Simplified Speed-Sensorless Vector Control for Induction Motors and Stability Analysis

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This paper presents a new simplified speed-sensorless vector control method for induction motors. The output voltage of the d-axis PI current controller with a decoupling control is used to compute the flux angle and to control the speed in correspondence with its reference. System stability is discussed by the root loci computed from a linear model. The effectiveness of the proposed method is demonstrated by nonlinear simulations and experiments.

Keywords: speed-sensorless vector control, induction motor, root loci, linear model, stability analysis

1. Introduction

Speed-sensorless vector control of induction motors (IM) allows high performance control of torque and speed and the system is widely used. Many model reference adaptive system (MRAS) based methods are studied. Representative speed estimation schemes use flux simulator (11), full-order observer (3), reduced-order observer (10), or sliding-mode full-order observer (4). Manipulated design of the observers can improve the stability of the sensorless system even at low speed regenerating operation. However, the configurations of these systems are relatively complicated. It is because the MRAS based methods need state observer and many PI controllers (d-q currents, speed and speed estimation). On the other hand, some simplified speed sensorless vector control methods are proposed. Simplifying the system configuration by removing the current regulators is proposed (11), and the stability is improved by adding a flux stabilizing controller using derivative of magnetic current (12). However, these papers have no information about the stability of regenerating mode. A sensorless method using the induced d- and q-axis voltages obtained by a voltage model has been proposed (13), and a similar method is applied to railway vehicle traction (10). However, the stable region is not clear in these papers. Furthermore, a primary flux control method is proposed (15) and the stability is improved at regenerating mode (16). In this paper, we propose a new simplified speed-sensorless vector control method of IM based on rotor flux (15)(16). By considering a flux vector obtained by a voltage model in which the derivative term is neglected and a flux vector of a current model, the flux angle is estimated so as to align these vectors. Since the output voltage of d-axis PI current controller is used for both the flux estimation and speed control (q-axis voltage control), the system is simplified and stabilized at regenerating mode. This scheme is not reported in conventional simplified methods. A linear model of the proposed system is derived in state space equation. System stability is discussed by showing root loci of the linear model. By virtue of the stability analysis, we design the parameters of controller. Transient responses of the proposed system are demonstrated by nonlinear simulation and experiment.

2. Proposed Method

The d-q voltage model and current model of the induction motor which are used for the derivation of proposed system are described by (1)–(4) as follows:

voltage model:

\[ e^*_{sd} = (R^*_d + \sigma L^*_d)p^*_d - \omega^* \sigma L^*_s i^*_{sq} + \frac{M^p}{L^s} \psi^p_{rd} - \frac{\omega^* M^p}{L^s} \psi^p_{rq} \]

\[ e^*_{sq} = \omega^* \sigma L^*_s i^*_{rd} + (R^*_q + \sigma L^*_q)p^*_q + \frac{\omega^* M^p}{L^q} \psi^p_{rd} + \frac{M^p}{L^q} \psi^p_{rq} \]

current model:

\[ 0 = \frac{M^p}{\tau^r} \dot{i}^*_{rd} + \left( \frac{1}{\tau^r} + p \right) \psi^*_{rd} - (\omega^* - \hat{\omega}_r) \psi^*_{rq} \]

\[ 0 = -\frac{M^p}{\tau^r} \dot{i}^*_{rq} + (\omega^* - \hat{\omega}_r) \psi^*_{rd} + \left( \frac{1}{\tau^r} + p \right) \psi^*_{rq} \]

where, \( p = d/dt, \sigma = 1 - M^2/(L^s L^q), \tau^r_\omega = L^r/R^r_\psi \)

The \( \hat{\omega}_r \) is the estimated speed. We define the d-q axis which rotates synchronously with the rotor flux of the current model so as to satisfy that \( \psi^*_{rq} = 0 \) as shown in Fig. 1. By assuming that the d-axis current command \( i^*_{rd} \) is constant, we obtain (5) and (6) from (3) and (4) respectively.

\[ \psi^*_{rd} = M^p i^*_{rd} \]  
\[ \omega^* = \hat{\omega}_r + \omega_e \]

where, \( \omega_e = \frac{1}{\tau^r_{rd}} \)
where, $\omega$ as follows:

Then, from (7) we have

$$e_{sd} = R_{sd}^\prime i_{sd} - \omega^* \sigma L_s i_{sq} + e_{sd}'$$  \hspace{1cm} (7)

$$e_{sq} = \omega^* \sigma L_s i_{sd}^\prime + R_{sq}^\prime i_{sq} + \omega^* \sigma L_s i_{sd}^\prime$$  \hspace{1cm} (8)

