Range Extension Control of a Three-Wheel Electric Vehicle Prototype Based on Aggregation and Distribution

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(Manuscript received Jan. 15, 2021, revised May 8, 2021)
J-STAGE Advance published date : June 11, 2021

This paper presents an approach for the range extension control of a three-wheel electric vehicle prototype. By using the torque distribution vector to aggregate motor speeds, the physical model of the vehicle is mapped to an aggregation-and-distribution model (AaDM), which possesses the passivity property. Based on the AaDM, motion control and energy optimization can be designed separately. In particular, a speed controller was designed for the system to operate in the automatic cruise mode. A disturbance observer was designed to operate in the human driving mode. In this study, the conditions for the controllers were obtained to sufficiently ensure the L_2 stability of the control system. The conditions can be checked conveniently without establishing the dynamical equation of the overall system. Under the practically reasonable assumption on motor parameters, the analytical solutions of the optimal torque distribution ratios and d-axis currents were approximately derived in this study. Various test scenarios were considered to validate the proposed control systems. The test results show that in either operation modes, the system can prevent wheel slip, thereby simultaneously improving motion control and energy minimization.

Keywords: electric vehicle, in-wheel-motor, range extension control system, passivity, disturbance observer, L_2 stability

1. Introduction

1.1 RECS: A Successful Example of Motion Control

Thanks to the advances of electric motors, motion control of electric vehicle (EV) has grown up to be a mature research field. To mention just a few examples, we have anti-slip control⁽¹⁾⁽²⁾, slip ratio control⁽³⁾⁽⁴⁾, and driving force control⁽⁵⁾⁽⁶⁾. Recently, range extension control system (RECS) has been shown a great success^{(7)–(10)}. The key idea of RECS is to find the driving distribution ratios that minimizes the summation of the output powers, the copper losses, and the iron losses⁽⁷⁾. The effectiveness of this idea has been verified through the history of RECS^{(7)–(10)}. Literature review also shows several concepts which shared the common philosophy with RECS. For instance, Suzuki et al. developed a torque distribution algorithm for minimizing the tire dissipation energy⁽¹¹⁾. Chatzikomis et al. proposed a torque vectoring algorithm for minimizing the power consumption of EVs driven by in-wheel-motors⁽¹²⁾. A hierarchically decentralized RECS was recently proposed by providing for each wheel a disturbance observer (DOB) based anti-slip control⁽¹³⁾. Despite of great successes already achieved, several issues still needed to be investigated.

1.2 Issues Raised from Practical Point of View

First, it is well known that more energy would be required

by the vehicle if wheel slip was not prevented. A straightforward idea is to design for each motor actuator a local controller^{(1)–(4)(13)}. However, this approach introduces the local wheel slip prevention command to the energy optimization problem. This increases the complexity of the optimization problem and challenges the attempt to obtain its analytical solutions. Therefore, how to independently design the energy optimization algorithm and motion control algorithm, especially slip prevention, is still an open issue.

Second, RECS^{(7)–(10)(13)} only dealt with driving torque allocation under the assumption such that the d-axis currents of the motors were neglectable. However, it is well known that motor loss can also be reduced via the d-axis components of armature current⁽¹⁴⁾. Hence, it is reasonable to address this current component in the general optimization problem of RECS. This issue has been preliminarily discussed in a study of RECS⁽⁸⁾ in which the authors showed that the problem can be solved numerically. Certainly, the discussion needs to be continued. Is it possible to derive for the optimization problem an approximated solution with analytical expression to reduce the computational burden of RECS?

Third, EV is actually a complex dynamical system with both nonlinearities and uncertainties. The nonlinearities exist due to the tire force characteristics⁽¹⁵⁾. The main uncertainties exist due to the change of road condition. Thus, system stabilization is a nontrivial issue for not only RECS but also other vehicle motion controls. An attempt to deal with RECS stabilization was made by treating the system as four independent wheel speed control loops⁽⁹⁾. As shown recently, the physical interaction between motor actuators actually exist and must

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be considered to guarantee system stability⁽¹⁶⁾⁽¹⁷⁾. However, linearization of the vehicle dynamics was required for applying the “linear quadratic regulator”⁽¹⁶⁾ and the “generalized frequency variable”⁽¹⁷⁾. This increased the complexity of system design. Hence, the following question raises naturally: How to stabilize RECS by a practical design procedure?

1.3 Problem Setting and Contributions RECS has been developed for a class of four-IWM-EV^{(7)–(10)}. To extend the application range of RECS, this paper considered a three-wheel EV prototype with two front IWMs and a rear motor and gear mechanism, all the motors are permanent magnet synchronous motors (PMSMs).

Using passivity theory, this paper shows that the vehicle is a passive system from the motor torques to the motor speeds. By a pair of torque distribution and speed aggregation vectors, the physical model of the vehicle can be mapped to a single-input single output model which also possesses passivity notation. This model is named aggregation-and-distribution model (AaDM). Thanks to the passivity of AaDM, energy optimization and motion controller can be designed separately. The optimization law is only to update the distribution and aggregation vectors. Besides, motion control is designed directly for the AaDM.

This paper proposed two RECSs for automatic cruise mode (ACM) and human driving mode (HDM), respectively. Thanks to its simplicity and efficacy, DOB⁽¹⁾⁽¹⁸⁾ was intentionally selected for the HDM. Note that, the traditional way of DOB design treated the vehicle as a linear plant with norm-bounded perturbation Δ . However, as discussed by Bickel *et al.*⁽¹⁹⁾ and Sariyildiz *et al.*⁽²⁰⁾, this approach does not show system stability rigorously. Based on passivity theory, this paper presents the conditions for the controllers to sufficiently guarantee L_2 stability of RECS in both operation modes. The conditions do not require linearization, and can be checked easily via linear matrix inequality without establishing the dynamical equation of the overall system.

Moreover, the optimization problem was formulated by both the distribution ratios and d-axis currents. In particular, this paper shows that, under practically reasonable assumption on motor parameters, it is possible to (approximately) derive the analytical solution of the optimization problem.

1.4 The Rest of This Paper Section II described the model of the vehicle under study. Section III introduced the fundamentals of passivity theory and analyzed the passivity properties of the vehicle. Section IV presented the general design procedure of RECS for ACM and HDM. Section V presented the energy optimization problem, and under reasonable assumption obtained for it an analytical solution. The proposed approach was evaluated in Section VI through various test scenarios. The conclusions and future study were stated in Section VII.

