Multirate PWM Control of Precision Stage for Ultrahigh-Speed Nanoscale Positioning

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Motion control techniques are employed for nanoscale positioning in industrial equipment such as numerical control (NC) machine tools and exposure systems. The advanced motion control techniques are based on precise current control. However, speeding up the precise current response causes a serious limitation owing to the carrier period of the inverter. In addition, the position response has to be slower than the current response. In a previous paper, we designed and fabricated an experimental precision stage, achieving novel ultrahigh-speed nanoscale positioning based on multirate pulse width modulation (PWM) control. However, it was difficult to achieve faster and more precise positioning because of the resonance modes of the stage. In this paper, we propose a multirate PWM control in which the resonance mode is considered. Simulations and experiments are performed to demonstrate the advantages of the proposed method.

Keywords: multirate control, high-precision motion control, nanoscale servo system, PWM inverter

1. Introduction

Digital control of servomotors has been a subject of active research owing to the high-speed switching capability of PWM inverters and improved arithmetic processing by digital signal processors (DSPs). Nowadays, motion control techniques are employed for nanoscale positioning in precision mechanical equipment, for example, NC machine tools and exposure systems. High-precision motion control is especially required to achieve nanoscale positioning for exposure system stages. There is considerable research on high-precision positioning for wafer stages (1)–(4). The precision control of an XY table driven by a linear motor, a DC motor with a ball-screw stage, or a piezoactuator has also been widely studied (5)–(9). Feedback control design is described in (5), in which robust control is discussed; in (4) and (6), where nonlinear feedback control is addressed; and in (3) and (7), which focus on iterative learning control.

High-precision motion control is based on precise current control of the motor. The performance of the current control is very important. However, speed-up of the precise current response has a serious limitation because of the inverter’s carrier period. In addition, the position response has to be slower than the current response. There is some research that suggest that field programmable gate arrays (FPGAs) can be developed for motor drives to solve this problem of the limitation of the carrier period (10). Enabling higher speed motion control by using control technology without making the carrier frequency higher has the advantages of reduced cost and low switching loss.

In (11), the authors proposed multirate PWM control and achieved positioning within 20 ms by using a brushless DC motor with a rough encoder. The proposed method has a novel feedforward controller considered with the PWM pulse of the inverter and the current loop.

In this paper, we illustrate an ultrahigh-speed nanoscale positioning system based on the proposed method with an experimental precision stage. The position error is within 100 nm. The target positioning time is 2 ms, which is only 10 times the carrier period. This target was conventionally possible only by using a linear amplifier, which was too bulky and had very low energy efficiency. Herein, we try to achieve this target by using a PWM inverter that is appropriate especially for large-scale industrial positioning stages.

In Sect. 2, the constitution and characteristics of the stage are given. In Sect. 3, multirate PWM position control is described. In Sect. 4, we outline the design of a simple and effective suppression filter for the primary resonance mode to achieve ultrahigh-speed nanoscale positioning. Finally, in Sects. 5 and 6, respectively, simulations and experiments are performed to show the advantages of the proposed method.

2. Nanostage

An experimental high-precision stage was designed and fabricated to research ultrahigh-speed nanoscale positioning. The experimental stage is called the “nanostage” in the following.

2.1 Constitution

Figure 1 shows an overview of the nanostage. The nanostage is driven by a linear motor, and friction is almost zero because of the use of an air guide. Moreover, the nanostage has two linear encoders to measure both the motor part (the drive) and the stage part (the load). To achieve nanoscale positioning, the resolution of the linear encoders is 1 nm/pulse.

The nanostage can be switched between two modes: a rigid mode and a two-inertia mode. In the two-inertia mode, the motor and the stage parts are connected by leaf springs. The
resonance characteristics can be changing by replacing the leaf springs. The nanostage can be switched to the rigid mode because the motor and the stage parts are fixed by plates. In this paper, the nanostage is treated as the rigid mode. By calculating the barycentric position of the stage with upper and lower encoders, the position can be assumed to be the real position of the stage.

