Transformations of Reference Frames and Model Development for Multi-Phase Machines

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This paper presents a general set of transformations of stationary and rotating reference frames for all types of symmetrical multi-phase machines, as well as a generic approach for developing detailed electrical models of such machines. The electrical multi-phase model of an example 50 kW interior permanent-magnet synchronous machine (IPMSM) is developed. The transformations and model development procedures are illustrated in context of this example machine. The transformations are also used to reduce the computational complexity of obtaining model parameters via finite element (FEM) analysis. The results for the example machine are presented and the implications for model validity and associated challenges are discussed.

Keywords: motor drives, permanent magnet machines, multi-phase machines, modular construction

1. Introduction

Modern electrical drives in mobility applications often have to exhibit a high power density and high system reliability or capability for fault-tolerant operation. One approach of interest in this field is that of modular multi-phase drives. Multi-phase drives are a key enabler for fault-tolerant electrical drives and feature a reduced per-phase power rating and an increased number of degrees of freedom for design and control [1]. A high level of subsystem integration, such as integrated power electronics and gearing, may reduce interfaces and housing materials, resulting in an increased system power density. Modular construction may offer advantages in production due to potentially reduced winding complexity and better utilization of sheet metal [10]. A further cost reduction due to economy-of-scale effects may be applicable, especially for approaches that are scalable to different machine sizes. These benefits might offset an increase in system complexity and make these modular drives economically viable.

While these modular multi-phase drives may provide key advancements for modern applications, they lead to machine designs with a high number of degrees of freedom, requiring extensions to classical transformations and often an impractical computational effort to fully represent in a model. This work aims to provide an overview of the methodologies employed to mitigate these problems and presents a general approach for a multi-phase machine model. The full set of transformations for an arbitrary number of phases, as well as approaches to reduce computational complexity for acquiring model parameters, are layed out.

1.1 State of the Art

One notable example is the ‘Integrated Modular Motor Drive’ (IMMD) [4], showcasing a five-phase, modular machine consisting of single pole segments with integrated half-bridge inverters in a wye-configuration with floating neutral point.

Another holistic approach for highly integrated, modular multi-phase drives was developed in the project EMiLE [5]. It utilizes single tooth modules featuring a tooth-concentrated winding, a full-bridge converter as well as logic and communication circuitry all integrated into a so-called ‘smart stator tooth’, further outlined in [6]. The winding is in open-delta configuration, as this best fits the design philosophy of a self-contained single-phase segment, as well as allowing for potentially higher efficiency during operation, especially at low loads [7]. The integrated logic circuitry performs basic functions and handles communication with a dedicated master control module [8].

While a number of publications featuring models and transformations for multi-phase machines exist, they usually focus on specific phase configurations, e.g. a nine-phase machine presented in [9] or a six-phase multiplexed machine presented in [10]. A didactic presentation of a general set of transformations as derived in [11] is undertaken in this work.

1.2 Ongoing Research

Current research in the project VERSE [12] investigates further development of the concept of the smart stator tooth introduced in the project EMiLE. A new generation of power electronics in the form of a silicon-carbide (SiC) based full-bridge inverters are considered. The effects of the fast voltage transients resulting from SiC-based power electronics are also under investigation, as they may cause damage to the winding insulation [13]. A further reduction of single points of failure is pursued via
decentralized control, which poses particular challenges for this type of drive as outlined in sections 4 and 5.

1.3 Prototype Demonstrator Machine An early prototype permanent-magnet synchronous machine (PMSM) presented in (40) is used as an example for the development of the machine model and for the illustration of the transformations in this work. It consists of 12 individual stator segments, each with a single-phase tooth-concentrated winding in open delta configuration. The rotor features 10 poles (p = 5) of interior permanent magnets. The concept and basic geometry of the machine is illustrated in Fig. 1.

2. Modeling of the Machine

2.1 Electrical Model In general, the electrical model of electrical machines can be derived from the voltage equation \( \vec{u}_{ph} \) of the stator phases (1). \( \vec{i}_s \) is the vector of the phase currents and \( \vec{\psi}_s \) is the vector of the flux linkages of the individual phases.

\[
\vec{u}_{ph} = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt}
\]  

(1)

The flux-linkage vector \( \vec{\psi}_s \) is composed of the permanent-magnet flux linkage \( \vec{\psi}_f \) as well as flux linkage due to self- and mutual induction with other phases as given in (2).

\[
\vec{\psi}_s = L_s \cdot \vec{i}_s + \vec{\psi}_m
\]  

(2)

The matrix \( L_s \) comprises the self- and mutual inductances of the n phases and is defined in (3), where \( L_{x,y} \) is the self-inductance of phase x and \( L_{x,y} \) is the mutual inductance between phase x and phase y. These entries are dependent on the rotor position due to saliency as well as all n phase currents due to saturation effects.