2. Vehicle Model

Fig. 1 describes the model of the three-wheel EV. The vehicle mass is denoted by m and the wheel radius is r . The driving force and vertical force of wheel are F_i and Z_i , respectively. The torque and rotational speed of the motor are denoted as $T_{m,i}$ and $\omega_{m,i}$. As described in Fig. 2, the rear wheel is driven via the gear with the ratio N_3 . Assuming that the gear efficiency is 100%, the following relationship holds for

Fig. 1. Three-wheel EV model

Fig. 2. Rotational motion of rear wheel

the rear wheel

$$\omega_{m,3} = N_3 \omega_{w,3}, \quad T_{wm,3} \omega_{m,3} = T_{mw,3} \omega_{w,3} \dots \dots \dots (1)$$

Let the inertias of motor and wheel be $J_{m,3}$ and $J_{w,3}$, the rotational motion of the rear wheel and rear motor are

$$J_{w,3} \dot{\omega}_{w,3} = T_{mw,3} - T_{F_3} = T_{mw,3} - r F_3 \dots \dots \dots (2)$$

$$J_{m,3} \dot{\omega}_{m,3} = T_{m,3} - T_{wm,3} \dots \dots \dots (3)$$

Summarizing (1)–(3), the equivalent equation is expressed as

$$J_{eq,3} \dot{\omega}_{m,3} = T_{m,3} - N_3^{-1} r F_3 \dots \dots \dots (4)$$

where the equivalent inertia is

$$J_{eq,3} = J_{m,3} + N_3^{-2} J_{w,3} \dots \dots \dots (5)$$

Let the inertia of the front wheel and motor be $J_{eq,i}$ ($i = 1, 2$). Define $N_{1,2} = 1$, the rotational motion of the front motor actuators can be expressed similar to that of the rear wheel

$$J_{eq,i} \dot{\omega}_{m,i} = T_{m,i} - N_i^{-1} r F_i, \quad i = 1, 2 \dots \dots \dots (6)$$

The slip ratio of each wheel can be uniquely expressed as

$$\lambda_i = \frac{r \omega_{w,i} - v_x}{\max\{r \omega_{w,i}, v_x, \varepsilon\}} \dots \dots \dots (7)$$

where v_x is the speed of the vehicle; ε is a small positive number to prevent division by zero, and $\omega_{w,i} \triangleq \omega_{m,i}/N_i$ with $i = 1, 2, 3$. The nonlinear relationship between the driving force and the slip ratio (Fig. 3) is commonly described by “magic formula”⁽¹⁵⁾:

$$F_i = \begin{cases} f_i(\lambda_i) & \text{if } \lambda_i \geq 0 \\ -f_i(-\lambda_i) & \text{if } \lambda_i < 0 \end{cases} \dots \dots \dots (8)$$

where

Fig. 3. Relationship between driving force and slip ratio

$$f_i(\lambda_i) = A_i \sin\{B_i \tan^{-1}[C_i \lambda_i - D_i(C_i \lambda_i - \tan^{-1}(C_i \lambda_i))]\} \quad (9)$$

where $A_i = \mu_i Z_i$, μ_i is the friction coefficient; and B_i , C_i and D_i are the shape factors. Finally, the longitudinal motion of the vehicle is expressed as

$$m\dot{v}_x = \sum_{i=1}^3 F_i - \frac{1}{2} \rho C_D A_F v_x |v_x| \quad (10)$$

where ρ is air density, C_D is the aerodynamical drag coefficient, and A_F is the frontal area of the vehicle.

3. Passivity Analysis of the Three-wheel EV

3.1 Introduction to Passivity We consider a system H given by the state space equation

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases} \quad (11)$$

where $u \in \mathbb{R}^p$, $y \in \mathbb{R}^p$, and $x \in \mathbb{R}^n$.

Definition 1⁽²¹⁾: The system H is passive from input u to output y if there exists a positive semidefinite function $S: \mathbb{R}^n \rightarrow \mathbb{R}_+$, called *storage function*, such that $\dot{S} \leq y^\dagger u \forall x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$. In addition, H is input strictly passive (ISP) if

$$\dot{S} \leq y^\dagger u - \delta_u \|u\|^2 \quad \forall x \in \mathbb{R}^n, u \in \mathbb{R}^p \text{ for some } \delta_u > 0,$$

and output strictly passive (OSP) if

$$\dot{S} \leq y^\dagger u - \delta_y \|y\|^2 \quad \forall x \in \mathbb{R}^n, u \in \mathbb{R}^p \text{ for some } \delta_y > 0.$$

Fig. 4 describes the system H which is the feedback connection of H_1 and H_2 . By Definition 1, if both H_1 and H_2 are passive, then system H with input (ξ_1, ξ_2) and output (y_1, y_2) is also passive. In addition, when $\xi_2 = 0$, system H with input ξ_1 and output y_1 is shown to be passive. Moreover, passivity notation is related to a type of input-output stability, namely, L_2 -stability.

Theorem 2⁽²¹⁾: If both H_1 and H_2 are OSP, then system H with input (ξ_1, ξ_2) and output (y_1, y_2) has a finite L_2 -gain. When $\xi_2 = 0$, the system H with input ξ_1 and output y_1 has a finite L_2 -gain if either (i) H_1 is passive and H_2 is ISP, or (ii) H_1 is OSP and H_2 is passive.

3.2 Passivity Analysis Based on (1)–(10), the vehicle is described by the EV system with the input and the output

$$\mathbf{T}_m = [T_{m,1} \ T_{m,2} \ T_{m,3}]^\dagger, \quad \omega_m = [\omega_{m,1} \ \omega_{m,2} \ \omega_{m,3}]^\dagger \quad (12)$$

where $\dagger$ is the transpose notation.