2.2 Nanostage Characteristics A proportional-integral current controller was designed based on pole-zero cancellation for the current loop to be a first-order system whose bandwidth was 1 kHz. Table 1 lists the nanostage parameters. Figure 2 shows the frequency response from the current reference to the velocity of the stage. Here, the solid line is the plant model whose primary resonance and antiresonance modes $G_{ab}(s)$ are

$$y(s) = \frac{1}{\tau s + 1} \cdot \frac{1}{Ms + B} \cdot G_{ab}(s), \quad \text{............... (1)}$$

$$G_{ab}(s) = K_p \cdot \frac{s^2 + 2\omega_n\omega_N s + \omega_N^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, \quad \text{............... (2)}$$

where $\tau = 1/(2\pi 1000)$ and $K_p = \omega_n^2/\omega_N^2$. Note that there is a resonance mode despite the stage being in its rigid mode. The resonance mode is considered in Sect. 4.

3. Multirate PWM Position Control

A multirate PWM control system is a kind of perfect tracking control (PTC) system designed for a plant model discretized based on PWM hold. Multirate PWM control is designed by considering a current loop and instantaneous values of the PWM pulse precisely. High-speed positioning can be achieved during several carrier sampling periods that are shorter than the response of the current loop. Perfect tracking was achieved for output voltage control of a single-phase inverter in (13), in which the output voltage tracks to an arbitrary waveform with zero tracking error every $T_r$.

3.1 Discrete Model Based on PWM Hold To discretize a plant model, a zero order hold is generally applied. However, for the single-phase inverter (or a four-quadrant chopper) (Fig. 3) that actuates a DC motor, the inverter cannot output an arbitrary output voltage $V[k]$ but can output only 0 or $\pm E [V]$, as shown in Fig. 4. Therefore, to control instantaneous values precisely, the zero order hold is unsuitable because the precise discrete model is based on PWM hold of Fig. 4. The plant model of a motor actuated by an inverter can be discretized based on PWM hold as follows.

A continuous-time state equation for a plant can be represented by

$$\begin{cases} x(t) = A_x x(t) + b_x u(t), \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdot (3) \\ y(t) = c_x x(t) \end{cases}$$

The conventional discrete model with a zero order hold in which the input $u[k]$ is assumed to be a constant value $V[k]$ during one control period $T_u$ is given as

$$\begin{cases} x[k + 1] = A_x x[k] + b_x V[k], \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdot (4) \\ y[k] = c_x x[k], \quad \end{cases}$$

where

$$A_u = e^{A_T T_u}, \quad b_u = \int_0^{T_u} e^{A_T \tau} b \, d\tau, \quad c_u = c_v \quad \text{............ (5)}$$

Because of this assumption, (4) is an approximate model that evaluates the average value during $T_u$.

In contrast, the precise discrete model with PWM hold in which the input $u[k]$ is the switching time $\Delta T[k]$ can be given as

$$\begin{cases} x[k + 1] = A_x x[k] + b_x \Delta T[k], \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdot (6) \\ y[k] = c_x x[k] \end{cases}$$
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(Recent and up-to-date text)

3.2 Perfect Tracking Control

The perfect tracking control system comprises a two-degree-of-freedom control system, as shown in Fig. 5. This system has two samplers \( S(T_s) \) and \( S(T_p) \) for the reference signal \( r(t) \) and the output \( y(t) \) and one holder for the input \( u(t) \). Therefore, there exist sampling periods \( T_r \), \( T_y \), and \( T_a \), which represent the periods of \( r(t), y(t), \) and \( u(t) \), respectively. PTC applies the multirate feedforward control in which the control input \( u(t) \) is changed \( n \) times during one sampling period \( T_r \) of reference input \( r(t) \), as shown in Fig. 6. Here, \( n \) is the plant order. \( H_M \) in Fig. 5 is the multirate holder, which outputs the input \( u(t) = u_0[i], i T_r + (k - 1) T_a \leq t < i T_r + k T_a \), where the vector \( u_0[i] = [u_0[i], \ldots, u_0[i]]^T \) is generated at every long sampling period \( T_r \).

From (6), the matrices \( A, B, C, \) and \( D \) are given as

\[
\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_0 & A_1 & \cdots & A_n \\ B & 0 & \cdots & 0 \\ c_c & c_e A_{0,1} & \cdots & c_e A_{0,n} \end{bmatrix}, \quad \cdots \cdots \cdots \cdots (8)
\]

Since matrix \( B \) of (8) is nonsingular, PTC can be designed as

\[
u_0[i] = B^{-1} (I - z^{-1} A) x_d[i + 1]
\]

\[
ym_0[i] = z^{-1} C x_d[i + 1] + D u_0[i], \quad \cdots \cdots \cdots \cdots (10)
\]

Equation (9) is the stable inverse system of the plant as the references are state variables \( x_d[k + 1] \). Therefore, perfect tracking is assured during the sampling period \( T_r \).

