Lagrangian-Based Derivation of a Novel Sliding-Mode Control for Synchronous Buck Converters

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Sliding-mode control for power converters is gaining significant research interest for achieving a fast transient response to a step load change in a wide operating range. However, power converters also require better dynamic load regulation against load current fluctuations slower than the step change. This paper addresses this issue by deriving a novel control method for synchronous buck converters using Lagrangian dynamics. Simulation results verified improvement in dynamic load regulation against slow sinusoidal load current fluctuations.

Keywords: buck converter, sliding-mode control, dynamic load regulation, Lagrangian dynamics

1. Introduction

Synchronous buck converters have been utilized in a wide application area, such as communication, robotics, and consumer electronics. These converters are generally required to stabilize the output voltage against load change. A well-known specification for this stability is transient response to a step load change, which is a faster change beyond the possible response speed of the converter. However, in many practical applications, the load is designed to change within the response speed. In these applications, the dynamic load regulation against comparatively slow load changes, or the output response speed. In these applications, the dynamic load regulation against comparatively slow load changes, or the output response speed of the converter. However, in many practical applications, the load is designed to change within the response speed. In these applications, the dynamic load regulation against comparatively slow load changes, or the output response speed of the converter. However, in many practical applications, the load is designed to change within the response speed. In these applications, the dynamic load regulation against comparatively slow load changes, or the output response speed of the converter.

With respect to the output voltage stability, the PWM-based sliding-mode control has been attracting great interest because it can offer fast transient response in wide operating range\textsuperscript{3,4}. However, as shown later, the dynamic load regulation can be further improved by a novel sliding-mode control method proposed in this paper.

We utilized Lagrangian modeling\textsuperscript{5,6} to derive this control method because it can convert complicated energy conserving systems into simple dynamically equivalent systems\textsuperscript{7,8,9}. In fact, this paper converts the synchronous buck converter into two independent systems, which enables decoupling between the output voltage and the load current. Along with theoretical derivation of the proposed control, this paper also presents simulation results that support improvement of the dynamic load regulation.

2. Proposed Control Method

2.1 Lagrangian Modeling

This section derives Lagrangian model of a synchronous buck converter system shown in Fig. 1(a). To model this system as an energy conserving system, we regard the load as an imaginary synchronous boost converter that emulates the load by extracting the load current, as shown in Fig. 1(b). (We neglect the current ripple of this converter.) We regard that switch S4 operates at duty cycle \(D_2\), which is unknown to the controller of the buck converter. For convenience, we assume that inductors L1 and L2 are the same. Let \(N\) and \(R\) be the number of their winding turns and the reluctance of their cores. We regard the switching-state indicators presented in Sect. 3.1 in the previous work\textsuperscript{3} as duty cycles to discuss a state-averaged model. Then, Lagrangian model\textsuperscript{3} \(\mathbf{L}\) for this system is

\[
\mathbf{L} = N \phi_1 + Nq_1 \phi_2 - R \phi_1^2/2 - R \phi_2^2/2 - (Q + qC)^2/2C + Eq \times \mathbf{E} + \lambda_1(\theta_1 - D_1 q_1) + \lambda_2(\theta_1 - D_2 q_2) \quad \cdots \quad (1)
\]

where \(\phi_1\) and \(\phi_2\) are the fluxes in L1 and L2, \(q_1\) and \(q_2\) are the charge flowing through L1 and L2, \(Q\) is the initial charge of C1, \(qC\) is the charge flowing into C1, \(E\) is the voltage of the power source, \(D_1\) is the duty cycle of S1, \(\lambda_1\) and \(\lambda_2\) are the Lagrangian multipliers. A dot over a variable represents its time derivative.