Proposition 3: EV is passive with the storage function

Fig. 4. Feedback connection of two sub-systems

$$S = S_V + \sum_{i=1}^3 S_{W,i} \text{ where } S_V = \frac{1}{2} m v_x^2, \quad S_{W,i} = \frac{1}{2} J_{eq,i} \omega_{m,i}^2 \quad (13)$$

Proof: From (4), (6), (10) and (13), we have

$$\dot{S}_V = m \dot{v}_x v_x = \sum_{i=1}^3 F_i v_x - \frac{1}{2} \rho C_D A_F v_x^2 |v_x| \quad (14)$$

$$\dot{S}_{W,i} = J_{eq,i} \dot{\omega}_{m,i} \omega_{m,i} = \omega_{m,i} T_{m,i} - N_i^{-1} r \omega_{m,i} F_i \quad (15)$$

From (7), (8), (14) and (15), we have

$$\begin{aligned} \dot{S} &= \omega_m^\dagger \mathbf{T}_m - \sum_{i=1}^3 \max\{r \omega_{m,i}, v_x, \varepsilon\} \lambda_i f_i(\lambda_i) \\ &\quad - \frac{1}{2} \rho C_D A_F v_x^2 |v_x| \quad (16) \end{aligned}$$

As can be seen from the tire force characteristics in Fig. 3, the term $\lambda_i f_i(\lambda_i)$ is always non-negative. Consequently, it can be shown from (16) that the following inequality holds true, which also shows that the system EV is passive by Definition 1:

$$\dot{S} \leq \omega_m^\dagger \mathbf{T}_m \quad (17)$$

4. General Design of RECS

4.1 Aggregation-and-distribution Model The proposed aggregation-and-distribution model (AaDM) is described in Fig. 5 with the scalar input T_a and the scalar output ω_a . The idea of the model is explained as follows. Since the front wheels are directly driven by in-wheel-motors, $\omega_{m,1}$ and $\omega_{m,2}$ also represent the rotational speeds of the front wheels. However, the rear wheel is not directly connected to the motor, but via a gear mechanism with the ratio N_3 . As shown in (1), the rotational speed of the rear wheel N_3 is smaller than that of the rear motor. Therefore, the motor speeds should be scaled down before aggregation. By applying a pair of scaling matrix $\text{diag}\{N_i^{-1}\}$ to EV, we obtain in Fig. 5 a system EV_C with the input and the output

$$\omega_c = \text{diag}\{N_i^{-1}\} \omega_m = [\omega_{m,1} \ \omega_{m,2} \ N_3^{-1} \omega_{m,3}]^\dagger \quad (18)$$

$$\mathbf{T}_m = \text{diag}\{N_i^{-1}\} \mathbf{T}_c = [T_{c,1} \ T_{c,2} \ N_3^{-1} T_{c,3}]^\dagger \quad (19)$$

where $\mathbf{T}_c$ is distributed from T_a , and ω_c is aggregated to be ω_a . Considering only the longitudinal motion of the vehicle, the aggregation/distribution vector is defined as

$$\mathbf{k} = [k_1 \ k_2 \ k_3]^\dagger \quad (20)$$

where the distribution ratios satisfy the following constraints

$$\begin{cases} k_1 = k_2, \ k_1 + k_2 + k_3 = 1 \\ 0 \leq k_{1,2,3} \leq 1 \end{cases} \quad (21)$$

Fig. 5. Block diagram of AaDM

With respect to (17), (18) and (19), we have

$$\omega_c^\dagger T_c = [\text{diag}\{N_i^{-1}\} \omega_m]^\dagger T_c = \omega_m^\dagger T_m \geq \dot{S} \dots \dots (22)$$

which means $\mathbf{EV}_C$ is also passive with the storage function (13). Hence, the scaling matrix $\text{diag}\{N_i^{-1}\}$ has three meanings. First, it is a map to transform motor speed to wheel speed. Second, it is a map to transform wheel driving torque to motor torque command. Last but not least, it maintains the passivity property of the vehicle.

With respect to (21), ω_a is almost close to the rotational speed of any single wheel if the slip ratio is small. In the control space, we can treat the AaDM as a “virtual EV” with the input torque T_a and the wheel speed ω_a . By applying motion control to AaDM, it is possible to separate the motion control algorithm and energy optimization algorithm which is designed for the inner of AaDM.

Proposition 4: AaDM is passive with the storage function (13).

Proof: From (22) and the structure of AaDM, we have

$$\omega_a^\dagger T_a = (\mathbf{k}^\dagger \omega_c)^\dagger T_a = \omega_c^\dagger (\mathbf{k} T_a) = \omega_c^\dagger T_c \geq \dot{S} \dots \dots (23)$$

which shows the passivity of AaDM.

Note that, the passivity notation of AaDM holds even when the distribution ratios are time-varying. This motivates us to consider the following design approach of RECS.

4.2 ACM-RECS The control system for the ACM was presented in Fig. 6 in which the reference speed is represented by a scalar signal ω_r . Its design procedure is presented as follows.

ACM-RECS design procedure

Stage 1: Establish the algorithm to find the distribution ratios and the d-axis currents that minimize be the summation of motor output powers, copper loss, and iron loss. The optimization problem is solved under the constraint (21).

Stage 2: Design the controller $C_a(s)$ which is OSP.

Proposition 5: The ACM-RECS design procedure sufficiently guarantees that the overall system in Fig. 6 with input ω_r and output T_a has a finite L_2 gain.

Proof: The system in Fig. 6 is actually the feedback connection of the passive system AaDM and $C_a(s)$. According to Theorem 2, if $C_a(s)$ is OSP then the system (with input ω_r and output T_a) has a finite L_2 gain and is L_2 stable. This completes the proof.

4.3 HDM-RECS The control system for HDM was described in Fig. 7(a) in which the DOB is established with the nominal model $P_n(s)$ and the low-pass filter $Q(s)$. T_a is the summation of the driving command given by the

Fig. 6. Block diagram of ACM-RECS

(a) Original block diagram.

(b) Equivalent expression.

Fig. 7. Block diagram of HDM-RECS

driver (T_{cmd}) and the anti-slip command given by DOB (T_d). Fig. 7(b) describes the equivalent block diagram of HDM-RECS with

$$G_1(s) = \frac{1}{1 - Q(s)}, \quad G_2(s) = \frac{Q(s)P_n^{-1}(s)}{1 - Q(s)} \dots \dots (24)$$

HDM-RECS design procedure

Stage 1: Establish the algorithm to find the distribution ratios and the d-axis currents that minimize be the summation of motor output powers, copper loss, and iron loss. The optimization problem is solved under the constraints (21).