\[
L_s = \begin{bmatrix}
L_{1,1} & L_{1,2} & L_{1,3} & \cdots & L_{1,n} \\
L_{2,1} & L_{2,2} & L_{2,3} & \cdots & L_{2,n} \\
L_{3,1} & L_{3,2} & L_{3,3} & \cdots & L_{3,n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
L_{n,1} & L_{n,2} & L_{n,3} & \cdots & L_{n,n}
\end{bmatrix}
\]  

(3)

By substituting (2) into (1), the voltage equation can now be written as (4).

\[
\vec{u}_{ph} = R_s \cdot \vec{i}_s + \frac{d\vec{\psi}_s}{dt} + \vec{i}_s \frac{d\vec{\psi}_s}{dt} + \frac{d\vec{\psi}_s}{dt}
\]  

(4)

This expression sufficiently describes the electrical side of the machine model and allows to represent the machine via an equivalent circuit as illustrated in Fig. 2. \( L_s \) and \( \vec{\psi}_s \) can be parametrized from measurements or FEM analysis and implemented as operating point dependent lookup-tables. Alternatively, field oriented models like those based on the ideal rotating transformer (IRTF) (41) are also viable, by extending the three-phase IRTF based models according to the transformations presented in section 3.

2.2 Torque Production The torque produced by the machine can be calculated in multiple ways. One method is to calculate the partial derivative of the magnetic co-energy with respect to the mechanical rotor angle \( \theta_{mech} \). This does not rely on further transformations and can be performed purely in the stator reference frame. The co-energy is separated into two components. One component reflects the interaction between the phase currents, while the other component reflects the interaction of the phase currents with the permanent-magnet flux linkage. The resulting expression for the mechanical torque is given in (5).

\[
T_m = \frac{p}{2} \cdot \sum_{a=1}^{n} \sum_{b=1}^{n} \left( i_{a,b} \cdot \frac{\partial(L_{a,b})}{\partial \theta_{mech}} + p \cdot \sum_{a=1}^{n} i_{a,a} \cdot \frac{\partial \vec{\psi}_{a,a}}{\partial \theta_{mech}} \right)
\]  

(5)

Note that the effects of cogging torque are not included in this expression.

3. Transformation of Reference Frames

In the following, transformations into stationary and rotating reference frames for any n-phase machine are pursued. One approach found in literature, such as (40), is to apply classical \( dq0 \)- and \( d0-q0 \)-transformations by extending the relevant matrices to the right to accomodate the higher number of phases. This leads to a reduction of degrees of freedom by the transformation and, therefore, a loss of information, resulting in ambiguity of the transformation. It is of critical importance, that the number of degrees of freedom cannot be reduced by these transformations without a loss of information. Thus a more complete transformation, as derived in (41), is presented here.

3.1 Stationary Reference Frames For symmetrical systems, the matrix \( M_{q0} \) for transformation into a stationary reference frame is given by (6).

\[
M_{q0}(1,k) = \frac{1}{n}
\]  

(6a)

\[
M_{q0}(2,m,k) = \frac{2}{n} \cdot \cos \left( m \cdot \frac{2\pi}{n} \cdot (k - 1) \right)
\]  

(6b)

\[
M_{q0}(2,m+1,k) = \frac{2}{n} \cdot \sin \left( m \cdot \frac{2\pi}{n} \cdot (k - 1) \right)
\]  

(6c)

\[
M_{q0}(n,k) = \frac{(-1)^{k-1}}{n}, \text{ if } n \text{ is even}
\]  

(6d)
The transformation separates the discrete distribution of phase values into different spatial harmonics. The first row (6a) and represents the common-mode (CM) component of the system. The last row (6d) only exists for systems with even number of phases \( n \) and represents the systems differential-mode (DM) component. Both these values are also called zero-systems \( \alpha\beta^{0,1} \) and \( \alpha\beta^{0,2} \). The \( \frac{n - 1}{2} \) pairs of rows (6b) and (6c) each project into an \( \alpha\beta \)-frame \( \alpha\beta^{m} \) which represents the \( m \)-th spatial harmonic of the system.

To illustrate this transformation, exemplary distributions of phase values \( \bar{x}_n \) for the 12-phase demonstrator machine, resulting in a value of \( x^{(0)}_{\alpha\beta} = 1 \) or \( x^{(m)}_{\alpha\beta} = (1, 0) \) respectively in the transformed systems, are given in Fig. 3. The inner, mid and outer circles represent values of -1, 0 an 1 respectively. Phase one is marked by an arrow.

The inverse transformation matrix \( M_{\alpha\beta}^{-1} \) is given by (7).