Now, we apply a coordinate transformation\textsuperscript{3} to \(1\) to obtain Lagrangian of a dynamically equivalent system. First, we eliminate Lagrangian multiplier terms by substituting \(qE = D_1 q_1 - (1 - D_2) q_2\) and \(q_1 = qC + q_2\) into \((1)\). Second, we introduce new variables \(\phi_A, \phi_C\) and \(qA\) defined as \(\phi_A = (\phi_1 + \phi_2)/2, \phi_C = \phi_1 - \phi_2\) and \(qA = (q_1 + q_2)/2\). Eliminating \(\phi_1, \phi_2, q_1\) and \(q_2\) from \(1\) yields

\[
\mathbf{L} = 2NqA \phi_A - R = \phi_A^2/4 - (Q + qC)^2/2C + Eq \times \mathbf{E} \quad \cdots \quad (2)
\]

where \(D_A\) and \(D_C\) are imaginary duty cycles defined as \(D_A = D_1 + D_2 - 1\) and \(D_C = (D_1 - D_2 + 1)/2\). Note that \(D_A\) takes from 1 to 1; and \(D_C\) takes from 0 to 1. Hence, \(2\) can be translated into an equivalent system of two independent converters shown in Fig. 2, according to the method presented in the previous work\textsuperscript{3}.

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voltage and current, to generate Step 1 observes the C1 voltage and current, which are the C2 voltage in Fig. 2(a), controlling have 

$$V_{C1}$$

of the load current on circuit, however, we can only adjustimaginary converter, i.e. Fig. 2(a), and 2. Determine $$D_1$$ from the required value for DC.

According to the voltage relation at the inductor L2, we have

$$d\cdot (di_{out}/dt) = V_{out}/L_2 - (1 - D_2)/E/L_2,$$  \hspace{0.5cm} (3)

where $$L_2$$ is the load current, and $$t$$ is the time. Substituting (3) into $$D_1 = 2D_2 + D_2 - 1$$ and noting that L1 and L2 have the same inductance, we have

$$D_1 = 2D_2 - V_{out}/E + (L_1/E) \cdot (d\cdot (di_{out}/dt)),$$  \hspace{0.5cm} (4)

2.2 Proposed Control Based on the above discussion, we can formulate the proposed control of Fig. 1(a). The control consists of the following two steps: 1. Determine $$D_1$$ according to the PWM-based sliding mode control of the imaginary converter, i.e. Fig. 2(a), and 2. Determine $$D_1$$ from $$D_1$$ according to (4).

Figure 3 illustrates an example of the control algorithm. Step 1 observes the C1 voltage and current, which are the C2 voltage and current, to generate $$D_1$$ for control of Fig. 2(a). Then, Step 2 calculates $$D_1$$ from $$D_1$$ according to (4). Gain operators can replace division operators, i.e. $$\div E$$, if the input voltage is almost constant.

The proposed control is an extension of the PWM-based sliding-mode control shown in Fig. 4 because STEP2 is the only difference.

3. Simulation Simulation was carried out to confirm effectiveness of the proposed control shown in Fig. 3 in comparison with the PWM-based sliding-mode control shown in Fig. 4. The simulation parameters are presented in Fig. 3. The simulator is PSIM9.3 (Myyaw Plus Corp.).

Figure 5 shows the transient response when the load resistance is switched between 1 $$\Omega$$ and 2 $$\Omega$$. The result shows that the proposed control shows almost the same response as the PWM-based sliding-mode control. Therefore, the proposed control showed no improvement in the transient response to a step load change.

Figure 6 shows the output voltage fluctuations when the load is sinusoidal current sink with 5 A_peak 10 kHz. The output voltage fluctuations were effectively suppressed in the proposed control. Therefore, the proposed converter improved the dynamic load regulation against slow load current fluctuations.

4. Conclusions Buck converters are generally required to stabilize the output voltage. To improve the stability, this paper proposed a novel control method for synchronous buck converters. The proposed control can improve the dynamic load regulation against slow load current fluctuations. Simulation results revealed successful suppression of the output voltage fluctuations under sinusoidal load current, whereas no improvement was found in the transient response to a step load change. In future works, the author will evaluate the performance of the proposed control experimentally.

References