Stage 2: Select the nominal model and the low pass filter such that $G_1(s)$ is a stable transfer function and $G_2(s)$ is ISP.

Proposition 6: The HDM-RECS design procedure sufficiently guarantees that the feedback connection in Fig. 7(b) with input T_1 and output ω_a has a finite L_2 gain.

Proof: The stability of $G_1(s)$ is to ensures the internal stability from T_{cmd} to T_1 . As AaDM is passive, if $G_2(s)$ is ISP then the system in the green-dashed rectangle of Fig. 7(b) has a finite L_2 gain and is L_2 stable. This completes the proof.

As shown in Fig. 7, instead of designing for each motor a DOB, we only design a DOB for AaDM. In the Section 6, we will show that this control configuration is enough to prevent the wheel from the slip phenomenon. This control configuration has another merit in comparison with the decentralized DOB⁽¹³⁾: The optimization problem was not intervened by the DOB command. This allows us to separate the motion control and optimization algorithms. Hence, ACM-RECS and HDM-RECS can share the same optimization algorithm designed insight AaDM.

4.4 Remark Several points should be noticed when applying the proposed design approach.

Remark 1: If the gear efficiency η_3 should be addressed, the EV system is still passive from T_m to the output $\text{diag}\{1, 1, \eta_3\} \omega_m$ with the storage function $S = S_V + S_{W,1} + S_{W,2} + \eta_3 S_{W,3}$. Hence, the proposed approach is still applicable without special difficulty.

Remark 2: If the dead-time of the motor driver is large enough, and the saturation of the motor torque/current should be addressed, then EV and AaDM are no longer passive systems. Fortunately, we might benefit from other passivity properties of the vehicle. For instance, the map from the slip ratio to the driving force is passive since $\lambda_i f_i(\lambda_i)$ is non-negative. Thus, it is possible to lump the nonlinear map and the torque saturation operator into a memoryless map with sector-bounded. This allows us to further examine the “absolute stability” of RECS by using Circle-Popov criteria⁽²¹⁾.

5. Optimization Problem

5.1 Derivation of the Optimization Problem Given a torque command T_a and a vehicle speed v_x , HDM-RECS and ACM-RECS share the same optimization problem which will be examined in this Section. Since L_2 stability is ensured by the proposed design procedures, it is possible to assume that the slip ratio is maintained at small value. The driving force can be linearized as $F_i = D_{s,i} Z_i \lambda_i$ where $D_{s,i}$ is the driving stiffness coefficient. The motor speed can be approximated as⁽⁸⁾:

$$\omega_{m,i} \approx \frac{N_i v_x}{r} (1 + \lambda_i) \approx \frac{N_i v_x}{r} \left(1 + \frac{F_i}{D_{s,i} Z_i} \right) \dots \dots \dots (25)$$

The motor torque and the driving force can be expressed as

$$T_{m,i} = \frac{k_i T_a}{N_i} \dots \dots \dots (26)$$

$$F_i \approx \frac{N_i}{r} T_{m,i} \approx \frac{N_i}{r} \frac{k_i T_a}{N_i} = \frac{k_i T_a}{r} \dots \dots \dots (27)$$

From (25)–(27), the output power $P_{M,i}$ is derived as in Table 1. Next, this paper examines in Fig. 8 the equivalent circuit of PM motor⁽¹⁴⁾. In this figure, the d- and q-axis components of armature current are $I_{d,i}$ and $I_{q,i}$, respectively. $I_{cd,i}$ and $I_{cq,i}$ represent the d- and q-axis components of iron loss current, respectively. The d- and q-axis components of the terminal voltage are denoted as $U_{d,i}$ and $U_{q,i}$, respectively. The armature winding resistance per-phase and the iron loss resistance are $R_{a,i}$ and $R_{c,i}$, respectively. $L_{d,i}$ and $L_{q,i}$ are the d- and q-axis components of armature self-inductance, respectively. Let $\Psi_{e,i}$ be the flux linkage of permanent magnet per-phase, we define

$$\Psi_{a,i} = \sqrt{3} \Psi_{e,i} \dots \dots \dots (28)$$

As shown in (14), we have

$$I_{od,i} = I_{d,i} - I_{cd,i}, \quad I_{oq,i} = I_{q,i} - I_{cq,i} \dots \dots \dots (29)$$

$$I_{cd,i} = -\frac{\omega_{e,i} \rho_i L_{d,i} I_{oq,i}}{R_{C,i}}, \quad I_{cq,i} = \frac{\omega_{e,i} (\Psi_{a,i} + L_{d,i} I_{od,i})}{R_{C,i}} \dots \dots \dots (30)$$

$$\omega_{e,i} = p_{n,i} \omega_{m,i} \approx \frac{p_{n,i} N_i v_x}{r} \dots \dots \dots (31)$$

where $p_{n,i}$ is the number of pole pairs. Define $\rho_i \triangleq L_{q,i}/L_{d,i}$ as the salient coefficient, the motor torque is expressed as

$$T_{m,i} = p_{n,i} \Psi_{eq,i} I_{oq,i} \dots \dots \dots (32)$$

$$\Psi_{eq,i} = \Psi_{a,i} + (1 - \rho_i) L_{d,i} I_{od,i} \dots \dots \dots (33)$$

The copper loss and the iron loss are

Table 1. Derivation of motor powers

Power	Formula
Output	$P_{M,i} = \frac{v_x}{r} \left(1 + \frac{k_i T_a}{r D_{s,i} Z_i} \right) k_i T_a$
Copper loss	$P_{Cu,i} = R_{a,i} \left\{ \left(I_{od,i} - \frac{v_x \rho_i L_{d,i}}{r R_{C,i}} \frac{k_i T_a}{\Psi_{eq,i}} \right)^2 + \left[\frac{1}{p_{n,i} \Psi_{eq,i}} \cdot \frac{k_i T_a}{N_i} + \frac{p_{n,i} N_i v_x (\Psi_{a,i} + L_{d,i} I_{od,i})}{r R_{C,i}} \right]^2 \right\}$
Iron loss	$P_{Fe,i} = \frac{p_{n,i}^2 N_i^2 v_x^2}{r^2 R_{C,i}} \left\{ \rho_i^2 L_{d,i}^2 k_i^2 T_a^2 + (\Psi_{a,i} + L_{d,i} I_{od,i})^2 \right\}$

