Switched Capacitor Discrete Control of Class E Amplifier to Achieve Nominal Operation

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This paper introduces the concept of a discrete control method of class E amplifiers to achieve nominal operation for varying load resistance. In the proposed method, shunt capacitance and output resonant capacitance are varied electronically by setting the status of switches that are connected to the capacitances. The proposed method can maintain nominal operation, i.e., zero-voltage switching and zero-derivative switching (ZVS/ZDS), both when the load resistance is higher and lower than the designed value. Design equations are presented considering the parasitic capacitance of a MOSFET switch. A design example is also provided in the paper. Finally, the operation of the proposed circuit is verified through experiments.

Keywords: switching power amplifier, class E, nominal operation

1. Introduction

Although class E amplifiers are highly efficient at operating frequencies above 10 MHz, they are vulnerable to variations in load impedance because nominal operation is impossible when the load impedance differs from its designed value(1)–(6). Because the relationship between the circuit elements of a class E amplifier and its operating waveform is complicated and tightly coupled, frequency modulation (FM), which uses the narrow band frequency characteristics of the output filter, has typically been used to control such devices. However, this control method sacrifices nominal operation as high efficiency cannot be achieved for variable load resistance. The nominal condition of a class E amplifier is often called the “one point condition”, meaning that all element parameters must coincide with their designed values simultaneously; if this is not so, the amplifier will not be able to maintain nominal operation. Because the circuit waveform of a class E amplifier is intricately related to each element parameter, simple control methods—such as pulse width modulation (PWM), FM, or pulse frequency modulation (PFM)—in which single parameters are manipulated, are unable to maintain nominal operation when the load impedance is varied from its designed value(5)–(7). In such circumstances, it is necessary to maneuver several circuit parameters simultaneously.

(8)–(10) introduced discrete control methods for class E amplifier operation. (8) and (9) describe so-called discrete pulse width modulation (DPWM) methods in which the duty ratio is altered discretely using a digital control circuit. (10) introduced a new discrete control approach in which plural

shunt capacitors that can be connected and disconnected by switches are connected in parallel to the transistor. Using various combinations of plural capacitance, the required shunt capacitance can then be approximately achieved for a given load resistance. Although this method allows for the manipulation of shunt capacitance via electronic signaling, it remains unsuitable over a wide range of load resistance for two primary reasons. First, as only sub-nominal operation can be maintained when the load resistance is altered, only zero-voltage switching (ZVS) can be achieved. Therefore, while switching loss can be eliminated, power loss owing to the transition time of the switching device cannot; this causes a deterioration of efficiency at high operating frequencies. Second, sub-nominal operation cannot be achieved when the load resistance is higher than its designed value; in order to achieve sub-nominal operation, the circuit must be operated at a lower load resistance than the resistance at nominal operation. This means that the highest efficiency can be achieved only at the highest load resistance, even though such situations do not always happen and it is difficult to predict the highest possible load resistance with high precision during the design stage. The problem with the control method in(10) is that only one circuit parameter is manipulated in order to address load variations.

In this paper, we review the operational equation of a class E amplifier outside of its designed condition. By rearranging the design equations, we develop a novel discrete control method to achieve nominal operation under a wide range of load resistances. In this method, both the shunt and resonant capacitances are varied by connecting and disconnecting multiple capacitors in parallel using switched capacitor legs. It should be noted that this method does not involve the manipulation of the resonant inductance value, which is difficult to do electronically. By using the proposed method, it is possible to approximately achieve nominal operation, i.e.,

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ZVS and zero-derivative switching (ZDS), even if the load resistance is higher than the designed value. Nominal operation can be also achieved if the load resistance is lower than the designed value. The proposed method is designed based on the analytical equation of the voltage waveforms of a class E amplifier. Correspondingly, the voltage waveforms are conserved even when the load resistance is changed from the designed value; i.e., the amplitudes of the output and peak switch voltages remain at their designed values. In this way, use of the proposed method can avert the destruction of a switching device when excessively high voltage is imposed upon its transistor during significant load resistance changes. The proposed concept of control operation was verified using a PSIM simulation. In this study, it was shown that nominal operation (ZVS/ZDS) can be approximately obtained for several values of load resistance; in each case, the switch and output voltage waveforms were approximately equal. Practical circuit design equations that take into account MOSFET parasitic capacitance were developed and a design example based on the parasitic-capacitance effect was experimentally demonstrated. The experimental results indicate that nominal operation can be achieved at two different load resistances.