### Table 1. Harmonics projected to \( \alpha\beta \)-frames with \( a \in \mathbb{N}^{(1)} \)

<table>
<thead>
<tr>
<th>System</th>
<th>Harmonics</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha\beta^{0,1} )</td>
<td>( \pi(n \cdot a) )</td>
</tr>
<tr>
<td>( \alpha\beta^{0,2} )</td>
<td>( \pi(n \cdot a \pm m) )</td>
</tr>
<tr>
<td>( \alpha\beta^{0,3} )</td>
<td>( \pi(n \cdot a \pm m/2) )</td>
</tr>
</tbody>
</table>

Fig. 4. Aliasing between the \( \alpha\beta^{5,0} \)-system and the 7th harmonic

\[
M_{\alpha\beta}^{-1}(\alpha\beta^{(k,1)}) = \begin{pmatrix} 1 & \cdots & \cdots & \cdots & 0 \\ 0 & M_{\alpha\beta}^{(1)} & \cdots & \cdots & 0 \\ 0 & 0 & M_{\alpha\beta}^{(2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & M_{\alpha\beta}^{(L - 1)} \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix} \quad (8a)
\]

\[
M_{\alpha\beta}^{(m)} = \begin{pmatrix} \cos(m \cdot \theta_{\text{mech}}) & \sin(m \cdot \theta_{\text{mech}}) \\ -\sin(m \cdot \theta_{\text{mech}}) & \cos(m \cdot \theta_{\text{mech}}) \end{pmatrix} \quad (8b)
\]

It should be noted that the 0 entries in (8a) are zero-matrices of dimensions \((1 \times 2)\) for the first and last row, \((2 \times 1)\) for the first and last column and \((2 \times 2)\) for the inner entries.
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The transformed system is again composed of one or two zero-systems containing the harmonics noted in Table 1 and \( \frac{\hat{\alpha}}{2\pi} \) dq-frames \( \hat{d}^m \), each rotating with their fundamental frequency \( m \cdot \omega_{mech} \). It is possible to separate the different harmonics projected onto the same \( \hat{\alpha}\beta \)-frames into separate subsystems of the corresponding rotating frames by substituting \( m \cdot \theta_{mech} \) in (8b) with \( (n \cdot a_n + m) \cdot \theta_{mech} \). In this work, the factor \( a_n \) is chosen as 0. Therefore, all harmonic components of the \( \hat{\alpha}\beta^m \)-frames are projected onto the fundamental frequency of the corresponding rotating frame.

The set of transformations allows expressing the vector of phase currents \( \vec{i} \), as a vector \( \vec{i}_{dq} \) in rotating reference frames, which is composed of the two zero- and five dq-systems in the form of (9).

\[
\begin{align*}
\vec{i}_{dq} &= \begin{pmatrix} i_{dq}^{(0,1)} & i_{dq}^{(1)} & i_{dq}^{(2)} & i_{dq}^{(3)} & i_{dq}^{(4)} & i_{dq}^{(5)} & i_{dq}^{(0,2)} & \cdots \end{pmatrix} \quad \cdots (9)
\end{align*}
\]

The inductance matrix \( L_s \) can be transformed into a new matrix \( L_{dq} \) in the rotating reference frame. If the dq-frames are decoupled from each other, the matrix will assume a form as given by (10).

\[
L_{dq} = \begin{pmatrix} L_{dq}^{(0,1)} & 0 & 0 & \cdots & 0 & 0 \\
0 & L_{dq}^{(1)} & 0 & \cdots & 0 & 0 \\
0 & 0 & L_{dq}^{(2)} & \cdots & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & L_{dq}^{(m)} & 0 \\
0 & 0 & 0 & \cdots & 0 & L_{dq}^{(0,2)} \end{pmatrix} \quad \cdots (10a)
\]

\[
L_{dq}^{(m)} = \begin{pmatrix} L_{dq}^{d} & L_{dq}^{d} \\
L_{dq}^{d} & L_{dq}^{q} \end{pmatrix} \quad \cdots \cdots \cdots (10b)
\]

Whenever a residual coupling between dq-frames persists, additional coupling terms will appear outside the main- and secondary-diagonals. The dimensions of the \( \theta \) entries are to be treated like in (8a).

4. Model Parametrization

4.1 Reduction of Operating Points To obtain the model parameters for a complete model of the machine, the inductance matrix \( L_s \) has to be obtained for every operating point, either from measurements or from computational methods such as FEM.

Due to the potentially high number of phases and, therefore, number of degrees of freedom, determining the inductance values for a reasonably high number of operating points in all phases or dq- and zero-frames may not be practical. For the 12-phase machine considered here, even in a rotating reference frame, the system has 13 input variables (12 current components and the rotor position). Even with very low resolution of operating points in each zero- or dq-frame, the number of FEM simulations required far exceeds any acceptable computational effort. For example, just 10 operating points in each degree of freedom would result in \( 10^{13} \) total operating points. Therefore, the number of operating points considered must be reduced.