In Fig. 1, the rotor flux space vector obtained by the voltage model is denoted by $\psi_{r_e}$. When the rotor speed is unknown, the voltage model flux $\psi_{r_e}$ can be used to estimate the flux direction. The flux angle $\theta^*$ is adjusted by changing $\omega^*$ such that the flux $\psi_{r_e}$ becomes zero. When $\psi_{r_e}^r > 0$ and $\psi_{r_e}^q$ is leading than $d$-axis as shown in (a), the controller must increase the value of $\omega^*$. When $\psi_{r_e}^r < 0$ and $\psi_{r_e}^q$ is lagging than $d$-axis as shown in (b), the controller must decrease the value of $\omega^*$. We propose a speed sensorless vector control system as shown in Fig. 2.

We define the induced voltage as

$$e_{sq} = \frac{\omega^* M}{L_s} \psi_{r_e}$$  \hspace{1cm} (9)

Then, from (7) we have

$$e_{sd} = R_{sd}^\prime i_{sd} - \omega^* \sigma L_s i_{sq} + e_{sd}'$$  \hspace{1cm} (10)

The $e_{sd}'$ is obtained by the output of $d$-axis PI current controller with a decoupling control as shown in Fig. 2.\textsuperscript{19}. We estimate the $\omega^*$ by using (9) to converge the $q$-axis flux zero\textsuperscript{17} as follows:

$$\omega^* = \omega_{sd}^* + \omega_{sp} - \omega_{sm}$$  \hspace{1cm} (11)

where, $\omega_{sd} = K_{sd} e_{sd}'$, $K_{sp} = \text{sign}(\omega^*)|K_{sp}|$.

The sign($\omega^*$) is 1 when $\omega^* > 0$ and -1 when $\omega^* < 0$.

By assuming that $\psi_{r_e}^q$ is equal to $\psi_{r_e}^r$, we compute $e_{sq}'$ as follows

$$e_{sq}' = (\omega^* + \omega_{sp} + \omega_{sm}) L_{r_e} i_{sd}^\prime$$  \hspace{1cm} (12)

where, $\omega^* = \frac{K_{sp}}{L_{r_e}} e_{sd}'$  \hspace{1cm} (13)

The term $\omega^*$ compensates the $q$-axis stator resistance voltage drop in steady-state condition. By the integral control of (13), $e_{sq}'$ becomes zero in steady-state, therefore, the actual speed $\omega_{r}$ is equal to the reference speed $\omega_{r}^*$ when motor constants are correct. From (6) and (11), we have

$$\omega_{sd} = \omega_{r}^* - \Delta \omega_r$$  \hspace{1cm} (14)

Therefore, the speed control is accomplished by (13) even in transient state.

In proposed system of Fig. 2, any observer is not used and $d$-axis current controller (PI) and flux frequency controller (P) and speed controller (I) are used. Therefore, the proposed system is much simpler than conventional observer based systems.

3. Stability Analysis

To describe the IM shown in Fig. 2, we chose the $d-q$ axis which rotates synchronously with $\theta^*$. Then, well-known induction motor $d-q$ model is obtained. In this paper, the following assumptions are made:

1) Voltage control is ideal, therefore

$$e_{ad} = e_{d}^r, \hspace{0.5cm} e_{sd} = e_{d}^r$$  \hspace{1cm} (15)

2) The machine constants of controller are equal to those of actual machine.

The state variable $e_{cd}$ is defined as

$$pe_{cd} = K_{i} (\hat{i}_{sd} - i_{sd})$$  \hspace{1cm} (16)

The equation of motion is described as

$$\frac{2}{P} \int \omega_{r}^* \omega_{r} = \frac{P M}{2 L_{r}} (i_{sq} \psi_{r}^d - i_{sd} \psi_{r}^q) - T_{L}$$  \hspace{1cm} (17)

where, $P$: number of poles, $T_{L}$: load torque.

By using (9)–(13), (15)–(17), and the d-q model of IM\textsuperscript{17}, the following non-linear state equation is obtained.

$$px = f(x, \dot{x}, \omega_{r}^*, T_{L})$$  \hspace{1cm} (18)

where,

$$x = [i_{sd}, i_{sq}, \psi_{r}^d, \psi_{r}^q, \omega_{e}, e_{cd}, \omega_{e}]^T$$  \hspace{1cm} (19)

Transient responses are computed by solving (18).