Fig. 8. Equivalent circuit of the PM motor

$$P_{Cu,i} = R_{a,i} (I_{d,i}^2 + I_{q,i}^2), \quad P_{Fe,i} = R_{c,i} (I_{cd,i}^2 + I_{cq,i}^2) \dots \dots \dots (34)$$

Substitute (26) and (29)–(32) into (34), the copper loss and iron loss are finally expressed as in Table 1. In this study, the inverter loss and other power losses are assumed to be uncontrollable. We can define the summation of the powers in Table 1 as

$$P_{in} = \sum_{i=1}^3 (P_{M,i} + P_{Cu,i} + P_{Fe,i}) \dots \dots \dots (35)$$

Note that, $P_{Cu,i}$ and $P_{Fe,i}$ are the convex functions of both k_i and $I_{od,i}$, and $P_{M,i}$ is a convex function of k_i . Therefore, the following optimization problem (OP) can be considered for RECS:

$$\text{OP: } \min_{\{k_i, I_{od,i}\}} P_{in} \text{ s.t. (21)} \dots \dots \dots (36)$$

The OP (36) is three-wheel version of the optimization problem discussed by Fujimoto *et al.*⁽⁸⁾ when considering an EV driven by four IWMs. In (21), we can let

$$k_1 = k_2 = k_f, \quad k_3 = 1 - 2k_f \quad \text{where} \quad 0 \leq k_f \leq 0.5 \dots \dots \dots (37)$$

Substituting (37) into (36), it can be shown that

$$P_{in} = P_{in}(k_f, I_{od,1}, I_{od,2}, I_{od,3}) \dots \dots \dots (38)$$

Table 2. Practical algorithm for the energy optimization problem of RECS

Step	Purpose	Detail algorithm
1	Update the values of $\alpha_{n,i}$, β_i and γ_i using the estimated values of vertical forces, driving stiffness, and a speed v_x	$\alpha_{n,i} = R_{a,i} \left[\left(\frac{v_x \rho_i L_{d,i}}{r R_{c,i} \Psi_{a,i}} \right)^2 + \left(\frac{1}{N_i p_{n,i} \Psi_{a,i}} \right)^2 \right] + \frac{v_x^2 \rho_i^2 L_{d,i}^2}{r^2 R_{c,i}^2 \Psi_{a,i}^2} + \frac{v_x}{r^2 D_{s,i} Z_i}$
		$\beta_i = R_{a,i} + (R_{a,i} + R_{c,i}) \left(\frac{p_{n,i} N_i v_x}{r R_{c,i}} \right)^2 L_{d,i}^2, \quad \gamma_i = 2(R_{a,i} + R_{c,i}) \left(\frac{p_{n,i} N_i v_x}{r R_{c,i}} \right)^2 \Psi_{a,i} L_{d,i}$
2	Obtain the optimal value of $I_{od,i}$ and k_f	$I_{od,i}^* = -\frac{\gamma_i}{2\beta_i}, \quad k_f^* = \frac{2\alpha_{n,3}}{\alpha_{n,1} + \alpha_{n,2} + 4\alpha_{n,3}}$
3	Update the distribution ratios and the d- and q-axis currents	$k_1 = k_2 = k_f^*, \quad k_3 = 1 - 2k_f^*$
		$I_{d,i} = I_{od,i}^* - \frac{v_x}{r} \frac{\rho_i L_{d,i}}{R_{c,i}} \left[\frac{k_i T_a}{\Psi_{a,i} + (1 - \rho_i) L_{d,i} I_{od,i}^*} \right], \quad I_{q,i} = \frac{k_i T_a}{p_{n,i} N_i [\Psi_{a,i} + (1 - \rho_i) L_{d,i} I_{od,i}^*]} + \frac{p_{n,i} N_i v_x (\Psi_{a,i} + L_{d,i} I_{od,i}^*)}{r R_{c,i}}$

As discussed by Fujimoto *et al.* ⁽⁸⁾, the optimal values of the distribution ratios and the d-axis currents can be obtained by numerically solving a set of equations

$$\frac{\partial P_{in}}{\partial k_f} = 0, \quad \frac{\partial P_{in}}{\partial I_{od,1}} = 0, \quad \frac{\partial P_{in}}{\partial I_{od,2}} = 0, \quad \frac{\partial P_{in}}{\partial I_{od,3}} = 0 \dots (39)$$

5.2 A Practical Solution of OP The numerical algorithm, however, will increase the computational burden. This paper aims to find an approximated solution of OP, which can be expressed analytically. Based on Table 1, we notice that P_{in} can be represented as

$$P_{in} = P^U + \sum_{i=1}^3 P_i^L \dots (40)$$

$$P^U = \sum_{i=1}^3 P_i^U \dots (41)$$

$$P_i^U = \alpha_i (k_i T_a)^2 + \frac{v_x}{r} \left(1 + \frac{2R_{a,i}}{R_{c,i}} \right) k_i T_a \dots (42)$$

$$\alpha_i = R_{a,i} \left[\left(\frac{v_x \rho_i L_{d,i}}{r R_{c,i} \Psi_{eq,i}} \right)^2 + \left(\frac{1}{N_i p_{n,i} \Psi_{eq,i}} \right)^2 \right] + \frac{v_x^2 \rho_i^2 L_{d,i}^2}{r^2 R_{c,i}^2 \Psi_{eq,i}^2} + \frac{v_x}{r^2 D_{s,i} Z_i} \dots (43)$$

$$P_i^L = \beta_i I_{od,i}^2 + \gamma_i I_{od,i} + (R_{a,i} + R_{c,i}) \left(\frac{p_{n,i} N_i v_x}{r R_{c,i}} \right)^2 \Psi_{a,i}^2 \dots (44)$$

$$\beta_i = R_{a,i} + (R_{a,i} + R_{c,i}) \left(\frac{p_{n,i} N_i v_x}{r R_{c,i}} \right)^2 L_{d,i}^2 \dots (45)$$

$$\gamma_i = 2(R_{a,i} + R_{c,i}) \left(\frac{p_{n,i} N_i v_x}{r R_{c,i}} \right)^2 \Psi_{a,i} L_{d,i} \dots (46)$$

Assumption: We assume that

$$\frac{2R_{a,i}}{R_{c,i}} \ll 1, \quad \forall i \in \{1, 2, 3\} \dots (47)$$

$$\Psi_{eq,i} \approx \Psi_{a,i} \quad \forall i \in \{1, 2, 3\} \dots (48)$$

The inequality (47) is practically reasonable. In many PMSM drives, we notice that $R_{a,i}$ is commonly small in comparison with $R_{c,i}$. For instance, the PMSM studied by Mori-moto *et al.* ⁽¹⁴⁾ has

$$2R_a/R_c = 2 \times 0.57/240 = 0.0081$$

From a practical point of view, we know that $L_{d,i} I_{od,i}$ is relatively small in comparison with $\Psi_{a,i}$. Hence, (48) will hold almost surely if the salient coefficient is closed to 1. In addition, $\Psi_{eq,i} = \Psi_{a,i}$ holds true if the motor has not saliency.