2. Analytical Result of Class E Amplifier Outside the Designed Condition

Figure 1 shows the basic circuit design of a zero voltage switching type class E amplifier. Figure 2 shows the waveforms of the switch (drain) voltage and the output voltage of the class E amplifier when nominal operation is achieved. As seen in Fig. 2, if the class E amplifier achieves nominal operation, both the amplitude and slope of the switch voltage waveform will be zero when the switch turns on. In this circuit, the switch is in the ON-state for $0 < \theta \leq 2\pi D$ and the OFF-state for $2\pi D < \theta \leq 2\pi$, where $D$ is the duty ratio ($0 < D < 1$) and $\theta = \omega t$ is the phase angle, with $\omega$ denoting the angular switching frequency and $t$ the time. The steady-state operation of a class E amplifier was analyzed in (7), where it was shown that the amplitudes of the output and input currents $I_m$ and $I_{DD}$, respectively, can be described as

$$I_m = \frac{2U_1}{U_3} \quad \text{(1)}$$

$$I_{DD} = \frac{\pi \omega C_f}{2(1-D)^2} + \frac{U_1(U_2/U_3)V_{DD}}{\pi^2} \quad \text{(2)}$$

where

$$U_1 = \left[-2\pi(1-D) - \sin 2\pi D\right] \cos \phi + (1 - \cos 2\pi D) \sin \phi \quad \text{(3)}$$

$$U_2 = \left[-2\pi(1-D) \cos 2\pi D - \sin 2\pi D\right] \cos \phi$$

$$+ \left[2\pi(1-D) \sin 2\pi D - \cos 2\pi D + 1\right] \sin \phi \quad \text{(4)}$$

$$U_3 = 2\pi \omega C_1 + (\cos 2\pi D - 1)[\cos(2\pi D + 2\phi) - 1] \quad \text{(5)}$$

$$\phi^c = \tan^{-1} \frac{A R}{B} \quad \text{(6)}$$

$$A = [2\pi(1-D) + \sin 2\pi D]$$

$$\times [2\pi(D - 1) - 0.5 \sin 4\pi D + 2\pi \omega C_1 X]$$

$$+ (\cos 2\pi D - 1)[2 \cos 2\pi D]$$

$$- 1.5 - 0.5 \cos 4\pi D - 2\pi \omega C_1 R] \quad \text{(7)}$$

$$B = [2\pi(1-D) + \sin 2\pi D]$$

$$\times [0.5 \cos 4\pi D - 0.5 - 2\pi \omega C_1 R]$$

$$+ (\cos 2\pi D - 1)[2 \sin 2\pi D - 2\pi(D - 1)]$$

$$- 0.5 \sin 4\pi D - 2\pi \omega C_1 X] \quad \text{(8)}$$

$$X = \frac{\omega L}{\omega C_f} \quad \text{(9)}$$

The switch- and output-voltage waveforms are described using $I_m$ and $I_{DD}$,

$$v_S = 0 \quad \text{for} \quad 0 < \theta \leq 2\pi D \quad \text{(10)}$$

$$v_S = \frac{1}{\omega C_1} \left[I_{DD}(\theta - 2\pi D) + I_m \cos(\theta + \phi)ight]$$

$$- I_m \cos(2\pi D + \phi)] \quad \text{for} \quad 2\pi D < \theta \leq 2\pi \quad \text{(11)}$$

$$v_o = R I_m \sin(\theta + \phi) \quad \text{(12)}$$

From the above equations, it is seen that the switch voltage $v_S$ is governed by only four parameters: $\omega C_1 R$, $\omega C_1 X$, $D$, and $V_{DD}$. Therefore, if $V_{DD}$ and $D$ are given as constants, only $\omega C_1 R$ and $\omega C_1 X$ govern the switch-voltage waveform. In order to maintain nominal operation, $\omega C_1 R$ and $\omega C_1 X$ should have values matching their respective design values, even if some of their constituent parameters, for instance, $C_1$, $R$, or $X$, change in value.