Taking a small perturbation of state variables at steady-state operating point, a linear model of proposed system is derived from (18). A linear model of the proposed system is derived in form (Refer to the Appendix.)

$$pA\Delta x = A\Delta x + B\Delta \omega_{e} + B_{L}\Delta T_{L}$$  \hspace{1cm} (20)

where, $\Delta x = [\Delta i_{sd}, \Delta i_{sq}, \Delta \psi_{r}^d, \Delta \psi_{r}^q, \Delta \omega_{e}, \Delta e_{cd}, \Delta \omega_{e}]^T$.

System stability is discussed by using root loci by computing the eigenvalues of the matrix $A$. We confirmed that the transient responses of linear model are close to those of the nonlinear model during small displacements about a steady state.
Therefore the derivation of linear model is reliable.

System is stable when all eigenvalues of the matrix $A$ of linear model (20) lie inside the left half of $s$-plane.

The $d$-axis PI current controller is designed as $K_p = 14.7$, $K_i = 3395$ having cut off frequency $1500$ rad/s. The control parameters $K_{\omega}$ and $T_{\omega}$ are studied by using the root loci. As the result explained below, we have selected $|K_{\omega}| = 5.0$ and $T_{\omega} = 0.05$ as designed parameters.

The constants of induction motor used for stability analysis are: number of poles $P = 4$, stator resistant $R_s = 1.54 \Omega$, rotor resistance $R_r = 0.787 \Omega$, stator and rotor inductance $L_s = L_r = 0.115 \text{H}$, mutual inductance $M = 0.11 \text{H}$, and moment of inertia including load $J = 0.0126 \text{kg-m}^2$.

Figure 3 shows the trajectories of dominant poles at speed command $50 \text{ min}^{-1}$ for the change of slip speed $N_{sl}$. Slip speed $80 \text{ min}^{-1}$ almost corresponds to the rated torque. Although the system is unstable at plugging region in which the slip speed is less than $-50 \text{ min}^{-1}$, the system is stable at regenerating region when $-50 \text{ min}^{-1} < N_{sl} < 0 \text{ min}^{-1}$.

When the speed commands are $100 \text{ min}^{-1}$, $1000 \text{ min}^{-1}$ and $1500 \text{ min}^{-1}$, the trajectories are shown in Fig. 4, Fig. 5 and Fig. 6 respectively. It is observed that the system is stable both in regenerating and in motoring operations. The roots move left side of $s$-plane and the system becomes more stable as the speed command increases.

Figures 7 and 8 shows the root loci when the gain $|K_{\omega}| = 2.0$ at speed $100 \text{ min}^{-1}$ and $1500 \text{ min}^{-1}$ respectively. It is observed that the system becomes unstable (Fig. 7) or weakly damped (Fig. 8) in regenerating region when $K_{\omega}$ is small. It is noted that small value of $|K_{\omega}|$ can be relatively acceptable in motoring region.

Figure 9 shows the unstable region when $|K_{\omega}| = 5.0$ and $T_{\omega} = 0.05$ for the changes of speed command and slip speed. The shaded area means unstable region. The unstable region is observed in plugging region and very low speed motoring region. However, all regenerating region is stable.

Figure 10 shows the unstable region when $|K_{\omega}| = 5.0$ and $T_{\omega} = 0.2$. Although the large value of $T_{\omega}$ causes unstable region of regenerating operation, the stable region of motoring and plugging operations is expanded. So if we change the
Fig. 7. Trajectories of poles at speed $N^*_r = 100 \text{ min}^{-1}$ for the change of slip speed with $|K_\omega| = 2.0$ and $T_\omega = 0.05$.

Fig. 8. Trajectories of poles at speed $N^*_r = 1500 \text{ min}^{-1}$ for the change of slip speed with $|K_\omega| = 2.0$ and $T_\omega = 0.05$.

Fig. 9. Unstable region with parameters $|K_\omega| = 5.0$ and $T_\omega = 0.05$.

Fig. 10. Unstable region with parameters $|K_\omega| = 5.0$ and $T_\omega = 0.2$.

Fig. 11. Unstable transient responses with parameters $|K_\omega| = 5.0$, $T_\omega = 0.05$ and $T_L = -4.0 \text{ N-m}$ for the step change of speed command from $20 \text{ min}^{-1}$ to $22 \text{ min}^{-1}$.