Under the above assumption, we can approximate

$$P_{in} \approx \bar{P}_{in} = \bar{P}^U + \sum_{i=1}^3 P_i^L \dots (49)$$

With respect to (37), we have in (49)

$$\bar{P}^U = T_a^2 [(\alpha_{n,1} + \alpha_{n,2} + 4\alpha_{n,3}) k_f^2 - 4\alpha_{n,3} k_f + \alpha_{n,3}] + \frac{T_a v_x}{r} \dots (50)$$

$$\alpha_{n,i} = R_{a,i} \left[\left(\frac{v_x \rho_i L_{d,i}}{r R_{c,i} \Psi_{a,i}} \right)^2 + \left(\frac{1}{N_i p_{n,i} \Psi_{a,i}} \right)^2 \right] + \frac{v_x^2 \rho_i^2 L_{d,i}^2}{r^2 R_{c,i}^2 \Psi_{a,i}^2} + \frac{v_x}{r^2 D_{s,i} Z_i} \dots (51)$$

We notice that, given a command T_a and a speed v_x :

- Each P_i^L is a quadratic function of only $I_{od,i}$.

- $\bar{P}^U$ is a quadratic function of only k_f .

Instead of solving (36) directly, we consider the minimization of the approximated function (49) by solving

$$\frac{\partial \bar{P}_{in}}{\partial k_f} = 0, \quad \frac{\partial \bar{P}_{in}}{\partial I_{od,1}} = 0, \quad \frac{\partial \bar{P}_{in}}{\partial I_{od,2}} = 0, \quad \frac{\partial \bar{P}_{in}}{\partial I_{od,3}} = 0 \dots (52)$$

In summary, a practical algorithm for obtaining an approximated solution of the energy optimization problem was shown in Table 2. As can be seen from (51), each $\alpha_{n,i}$ is a positive number. Hence, the distribution ratios calculated in Table 2 will certainly satisfy the condition (21). With respect to (29), (30) and (31), the d- and q-axis currents are finally calculated as in Table 2.

Notice that, the driving stiffness can be estimated easily using the motor torque and motor speed ⁽⁵⁾. Also, the vertical force can be calculated using on-board sensors ⁽⁸⁾. Since this paper is to demonstrate the idea of aggregation-distribution for RECS, we did not present such estimation algorithms which can be implemented in real-time without special difficulty.

6. Evaluation and Discussion

6.1 Vehicle Model Three-wheel vehicles have been developed from the beginning of automotive history. They have been used for various purposes, such as personal transportation and racing. For instance, Toyota has developed the concept of i-ROAD EV with two front wheels. In this study, we demonstrated the proposed approach by a three-wheel recreational EV prototype⁽²²⁾. The main physical parameters of the vehicle are presented in app. Table 1 of the Appendix.

6.2 Control System Design First, we presented the condition for selecting the controller $C_a(s)$ of the ACM-RECS. According to Proposition 5, $C_a(s)$ should be OSP. This paper considered the class of compensator

$$C_a(s) = K_a \frac{s + a_1}{s + a_2} \dots \dots \dots (53)$$

where $K_a > 0$, $a_1 > a_2 > 0$. As shown in Fig. 6, the above controller can be represented in state space as

$$\begin{cases} \dot{x}_a = -a_2 x_a + b e_a \\ T_a = K_a x_a + K_a e_a \end{cases} \dots \dots \dots (54)$$

where $b = a_1 - a_2$. Applying the Kalman–Yakubovich–Popov (KYP) Lemma⁽²³⁾ to the system (54), $C_a(s)$ is OSP if there exist the positive numbers ϕ and δ such that the following matrix is negative semidefinite

$$Q_{ACM} = \begin{bmatrix} -2\phi a_2 + 2\delta K_a^2 & \phi b - K_a + 2\delta K_a^2 \\ \phi b - K_a + 2\delta K_a^2 & -2K_a + 2\delta K_a^2 \end{bmatrix} \leq 0 \dots \dots \dots (55)$$

The above condition can be checked without special difficulty by Matlab program. In practical application, the motor torque has its maximum value. Therefore, the gain K_a has a certain upper-bound. If K_a is bigger than this upper-bound, the system might suffer the fluctuation of the motor torque. Hence, the set of $\{K_a, a_1, a_2\}$ could be selected from a fine-tuning process with the condition (55).

Next, this paper considered the following nominal model and low pass filter for the DOB in HDM-RECS.

$$P_n(s) = \frac{1}{J_n s + D_n}, \quad Q(s) = \frac{K_f}{\tau_f s + 1} \dots \dots \dots (56)$$

where $P_n(s)$ represents the nominal dynamics of the AaDM. It consists of two parameters J_n and D_n . On the other hand, $Q(s)$ has the time constant τ_f and K_f is the tuning gain which should be between 0 and 1. With the above setting, we have

$$G_1(s) = \frac{\tau_f s + 1}{\tau_f s + 1 - K_f}, \quad G_2(s) = K_f \frac{J_n s + D_n}{\tau_f s + 1 - K_f} \dots \dots \dots (57)$$

which shows that $G_1(s)$ is stable for any positive time constant of the DOB, and all tuning gain selected between 0 and 1. Applying the KYP Lemma, $G_2(s)$ is ISP if there exist the positive numbers χ and β such that

$$Q_{HDM} = \begin{bmatrix} -2\frac{1-K_f}{\tau_f}\chi & \xi \\ \xi & -2\frac{K_f J_n}{\tau_f} + 2\beta \end{bmatrix} \leq 0 \dots \dots \dots (58)$$