The reduction of operating points needs to take into account the machine geometry and the desired application of the machine model, as the validity of the model will be limited to the chosen operating range. Typically, the main torque producing component of the current needs to be considered. As the demonstrator is a machine with pole-pair number \( p = 5 \), the torque production will be predominantly caused by currents in the \( 5^\text{th} \) dq-frame \( i_{dq}^{(5)} \), which rotates at the fundamental electrical frequency of the machine.

Furthermore, the range of operating points can be reduced to a single quadrant in the \( i_{dq}^{(5)} \) plane, since the direct-axis current will be negative or zero \( (\vec{i}_d \leq 0) \) during normal operation and the influence of sign of the quadrature-axis current can be derived from symmetry considerations, as shown in Fig. 5. This data was derived from a first, separate FEM analysis with only five current values over a full electrical rotation.

Additionally, the range of operating points for the rotor angle can be reduced to half an electrical period. A reduced set of operating points covering the second quadrant of the \( i_{dq}^{(5)} \)-plane up to amplitudes of 100 A with a current spacing of 10 A and with a resolution of 0.25° for the mechanical rotor angle consists of only 17545 operating points and could be evaluated via FEM simulation in less than 4 hours. Performing this analysis at the same resolution and without this reduction would result in \( 9.101 \cdot 10^{14} \) operating points, requiring a computation time of over 23 million years.

4.2 FEM Analysis To determine the inductance parameters for the model of the demonstrator, finite element method (FEM) simulation of the machine for a range of operating points \( i_{dq}^{(5)} \) is performed using the software Ansys Maxwell. All other current frames are kept at 0 A for all simulation steps.

The self- and mutual inductance values for one operating point, given by \( \vec{i}_{dq} \) and the rotor angle \( \theta_{mech} \), are obtained from the FEM analysis.

4.3 FEM Results The results of this procedure applied to the demonstrator machine are illustrated in Fig. 6 and in Fig. 7. Note that the results span one rotation of the example machine’s main torque producing system \( (m = 5) \). While the inductances in the stator reference frame (Fig. 6(a) and Fig. 7(a)) change significantly with the rotor angle, the inductances transformed into the rotating reference frame (Fig. 6(b) and Fig. 7(b)) appear almost constant over the rotor angle. The Figs. 6(b) and 7(b) also clearly show the low value of...
of the common-mode inductance $L_{dq}^{(0,1)}$.

5. Model Limitations and Challenges

By transforming the matrix of self- and mutual inductances into the respective rotating reference frames, the model complexity can be greatly reduced. It should be noted, that the machine model obtained by these simulations is only valid for operating points that are covered in the model parametrization. For the results presented here, only $\vec{i}_{(5)}$ was considered, as it is the primary torque producing system for the demonstrator machine.

While accurate control of the machine should force other current components to zero, development of these control algorithms might require a machine model that can accurately respond to other current components. Higher harmonics of the fundamental electrical frequency, which might also be injected to influence torque or acoustic characteristics, may also be considered. For the demonstrator, the flux linkage has relevant components in the 3rd, 5th and 9th harmonic of the fundamental electrical frequency. These components correspond to the 15th, 25th and 45th harmonic of the mechanical frequency and are projected into the systems $\alpha\beta^{(3)}$, $\alpha\beta^{(1)}$ and $\alpha\beta^{(3)}$ respectively according to Table 1. If significant current is desired in these systems, additional operating points in the corresponding systems can be considered.

Further current components might be required to accurately model fault conditions, e.g., a significant offset in the common-mode system by a current $i_{dq}^{(0)}$. Hence, a common-mode free pulse width modulation is a key requirement. Machines with a star-connected winding are naturally not susceptible to this current component.

6. Conclusion

The challenges of modeling multi-phase machines have been outlined on the basis of a generic electrical model. Transformations of stator values into stationary $\alpha\beta$- and rotating $dq$-reference frames for any symmetrical machine have been presented and visualized for the example demonstrator machine. With the aid of these transformations, a suitable reduction of the input parameters to valid operating points for the model parametrization has been performed. This allows the application of FEM analysis to derive model parameters. The results of the FEM analysis for a prototype machine are presented and the model validity is discussed. The low common-mode inductance can be considered a key challenge for the control of this type of machine, as even very small common-mode voltage components may induce high common-mode currents $i_{dq}^{(0)}$. Hence, a common-mode free pulse width modulation is a key requirement. Machines with a star-connected winding are naturally not susceptible to this current component.

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