3. Variation of Shunt Capacitance and Resonant Capacitance to Maintain Nominal Operation

Assume that the class E amplifier satisfies its nominal condition with $R$, $C_1$, $C_f$, $L$, and $\omega$ as shown in Fig. 3. If some $\phi$ is the phase-shift between the driving signal $D_t$ and the output current $i_o$ as shown in Fig. 2. However, as the amplitude of the driving signal does not affect the amplitude of the output current, there is no transadmittance relationship between the two. Therefore, parameters $A$ and $B$ do not have special physical meanings.
Similarly, assume that \( C_1 \) changes to \( C'_1 \) and \( X \) changes to \( X' \). In order to keep \( \omega C_1 R \) and \( \omega C_1 X \) constant, the following conditions should be satisfied:

\[
\omega C'_1 R' = \omega C_1 R \quad \text{and} \quad \omega C'_1 X' = \omega C_1 X \quad \text{constant}. \quad (14)
\]

Because \( R' = aR \),

\[
C'_1 = \frac{C_1}{a} \quad \text{constant}. \quad (16)
\]

Substituting (16) into (15),

\[
X' = aX \quad \text{constant}. \quad (17)
\]

In most cases, it is more difficult to change inductance than capacitance during circuit operation. Therefore, resonant capacitance \( C_f \) is altered instead of resonant inductance \( L \) in the proposed method. The resonant inductance can be equivalently divided into two series-connected inductors, \( L_f \) and \( L_r \), as shown in Fig. 3:

\[
L = L_f + L_r \quad \text{constant}. \quad (18)
\]

\( L_f \) is resonated at the switching frequency

\[
\omega = \frac{1}{\sqrt{C_f L_f}} \quad \text{constant}. \quad (19)
\]

Thus, \( L_r \) assumes the role of the required phase shift reactance:

\[
X = \omega L_r \quad \text{constant}. \quad (20)
\]

Because \( R' = aR \), \( X' \) should be given by \( X' = aX \). Therefore,

\[
L'_r = aL_r \quad \text{constant}. \quad (21)
\]

In order to keep the total inductance \( L \) constant,

\[
L'_f = L - aL_r = L_f + (1 - a)L_r \quad \text{constant}. \quad (22)
\]

Because \( L_f \) has changed to \( L'_f \), \( C_f \) must change to the new value \( C'_f \) in order to maintain resonance at the switching frequency:

\[
\omega = \frac{1}{\sqrt{C'_f L'_f}} = \frac{1}{\sqrt{C_f [L_f + (1 - a)L_r]}} \quad \text{constant}. \quad (23)
\]

Equating (19) and

\[
C'_f = \frac{C_f}{1 + (1 - a)\frac{L_r}{L_f}} \quad \text{constant}. \quad (24)
\]

4. Circuit Configuration and Concept of Proposed Control Method

In the previous section, inductance was kept constant while two capacitances were changed electronically. Several capacitances are connected in parallel. In the proposed setup, each capacitance is connected to a switch in series in a configuration called a switched capacitor leg. By turning on the switch, the capacitance is connected to the main circuit. Because the two capacitances are connected in parallel, the total capacitance is the sum of their ON-state capacitances. In order to change the two capacitances \( C_1 \) and \( C_f \), a circuit configuration as shown in Fig. 5 is proposed. In this circuit, the total capacitance of \( C_1 \) and \( C_f \) is adjusted by combining the switch states of their respective switched capacitor legs.

Figures 6 to 8 show typical configurations for the selected values of load resistances. This circuit was designed to achieve nominal operation at load resistance \( R = 18 \Omega \), switching frequency \( f = 13.56 \text{ MHz} \), and duty ratio \( D = 0.5 \). In this configuration, the shunt capacitance \( C_1 \) and resonant capacitance \( C_f \) are \( C_1 = 118.75 \text{ pF} \) and \( C_f = 76.25 \text{ pF} \), respectively. Figure 6(b) shows simulated waveforms of the switch voltage \( v_S \) and output voltage \( v_o \). As seen in Fig. 6(b), nominal operation can be achieved in this case.