Moreover, the feedback control \( C_{z}[z] \) suppresses the error between the output \( y(k) \) and the nominal output \( y_0[k] \) to assure robustness only when disturbances or plant variations exist.

3.3 Original PTC System

Conventional PTC is designed for a third-order plant for which the current loop is assumed to be an ideal first-order system. This method will be referred to as “Conventional 2.” The design method for a second-order plant with the current loop ignored will be referred to as “Conventional 1” \((12)\).

The “Conventional 1” plant is described by

\[
y_{\text{ref}} = \frac{1}{M s^2 + B s} \cdot \cdots \cdots \cdots \cdots (11)
\]

In contrast, the “Conventional 2” plant is modeled as

\[
y_{\text{ref}} = \frac{1}{\tau s + 1} \cdot \frac{1}{M s^2 + B s} \cdot \cdots \cdots \cdots \cdots (12)
\]

These plants are discretized by using the conventional zero order hold model \((4)\), and then the PTC system is designed.

3.4 Constitution of Multirate PWM Position Control System

The \( q \)-axis model of the nanostage operating in the rigid mode is shown in Fig. 7. A control system can be designed for the model with vector control. The transfer function from \( V_{\text{inv}} \) to \( y \) is described by

\[
y = \frac{V_{\text{inv}}}{\tau s + 1} \frac{K_i}{M s^2 + (MR + LB)s^2 + (BR + K_K)s} \cdot \cdots \cdots \cdots \cdots (13)
\]

The controllable canonical form of (13) is given by

\[
\begin{bmatrix} A_c & b_c \\ c_c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{BR + K_K}{M} & -\frac{MR + LB}{M} & K_K \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \cdots \cdots \cdots \cdots (14)
\]

according to (3), where \( x = [y \; \dot{y} \; \ddot{y}]^T \).

In addition, the transfer function from \( V_{\text{inv}} \) to \( i \) is described by

\[
i \cdot V_{\text{inv}} = \frac{Ms^2 + B s}{MLs^3 + (MR + LB)s^3 + (BR + K_K)s} \cdot \cdots \cdots \cdots \cdots (15)
\]

The output equation of (15) can be represented by

\[
y = c_e^T x, \quad c_e = [0 \; \frac{B}{K} \; \frac{K}{K} \; \cdots] \cdot \cdots \cdots \cdots \cdots (16)
\]

A multirate PWM position control system can be designed, as depicted in Fig. 8, based on the precise PWM hold model \((6)\). The current controller \( C_P[z] \) and the position feedback controller \( C_{z}[z] \) operate only when errors between the nominal outputs and the actual outputs exist.

3.5 Input Generation from the Three-phase Inverter

For an AC motor, like a linear motor, the input comprises
the switching times $\Delta T_d$ and $\Delta T_q$ because the control system is designed by using the $dq$ model. However, to apply a three-phase inverter, input for the PWM pulse of the three-phase system has to be generated. In this section, we explain the generation method. Herein, coordinate transform matrices are represented as absolute transforms.

The input voltage of the three-phase inverter is defined as $V_{dc}$. The discrete model of (6) for the $dq$ system is designed as $E = V_{dc}$ in (7). $\Delta T_d$ and $\Delta T_q$ are transformed into $\Delta T_\alpha$ and $\Delta T_\beta$ by the $dq/2$ phase transform as follows:

$$
\begin{bmatrix}
\Delta T_\alpha \\
\Delta T_\beta
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\Delta T_d \\
\Delta T_q
\end{bmatrix}, \quad \cdots \cdots \quad (17)
$$

Then, $\Delta T_\alpha$ and $\Delta T_\beta$ are in $\alpha\beta$ coordinates, transformed into $\Delta T_{\alpha r}$, $\Delta T_{\alpha w}$, and $\Delta T_{\alpha u}$ by the two-phase/three-phase absolute transform. In Fig. 8, the $3/dq$ block represents the transformation from $dq$ coordinates to three-phase coordinates. Decoupling control is also applied, and this cancels the coupling term $L_d i_q$, where $L_q$ is the $q$-axis reluctance and the stage speed $\omega[k]$ is obtained with a differentiator $C_\nu(z) = (1 - z^-1)/T_y$.