As the results of stability analysis, large value of $|K_\omega|$ stabilize the system. Too small value of $T_\omega$ increases the unstable region in motoring mode and too large value of $T_\omega$ increases it in regenerating mode.

Figure 11 shows the transient responses when the system is operated at unstable region in Fig. 9 for the step change of speed command from $20 \text{ min}^{-1}$ to $22 \text{ min}^{-1}$ with parameters $|K_\omega| = 5.0$ and $T_\omega = 0.05$ and $T_L = -4.0 \text{ N-m}$ ($N_{sl} = -35.2 \text{ min}^{-1}$ at $t = 0$). Both the non-linear model of (18) and the linear model of (20) have similar responses in short period after the step change of speed reference.

4. Experimental Results

The proposed control system is implemented by a DSP (TMS320C32)-based PWM inverter. Sampling period used in this system is $200 \mu s$. Because of the dead time and the non-ideal features of IGBT influence on the output voltage of the inverter, a compensating algorithm is developed for the experimental system. The tested induction machine is $1.5 \text{ kW}$ and the constants have been described in chapter 3.

Figure 12 shows the experimental results for the step change of speed command from $20 \text{ min}^{-1}$ to $22 \text{ min}^{-1}$.
change of the speed command \( N_r^* \) in motoring operations. In case (a), \( N_r^* \) is stepped from 50 min\(^{-1}\) to 150 min\(^{-1}\) and then down to 50 min\(^{-1}\). In cases (b) and (c), the speed commands are similarly stepped as 500→600→500 min\(^{-1}\) and 1000→1100→1000 min\(^{-1}\) respectively. \( N_r \) is actual motor speed (\( \omega_r \)) and \( N^* \) is synchronous speed (\( \omega^* \)). The control parameters are set as \(|K_\omega| = 5.0\) and \( T_\omega = 0.05\). The load torque \( T_L \) is set to 4.0 N·m (half of rated torque) by a DC generator connected to IM. In any cases, quick responses of \( N_r \) are obtained and the changes of torque current \( i_{\text{dq}} \) are similar to those of vector control system with speed sensor. The non-linear simulation results computed by (18) are shown in Fig. 13. Figures 13(a), (b) and (c) corresponds to Figs. 12(a), (b) and (c) respectively. The experimental results agree well
Fig. 14. Transient responses for the step change of speed command (Regenerating operation $|K_{ω}| = 5.0; T_{ω} = 0.05; T_L = -4.0 \text{ N-m}$) respectively. Same change of speed commands are tested under the load torque $T_L = -4.0 \text{ N-m}$. In any cases, quick responses of $N_r$ are obtained. The experimental results agree well with the computed results too. The high frequency ripples of $N^*$ in Fig. 14(c) are smaller than those in Fig. 12(c). The reason for this difference can be considered that the amplitude modulation ratio of regenerating operation is smaller if the voltage

with the computed results except for high frequency ripples. The high frequency ripples of $N^*$ and $e_d^*$ are caused by PWM voltage control in experimental system. However, since the flux angle $θ^*$ is obtained by integrating $ω^*$, the actual rotor speed has little ripples.

Figures 14 and 15 show the experimental results and the non-linear simulation results in regenerating operations respectively. Same change of speed commands are tested under the load torque $T_L = -4.0 \text{ N-m}$. In any cases, quick responses of $N_r$ are obtained. The experimental results agree well with the computed results too. The high frequency ripples of $N^*$ in Fig. 14(c) are smaller than those in Fig. 12(c). The reason for this difference can be considered that the amplitude modulation ratio of regenerating operation is smaller if the voltage
control has allowance under limited DC-bus voltage; the distortion of stator currents is reduced. The larger we choose the gain $|K_a|$, the larger high frequency ripples of $N^*$ are induced. From this point, we have selected the gain $|K_a|$ to 5.0.

5. Conclusions

The conclusions drawn from this study are summarized as follows:

1. We have proposed a new simplified speed-sensorless vector control method of IM.
2. The flux angle computation and the torque and speed control are realized by using the output voltage of d-axis PI current controller with a decoupling control.
3. A linear model is derived and the root loci obtained the model is used to design the control parameters.
4. It is demonstrated that the proposed system can realize stable operation in both motoring and regenerating modes by the result of root loci.

5. The experimental results agree with those of non-linear simulation and the usefulness of proposed method is verified.

References

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