Figure 7(a) shows the circuit configuration at \( R = 15 \Omega \). Here, the load resistance is lower than its designed value. In this configuration, the shunt capacitance \( C_1 \) and resonant capacitance \( C_f \) are \( C_1 = 142.75 \text{ pF} \) and \( C_f = 73.125 \text{ pF} \), respectively. Figure 7(b) shows simulated waveforms of the switch voltage \( v_S \) and output voltage \( v_o \). As seen in Fig. 7(b), nominal operation can be achieved in this case as well.

Figure 8(a) shows the circuit configuration at \( R = 22 \Omega \); here, the load resistance is higher than its designed value. In this configuration, the shunt capacitance \( C_1 \) and resonant capacitance \( C_f \) are \( C_1 = 100 \text{ pF} \) and \( C_f = 79.375 \text{ pF} \), respectively. Figure 8(b) shows simulated waveforms of the switch...
Fig. 6. Circuit configuration at $R = 18\,\Omega$ and simulated waveforms. (a) Circuit configuration. (b) Simulated waveforms of $v_S$ and $v_o$.

Fig. 7. Circuit configuration at $R = 15\,\Omega$ and simulated waveforms. (a) Circuit configuration. (b) Simulated waveforms of $v_S$ and $v_o$.

Fig. 8. Circuit configuration at $R = 22\,\Omega$ and simulated waveform. (a) Circuit configuration. (b) Simulated waveforms of $v_S$ and $v_o$.

Fig. 9. Combination of capacitance of a switched capacitor leg with parasitic output capacitance of MOSFET $C_o > C_{ds}$. When the MOSFET switch is in the OFF-state, the total capacitance becomes

$$C_F = \frac{C_oC_s}{C_o + C_s}$$

(25)

where $C_o = C_{ds} + C_{ex}$. When the switch is in the ON-state, the total capacitance becomes

$$C_N = C_e$$

(26)

In a practical circuit, one switched capacitor leg can switch its total capacitance between $C_F$ and $C_N$ by changing its switch state.

5.1 Determination of Capacitance in Switched Capacitance Legs to Achieve a Specified Resolution

When the shunt capacitance $C_1$ needs to be adjusted between specified values, i.e., $C_{1\text{max}}$ and $C_{1\text{min}}$ with specified resolution $C_{1\text{step}}$, the minimum step of the shunt capacitance variation should be equal to $C_{1\text{step}}$. If we define $C_{1e} = xC_{1o}$

$$C_{1F} = \frac{x}{1 + x} C_{1o}$$

(27)

$$C_{1N} = C_{1o}$$

(28)

In other words, the capacitance can be changed by $yC_{1o}$, where

$$y = x = \frac{x^2}{1 + x}$$

(29)
In Eqs. (32) and (33), 1 is added because the parasitic shunt capacitance of the main switch must be included. Figure 11 shows the operation at only two values of load resistance, it must have x

\[ C_{1\text{min}} = 1 + \sum_{l=1}^{p} \frac{x}{1 + x} C_{10} \]

\[ C_{1\text{max}} = (1 + x)C_{10} \]

\[ C_{1\text{step}} = 2C_{10} \]

In Eqs. (32) and (33), 1 is added because the parasitic shunt capacitance of the main switch is intrinsic.

When the resonant capacitance \( C_{r} \) needs to be regulated between \( C_{\text{max}} \) and \( C_{\text{min}} \) with specified resolution \( C_{\text{step}} \),

\[ C_{\text{min}} = \sum_{k=1}^{p} \frac{x}{1 + x} C_{0} \]

\[ C_{\text{max}} = \sum_{k=1}^{p} x C_{0} \]

\[ C_{\text{step}} = 2C_{0} \]

### 5.2 Determination of Capacitance in Switched Capacitance Legs to Achieve Nominal Operation at Two States

In order for the class E amplifier to achieve nominal operation at only two values of load resistance, it must have two switched capacitor legs, namely, one for the shunt capacitance and the other for the resonant capacitance.