Symmetric pulses such as those shown in Fig. 9 are outputted in order to control precisely by using (6). The order of generation of the pulses is demonstrated in the following.

The region is decided by $\Delta T_\alpha$ and $\Delta T_\beta$ (see Fig. 10). The order and the switching times $\sqrt{3}/2 \Delta T_{\alpha j}$ of the output vectors $V_{i,j}$ in each region are decided by Tables 2 and 3. Here, $\sqrt{3}/2$ is the coefficient used to transform two-phase quantities into three-phase ones, and the output order is decided for the number of switching times to be fewer.

For example, the pulses of Fig. 9 are output in region VI. In this case, the switching times, which are $\Delta T_1 = \sqrt{3}/2 \Delta T_i$ and
ΔT_i = \sqrt{3/2} ΔT_j, are shared, as indicated in Table 3.

3.6 Design of Feedback Control A proportional-integral-derivative position controller was designed so that the closed-loop poles of the position loop were assigned to 100 Hz. The sensitivity characteristic is shown in Fig. 11. The steady-state position error is shown in Fig. 12. The 3σ standard deviation of the steady-state position error is 59.7 nm. In Fig. 12, the error is seen to have a sharp peak at 50 Hz, which is the measurement noise at the commercial frequency.

4. Resonance Mode Consideration

In general, the plant has resonance modes caused by pitching in positioning of a stage. In a large-scale stage, numerous resonance modes exist with frequencies ranging from several hertz to several kilohertz. Moreover, most of the resonance modes have antiresonance modes. Our nanostage has a primary resonance mode at \( \omega_r = 2\pi (670) \), \( \zeta_r = 0.004 \) and an antiresonance mode at \( \omega_a = 2\pi (690) \), \( \zeta_a = 0.004 \) (Fig. 2).

The characteristic \( G_{vb}(s) \) of the primary resonance mode and the antiresonance mode is represented by (2), which is the biproper minimum phase system. Therefore, a stable inverse model of \( G_{vb}(s) \) can be designed. The inverse model is defined as the vibration suppression filter (VSF)

\[
VSF(s) = G_{vb}^{-1}(s). \quad (18)
\]

VSF(z), which is discretized by shorter period \( T_d \) with a prewarp-Tustin transformation, is applied in the multirate PWM position control system depicted in Fig. 13. The resonance mode can be suppressed with the feedforward controller.

By using this method, one can suppress the resonance mode very well because the inverse model of the resonance modes is applied. It is not necessary to give the target trajectory beforehand, which has a small power spectrum at the primary vibration frequency, as was done by minimizing the primary vibration trajectory in (15).

Moreover, the PTC method ensures that output errors are exactly zero at every longer period \( T_r \) under ideal conditions in which the plant has no modeling error and no disturbance. The period \( T_r \) of the system-applied VSF is \( N_n \) periods \( (n_rT_r) \) shorter than that of vibration suppression PTC in (16), where \( n_r \) is the order of considered resonance modes. This difference is very important for positioning during several

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**Table 4. Sampling periods and target trajectory**

<table>
<thead>
<tr>
<th>Spec. 1</th>
<th>Tr (ms)</th>
<th>Tu (ms)</th>
<th>Ty (ms)</th>
<th>td (ms)</th>
<th>Are f (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional 1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
<td>20</td>
<td>1.5</td>
</tr>
<tr>
<td>Conventional 2</td>
<td>0.6</td>
<td>0.2</td>
<td>0.2</td>
<td>20</td>
<td>1.5</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.6</td>
<td>0.2</td>
<td>0.2</td>
<td>20</td>
<td>1.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spec. 2</th>
<th>Tr (ms)</th>
<th>Tu (ms)</th>
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</tr>
<tr>
<td>Conventional 2</td>
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<td>0.2</td>
<td>0.2</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.6</td>
<td>0.2</td>
<td>0.2</td>
<td>2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

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**Fig. 13. Diagram of multirate PWM position control system with VSF**

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**Fig. 14. Simulation results**

Simulation of Spec. 1.
Simulation 1 of Spec. 2.
Simulation 2 of Spec. 2.
sampling periods.

