Eq.(2)

Voltage equation in steady state conditions (direct current flowing on the d - q plane) is represented as follows:

$$\begin{cases} v_d = R_a i_d - \omega_e \Phi_q \\ v_q = R_a i_q + \omega_e \Phi_d \end{cases} \dots\dots\dots (A1)$$

If the dq -conversion is assumed unitary, the voltage amplitude on the d - and q -axes is equivalent to the effective value of the line to line voltage as follows:

$$V_a = \sqrt{(v_d^2 + v_q^2)} \dots\dots\dots (A2)$$

The first equation of (2) is obtained by substituting Eqs.(1) and (A1) into (A2). The output power is the input power minus the copper loss generated in the armature winding as follows:

$$P = v_d i_d + v_q i_q - R_a (i_d^2 + i_q^2) \dots\dots\dots (A3)$$

The second equation of (2) is obtained by substituting Eqs.(1) and (A1) into (A3). The reactive power is the product of the amplitude of the voltage, amplitude of the current, and the sine with the argument of the phase difference between the voltage and current, in other words, this is the vector product.

$$Q = (v_d, v_q) \times (i_d, i_q) = v_d i_q - v_q i_d \dots\dots\dots (A4)$$

The third equation of (2) is obtained by substituting Eqs.(1) and (A1) into (A4).

2. Saturation Function when the Number of Turns and the Length of the Iron Core are Changed

When the number of turns of the armature winding and the field winding is each increased by a factor of α_a and α_f , respectively, the magnetomotive forces of each winding are

each increased by a factor of α_a and α_f , respectively, and the magnetic flux linkage of the armature winding is increased by a factor of α_a . In addition, increasing the length of the iron core by a factor of α_c results in the magnetic flux linkage of the armature winding increasing by a factor of α_c . Summarizing the above points, the saturation functions $\phi_d'(i_d^\dagger, i_q, i_{fd})$ and $\phi_q'(i_d^\dagger, i_q, i_{fd})$ after the design changes are as follows:

$$\begin{cases} \phi_d'(i_d^\dagger, i_q, i_{fd}) = \alpha_a \alpha_c \phi_d(\alpha_a i_d^\dagger, \alpha_a i_q, \alpha_a i_{fd}) \\ \phi_q'(i_d^\dagger, i_q, i_{fd}) = \alpha_a \alpha_c \phi_q(\alpha_a i_d^\dagger, \alpha_a i_q, \alpha_a i_{fd}) \end{cases} \quad \dots\dots\dots (A5)$$

Here, the turn ratio following the change is $k' = (\alpha_f/\alpha_a)k$, and k' must be used in Eq. (4). The current condition of SMs with changing arbitrarily the number of turns or the length of the iron core can be identified from the converted saturation function of Eq. (A5).

3. Equation for Identifying for Current Conditions at Maximum Output

The maximum output power (pull-out torque) is the maximum value of the output power that can be taken out with a certain field current i_{fd} , and this is expressed by the following equation:

$$\begin{cases} f = \left\{ R_a (i_d^\dagger - i_{fd}) - \frac{P}{2} \omega_m \phi_q \right\}^2 \\ \quad + \left(R_a i_q + \frac{P}{2} \omega_m \phi_d \right)^2 - V_a^2 = 0 \\ g = \delta T = \frac{P}{2} \omega_m \delta \left\{ \phi_d i_q - \phi_q (i_d^\dagger - i_{fd}) \right\} = 0 \end{cases} \quad \dots\dots\dots (A6)$$

The first equation shows the voltage limit condition, and the second equation shows the condition that the torque becomes the extreme value. Eq. (A6) involves the two unknown variables $i_d^\dagger$ and i_q , and also two equations to be solved. Eq. (A6) can be transformed as follows by applying the method of Lagrange multipliers:

$$\begin{cases} F = \frac{\partial g}{\partial i_d^\dagger} - \lambda \frac{\partial f}{\partial i_d^\dagger} = 0 \\ G = \frac{\partial g}{\partial i_q} - \lambda \frac{\partial f}{\partial i_q} = 0 \\ H = -f = 0 \end{cases} \quad \dots\dots\dots (A7)$$

Here, λ is a newly introduced undetermined multiplier. If F , G , and H in Eq. (A7) are solved instead of f , g , and h in Eq. (8) to obtain $i_d^\dagger$, i_q , and λ , the current condition that maximizes the output can be identified.

4. Relationship Between the Amplitude of Armature Current and Saturation Function

The following relationship can be derived by rearranging Eq. (8) with respect to I_a :

$$R_a^2 (3I_a^2)^2 + 3I_a^2 (2R_a P - V_a^2) + P^2 + Q^2 = 0 \quad \dots\dots\dots (A8)$$

The saturation function is not included in Eq. (A8).

Therefore, I_a can be calculated as follows:

$$I_a = \sqrt{\frac{V_a^2 - 2R_a P - \sqrt{(V_a^2 - 2R_a P)^2 - 4R_a^2 (P^2 + Q^2)}}{6R_a^2}} \quad \dots\dots\dots (A9)$$

Since I_a does not depend on the saturation function, it is not affected by the number of divisions of the analysis point.

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