To calculate the shunt capacitance, the parasitic output capacitance of the main switch must be included. Figure 11 shows the equivalent circuit.

The total shunt capacitance is

\[ C_{1\text{min}} = \frac{x + 2}{1 + x} C_{10} \]

\[ C_{1\text{max}} = (1 + x)C_{10} \]

\[ C_{1\text{step}} = 2C_{10} \]

Fig. 11. Combination of capacitance of a switched capacitor leg for shunt capacitance

\[ C_{\text{min}} = \frac{x}{1 + x} C_{0} \]

\[ C_{\text{max}} = xC_{0} \]

Fig. 12. Combination of capacitance of a switched capacitor leg for resonant capacitance

\[ C_{1\text{min}} = C_{10} + \frac{x}{1 + x} C_{10} \]

\[ C_{1\text{max}} = C_{10} + (1 + x)C_{10} \]

when the switch is in the OFF-state and

\[ C_{1\text{min}} = C_{10} + C_{1e} = (1 + x)C_{10} \]

when the switch is in the ON-state. Rearranging (38) and (39) produces

\[ x = \frac{-(C_{1\text{max}} - C_{1\text{min}})}{C_{1\text{min}}} \]

Once again, this is the only solution for \( x \) that produces a non-negative capacitance.

\[ C_{10} = \frac{C_{1\text{max}}}{1 + x} \]

\[ C_{1e} = xC_{10} \]

\[ C_{1e} = C_{10} - C_{ds} \]

where \( C_{ds} \) is obtained as a catalog value of the MOSFET transistor.

The equivalent circuit for the resonant capacitance is given in Fig. 12. By using a process similar to that used for obtaining the total shunt capacitance, the total resonant capacitance is found to be

\[ C_{\text{min}} = \frac{x}{1 + x} C_{0} \]

\[ C_{\text{max}} = xC_{0} \]

Rearranging (44) and (45) produces

\[ x = \frac{C_{\text{max}}}{C_{\text{min}}} - 1 \]

\[ C_{0} = \frac{C_{\text{max}}}{x} \]

\[ C_{fe} = xC_{0} = C_{\text{max}} \]

\[ C_{ds} = C_{0} - C_{fe} \]
6. Design Example and Experimental Result

A 13.56 MHz class E amplifier was designed for experimental verification. The basic class E amplifier was designed using the equations shown in the book \(^{(4)}\), in which the output resistance and dc-supply voltage are given as \(R = 18 \Omega\) and \(V_{DD} = 12\) V. Based on this, one obtains

\[
P = \frac{8}{\pi^2 + 4} \frac{V_{DD}^2}{R} = \frac{8}{\pi^2 + 4} \frac{12^2}{18} = 4.69\, W\quad (50)
\]

\[C_1 = \frac{1}{\pi^2 + 4} \frac{1}{\omega R} = 120\, pF\quad (51)
\]

\[L = \frac{QR}{\omega} = 2\, \mu H\quad (52)
\]

\[C_f = \frac{1}{\pi^2 - 4} \frac{1}{\omega R} = 73.7\, pF\quad (53)
\]

\[L_{RFC} = \frac{7R}{f} = 9.26\, \mu H\quad (54)
\]

\[L_f = \frac{1}{\omega C} = 1.87\, \mu H\quad (55)
\]

\[L_r = L - L_f = 0.243\, \mu H\quad (56)
\]

In this case, the class E amplifier must achieve nominal operation at load resistance 15 \(\Omega\),

\[a = 15/18 = 0.833\quad (57)
\]

\[C_f' = C_f = 1 + (1 - a) \times L_r/L_f = 72\, pF\quad (58)
\]

\[C_f'' = C_1/a = 144\, pF\quad (59)
\]

In this case, \(C_{1\,\text{min}}\) and \(C_{1\,\text{max}}\) are

\[C_{1\,\text{min}} = 120\, pF\quad (60)
\]

\[C_{1\,\text{max}} = 144\, pF\quad (61)
\]