As previously mentioned, vibration suppression control can be achieved easily and efficiently by using \( G_{\text{vb}}(s) \). This method is very effective if \( G_{\text{vb}}(s) \) has only small variations in parameters such as stepper or scanner stages of exposure systems in which the environmental temperature is controlled and the stage mass is constant. For applications in which \( G_{\text{vb}}(s) \) has large parameter variation, real-time parameter estimation \(^{(17)}\) is needed to recover the best tracking performance. However, without using this on-line estimation, sufficient robustness might be obtained by setting the nominal damping parameter \( \zeta_a \) in (2) a little larger than the assumed value because VSF \( (s) \) can exhibit notch-like characteristics.

5. Simulations

Using the specifications of Table 4, we performed simulations of ultrahigh-speed nanoscale positioning. In the table, \( t_d \) is the positioning time and \( A_{\text{ref}} \) is the target position. The target position trajectory is based on a fifth-order polynomial. Here, the position error tolerance is defined as 100 nm. Conventional multirate PWM position control was compared with original PTC based on the approximated discrete model by using zero order hold \((4)\). The input voltage \( V_{\text{dc}} \) of the three-phase inverter is 60 V.

Spec. 1 simulation results are shown in Fig. 14(a) and (b). All methods can be seen to achieve the position error tolerance. In particular, “Proposed” is better in transient response because of the precise discrete model \((6)\).

The simulation results for Spec. 2 without VSF are shown in Fig. 14(c) and (d). With Spec. 2’s 2-ms positioning, all methods can also achieve the position error tolerance as obtained in Spec. 1. Given the more precise plant model, the transient response is better. However, residual vibrations exist in all methods because of the resonance mode at 670 Hz.

In Fig. 14(e) and (f) with VSF, good positioning can be achieved without residual vibrations. “Proposed + VSF” can achieve the best tracking control during the transient period and after positioning. These PWM pulses along the \( q \) axis are shown in Fig. 15. The difference between these PWM pulses occurred because of the modeling error in \((4)\).

In our simulations, only the primary resonance and the anti-resonance mode are considered. In a real stage, many high-order resonance modes exist. These resonance modes influence the positioning in experiments.

6. Experiments

Using the specifications of Table 4, we next performed ultrahigh-speed nanoscale positioning experiments.

The experimental results for Spec. 1 are shown in Fig. 16(a) and (b). All methods can be seen to achieve the position error tolerance.

The experimental results for Spec. 2 without VSF are shown in Fig. 16(c) and (d). With Spec. 2’s 2-ms positioning, “Proposed” and “Conventional 2” can achieve the position error tolerance. In Fig. 16(d), one can see that “Conventional
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1” has a large positive error around $t \approx 1$ ms, as was found in Fig. 14(d). This means that the position is delayed from the target trajectory because the current loop has been neglected in “Conventional 1.” In contrast, “Proposed” and “Conventional 2” exhibit smaller errors during transient periods because the current loop is taken into account. However, residual vibrations exist in all methods because of the resonance mode at 670 Hz.

In Fig. 16(e) and (f) with VSF, one can see that good positioning can be achieved without residual vibrations, as found in simulations. Moreover, “Proposed” with the precise zero order hold model (4) is better than “Conventional 2” with the conventional model (6) because the precise PWM hold and the current loop are considered in the same way as in the simulation. As shown in the measurement of Fig. 2, the largest resonance mode exists around 10–20 kHz. However, simulation.

In this paper, an ultrahigh-speed nanoscale positioning control system is developed with an instantaneous precise model of PWM hold. We also propose multirate PWM control by considering the resonance mode for more precise positioning. Simulations and experiments are performed to show the advantages of the proposed method. The proposed method can achieve vibration suppression control easily and efficiently by taking into account the antiresonance mode. The relation between machine and antiresonance characteristics for the design of precise stages would be a worthy subject of future research.

7. Conclusion

In this paper, an ultrahigh-speed nanoscale positioning control system is developed with an instantaneous precise model of PWM hold. We also propose multirate PWM control by considering the resonance mode for more precise positioning. Simulations and experiments are performed to show the advantages of the proposed method. The proposed method can achieve vibration suppression control easily and efficiently by taking into account the antiresonance mode. The relation between machine and antiresonance characteristics for the design of precise stages would be a worthy subject of future research.

References


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