Fig. 9. Comparison of $\Psi_{eq,i}$ and $\Psi_{a,i}$ for the three-wheel EV

Fig. 10. ACM-RECS: Simulation setting of Test 1

where

$$\xi = \left(\frac{D_n}{\tau_f} - \frac{1 - K_f}{\tau_f^2} J_n \right) \chi - K_f \dots \dots \dots (59)$$

6.3 Evaluation of ACM-RECS By fine-tuning process with the condition (55), we selected the controller $C_a(s)$ with $K_a = 295$, $a_1 = 21.2$, and $a_2 = 4.1$. We can verify that this controller is OSP. For instance, by selecting

$$\phi = \frac{K_a}{(a_1 - a_2)} = 17.2515 \dots \dots \dots (60)$$

we can show that matrix Q_{ACM} has two negative eigenvalues $\{-573.27; -123.38\}$ with the value $\delta = 10^{-4}$. Next, we notice that the ratio $2R_a/R_c$ is only 1.9×10^{-4} for the front motor and 9.5×10^{-4} for the rear motor. In addition, as can be seen in Fig. 9, even if $I_{od,i}$ reaches -50 [A] or $+50$ [A], $\Psi_{eq,i}$ only deviates 8% for the front motor, and 1.67% for the rear motor. This means the practical algorithm in Table 2 is applicable to the three-wheel EV under study.

Test 1: A simple speed pattern

Fig. 10 describes the setting of Test 1 in which the vehicle has to follow a given speed pattern v_r under the change of road friction. In this test, the reference speed ω_r is roughly calculated as v_r/r . The vehicle operates on the low friction surface from 2 to 4 seconds, and from 25 to 26 seconds.

Test 1-A: Instead of aggregating the motor speeds, we utilize the longitudinal speed measurement v_x directly (Fig. 11(a)). The speed response of this test case with the optimal torque-current distribution is shown in Fig. 11(b). Since the speed responses of the front-left and front-right wheels are almost the same, we only plotted the results of the front-left wheel and rear wheel. The vehicle suffers a large slip of the rear wheel. It consumes an energy of 60.1 [Wh] to cruise a range of 359.2 [m].

Test 1-B: In this case, the control system in Fig. 6 was utilized. The energy consumption with different distribution

Fig. 11. Test 1-A with vehicle speed sensor

Table 3. Energy consumption of Test 1-B

Torque distribution	Current distribution	
	$I_{od,i} = 0$	optimal $I_{od,i}$
Front drives ($k_f = 0.5$)	L = 351.3 [m]	L = 351.3 [m]
	E = 36.88 [Wh]	E = 32.60 [Wh]
Rear drives ($k_f = 0.0$)	L = 358.2 [m]	L = 358.2 [m]
	E = 30.77 [Wh]	E = 25.95 [Wh]
All wheel drives ($k_f = 1/3$)	L = 357.6 [m]	L = 357.6 [m]
	E = 30.58 [Wh]	E = 26.11 [Wh]
All wheel drives ($k_f = 0.1$)	L = 357.4 [m]	L = 357.4 [m]
	27.98 [Wh]	23.28 [Wh]
All wheel drives (optimal k_f)	L = 357.2 [m]	L = 357.2 [m]
	E = 27.53 [Wh]	E = 22.91 [Wh]

strategies are summarized in Table 3. Even with the worst distribution strategy, energy consumption of Test 1-B is much less than that of Test 1-A. This is because the control system in Test 1-B might prevent the wheel slip to a certain extent. Due to the limitation of paper space, we only presented in Fig. 12 the results of Test 1-B with the optimal torque-current distribution strategy. Again, we only showed the results of the front-left and rear wheels.

For better comparison, the results in Table 3 were normalized by energy consumption per kilometer (Fig. 13). For any torque distribution strategy, 15% of energy consumption can be saved by optimizing the d-axis current of the motors. In comparison with the front or rear drive strategy, 37.88% and 25.54% of energy consumption can be reduced by simultaneously optimizing the torque distribution to all wheels with the optimal d-axis currents.

Test 2: WLTC test

Test 1 shows that it is possible to aggregate the motor speeds for the cruise control purpose. However, rw_a is not exactly a representative of the vehicle speed. Fortunately, it is possible to estimate the slip ratio using the motor torque and motor speed⁽²⁴⁾. Using the definition of slip ratio (7), the vehicle speed can be calculated from the i th motor speed as

Fig. 12. Result of Test 1-B with optimal distribution

Fig. 14. Extended ACM-RECS for vehicle speed control

$$\hat{v}_{x,i} = r\rho_i N_i^{-1} \omega_{m,i} \dots \dots \dots (61)$$

where

$$\rho_i = \begin{cases} (1 - \hat{\lambda}_i) & \text{in acceleration mode} \\ (1 + \hat{\lambda}_i)^{-1} & \text{in deceleration mode} \end{cases} \dots \dots \dots (62)$$

An estimation of the vehicle speed is proposed as follows

$$\hat{v}_x = k_1 \hat{v}_{x,1} + k_2 \hat{v}_{x,2} + k_3 \hat{v}_{x,3} = r \mathbf{k} \text{diag} \{ \rho_i N_i^{-1} \} \boldsymbol{\omega}_m \dots \dots \dots (63)$$

From (63), an extended version of the ACM-RECS is established in Fig. 14. We did not present the slip ratio estimation algorithms which has been designed and verified⁽¹⁶⁾⁽²⁴⁾. Note that the scaling matrix $\text{diag} \{ \rho_i / N_i \}$ does not change the passivity of the extended AaDM system in the red-dashed rectangle of Fig. 14.