Therefore, we have

\[x = \frac{(144 - 120) + \sqrt{144(144 - 120)}}{120} = 0.69\quad (62)
\]

\[C_{10} = \frac{144}{1 + 0.69} = 85\, pF\quad (63)
\]

\[C_{1r} = 0.69 \times 85\, pF = 58\, pF\quad (64)
\]

Using Power MOSFET IRF510, the output capacitance is \(C_{ds} = 80\, pF\). Hence, \(C_{1r\,c}\) is

\[C_{1r\,c} = C_{10} - C_{ds} = 85\, pF - 80\, pF = 5\, pF\quad (65)
\]

Additionally, \(C_{f\,\text{min}}\) and \(C_{f\,\text{max}}\) are

\[C_{f\,\text{min}} = 72\, pF\quad (66)
\]

\[C_{f\,\text{max}} = 73.7\, pF\quad (67)
\]

Therefore, one obtains

\[x = \frac{73.7\, pF}{72\, pF} - 1 = 0.0236\quad (68)
\]

\[C_{fo} = 73.7\, pF \times 0.0236 = 3.1\, nF\quad (69)
\]

\[C_{fe} = 0.0236 \times 3.1\, nF = 73.7\, pF\quad (70)
\]

\[C_{fex} = 3.1\, nF - 80\, pF = 3.1\, nF\quad (71)
\]

![Fig. 13. Experimental Circuit](image1)

![Fig. 14. Experiments at \(R = 18\, \Omega\)](image2)

![Fig. 15. Experiments at \(R = 15\, \Omega\)](image3)

Table 1. Design values of class E amplifier for \(R = 15\, \Omega\)

<table>
<thead>
<tr>
<th></th>
<th>Calculated</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_{RFC})</td>
<td>9.26 (\mu F)</td>
<td>10 (\mu F)</td>
</tr>
<tr>
<td>(C_{1,\text{min}})</td>
<td>5 (pF)</td>
<td>5 (pF)</td>
</tr>
<tr>
<td>(C_{p})</td>
<td>80 (pF)</td>
<td>80 (pF)</td>
</tr>
<tr>
<td>(C_{1,\text{max}})</td>
<td>58 (pF)</td>
<td>58.7 (pF)</td>
</tr>
<tr>
<td>(L_f)</td>
<td>5 (pF)</td>
<td>5 (pF)</td>
</tr>
<tr>
<td>(L_{15})</td>
<td>1.87 (\mu F)</td>
<td>2.11 (\mu F)</td>
</tr>
<tr>
<td>(R)</td>
<td>15 (\Omega)</td>
<td>15 (\Omega)</td>
</tr>
<tr>
<td>(C_{p})</td>
<td>73.7 (pF)</td>
<td>73.7 (pF)</td>
</tr>
<tr>
<td>(V_{out})</td>
<td>3.1 (nF)</td>
<td>3.3 (nF)</td>
</tr>
</tbody>
</table>

Figure 13 shows the experimental circuit. The switch states of \(S_{C1}\) and \(S_{C_f}\) were manually set to OFF and ON at \(R = 18\, \Omega\) and ON and OFF at \(R = 15\, \Omega\), respectively. Table 1 provides calculated and measured design values of the class E amplifier for \(R = 15\, \Omega\). Figure 14 shows the experimental waveforms at \(R = 18\, \Omega\). Nominal operation could be achieved at 11.59 MHz. Figure 15 shows the experimental waveforms at \(R = 15\, \Omega\). As is seen in these two figures, almost identical waveforms were obtained for two different
values of load resistance. These results show that proposed discrete control method was successful in this experiment.

7. Conclusions

In this paper, the concept of discrete control of a class E amplifier that maintains nominal operation at varying load resistance was demonstrated. Using PSIM simulation, nominal operation was approximately achieved for a range of load resistances. Design equations that take the parasitic capacitance of the MOSFET into account were developed, and the experimental results produced by a design example demonstrated that the proposed control method could properly maintain the nominal operation of the class E amplifier at two different load resistances. The proposed method only took variation of resistance into account; in future research, a control method based on variation of reactance will be developed.

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