The worldwide harmonized light-duty vehicle test cycle (WLTC) is one of the standard driving-cycles to evaluate the performance of the vehicle. It is used to evaluate the extended ACM-RECS. The test results were summarized in Fig. 15. In this test, the vehicle enters the low friction surface in three periods, 200 ~ 300 seconds, 700 ~ 800 seconds, and 1160 ~ 1190 seconds. Even though, the longitudinal speeds of the wheels are almost close to that of the vehicle body. Similarly to Test 1, we only demonstrated the results of the

Fig. 13. Comparison of different strategies in Test 1-B

Fig. 15. Result of Test 2 using WLTC with optimal torque-current distribution strategy

Table 4. Energy consumption of Test 2

Torque distribution	Current distribution	
	$I_{od,i} = 0$	optimal $I_{od,i}$
All wheel drives ($k_f = 1/3$)	E = 1415 [Wh]	E = 1194 [Wh]
All wheel drives ($k_f = 0.1$)	E = 1427 [Wh]	E = 1196 [Wh]
All wheel drives (optimal k_f)	E = 1390 [Wh]	E = 1166 [Wh]

front-left wheel and rear wheel. The energy consumptions in Test 2 are summarized in Table 4. Since the estimated vehicle speed is controlled in this test, the driving range is

14990 [m] for all strategies. In comparison with the worst strategy ($k_f = 0.1$ and $I_{od,i} = 0$), the energy consumption can be reduced 18.29% by optimizing the torque distribution ratios and d-axis currents.

6.4 Evaluation of HDM-RECS By fine tuning process with the condition (58), we selected a candidate of DOB with the parameters $J_n = 84.1$, $D_n = 0.1$, $K_f = 0.8$, $\tau_f = 0.05$. The HDM-RECS was evaluated by two tests as follows.

Test 3: Short acceleration with sharp change of friction

The setting and results of this test are described in Fig. 16. To accelerates the vehicle, a large driving command T_{cmd} is given by the driver (Fig. 16(a)). As shown in Fig. 16(b), the vehicle enters the low friction surface from 2 to 4 seconds.

In Test 3-A, the DOB is not utilized, which means T_a always equals to T_{cmd} . As can be seen clearly in Fig. 16(c),

Fig. 16. Results of Test 3

Fig. 17. Result of Test 4 using CADC with optimal torque-current distribution strategy

the vehicle suffers a noticeable wheel slip phenomenon. Consequently, it requires an energy of 259.5 [Wh] to run a distance of 77.89 [m].

In contrast, Test 3-B utilized the whole control system in Fig. 7. As a result, the wheel slip phenomenon could be prevented to a certain extent (Fig. 16(d)). Certainly, the DOB slightly degrades the acceleration performance in Test 3-B. Therefore, the driving range of Test 3-A is 1.05 times longer than that of Test 3-B. However, the energy consumed by the vehicle in Test 3-A is 3.8 times as much as that of Test 3-B. The results showed that proposed control system can improve the safety level and reduce the energy consumption of the RECS.

Test 4: CADC test

In this test, the driver has to drive the vehicle in the rural schedule of the common Artemis driving cycle (CADC)⁽²⁵⁾. The setting and results of this test are summarized in Fig. 17. To generate the total driving command T_{cmd} , a driver model is established as in the study of Fujimoto *et al.*⁽⁸⁾. Since the

Table 5. Energy consumption of Test 4

Torque distribution	Current distribution	
	$I_{od,i} = 0$	optimal $I_{od,i}$
All wheel drives ($k_f = 1/3$)	E = 2017 [Wh]	E = 1710 [Wh]
All wheel drives ($k_f = 0.1$)	E = 2041 [Wh]	E = 1721 [Wh]
All wheel drives (optimal k_f)	E = 2000 [Wh]	E = 1690 [Wh]

DOB is utilized, the longitudinal speeds of the wheels and the vehicle body are almost the same. Similarly to the test with ACM-RECS, we only showed the results in accordance with the front-left wheel and rear wheel.

The energy consumption of Test 4 with several distribution strategies are summarized in Table 5, with the driving

range of 17150 [m]). In comparison with the worst case ($k_f = 0.1$ and $I_{od,i} = 0$), the energy consumption can be reduced 17.20% by optimizing the torque distribution ratios and the d-axis current.

7. Conclusions

This paper shows that to design RECS, energy optimization should be integrated with slip prevention. To this end, this paper introduces the aggregation-and-distribution model (AaDM) which maintains the passivity property of the vehicle. Based on the AaDM, this paper presents an ACM-RECS for automatic cruise mode and another HDM-RECS for operating in human driving mode. Using passivity theory, L_2 stability conditions and design procedures of two systems were proposed. The condition can be test conveniently via matrix inequality. Under practical assumptions on motor models, this paper derived a practical algorithm for optimizing the torque distribution ratios and d-axis currents of the motors. Although the algorithm only gives an approximated solution of the optimization problem, it can perform analytically, thereby alleviating the computational burden of RECS. The effectiveness of the proposal has been evaluated by various test scenarios using a three-wheel vehicle model. Test results show that, the proposed systems can prevent the slip of the wheel to a certain extent. Besides, given any torque distribution strategy, 15% of energy consumption can be saved by optimizing the d-axis currents. In comparison with the front drive or rear drive, 37.88% and 25.54% of energy consumption can be reduced thanks to the proposed algorithm.

In future study, we are interested in the application of RECS to other EV prototypes, including the vehicle driven by induction motors. We will continue to investigate the optimization problem for the general EV, especially the well-posedness of the problem. Moreover, how to design decentralized DOB while maintaining a simple optimization algorithm, is another interested issue.

Acknowledgment

We would like to thank Prof. João Pedro F. Trovão, Prof. Minh C. Ta, and Dr. Bao-Huy Nguyễn at e-TESC Lab., Department of Electrical and Computer Engineering, University of Sherbrooke, Sherbrooke, QC, J1K 2R1, Canada for their great supports of the electric vehicle model used in this study.

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Appendix

The main parameters of the three-wheel EV and its figures are listed in app. Table 1.

app. Table 1. Main parameters of the three-wheel EV

Vehicle mass	$m = 430$ [kg]
Wheel's radius	$r = 0.277$ [m]
Gear transformation ratio	$N_1 = N_2 = 1, N_3 = 5.033$
Equivalent inertia	$J_{eq,1} = J_{eq,2} = 0.7807$ [kgm ²] $J_{eq,3} = 0.1376$ [kgm ²]
Cross-sectional area of vehicle in air	$A_F = 2.37$ [m ²]
Drag coefficient	$C_D = 0.35$
Distance from center of gravity to front and rear axles	$L_f = 0.678$ [m] $L_r = 1.039$ [m]
Height of center of gravity	$H_g = 0.60$ [m]
Rear motor	13 kW, 120 N.m, $p_n = 10$
Front motor	4 kW, 150 N.m, $p_n = 12$

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