Disturbance Calculation based on Space Vector Dot Product: Applications to Compensators

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The ability to individually extract different types of disturbances present in AC line currents or voltages is desirable for reference signal calculation algorithms employed in power conditioners, in order to optimize the operation and design of power converters. This paper proposes a novel algorithm based on the dot product of three-dimensional space vectors in the *abc* coordinate system, for real-time calculation of the instantaneous positive-, negative- and zero-sequence components for individual harmonics. Certain applications are presented, including individually compensating the fundamental reactive power, selected current harmonics and current unbalances. This method is computationally simple and requires no coordinate transformation. The proposed algorithm is validated by simulation and experiments.

Keywords: amplitude and phase estimation, phase-locked loops (PLL), power quality, space vectors

1. Introduction

Many strategies are employed for the real time calculation of the reference signals for disturbance active compensators. One desirable feature for the reference calculation algorithm is the ability to separate the disturbance in relevant parts, since this characteristic has impact on the choice of the converter power rating and bandwidth. It is well known that reactive power and unbalance compensation requires a high power, low frequency converter. On the other hand, harmonics compensation requires high bandwidth for the converter controller, high switching frequency and operation of the converter at lower power ratings.

Among the strategies presented in the literature, it can be noted that the Instantaneous *pq* theory, in its original form⁽¹⁾, does not require a phase-locked loop (PLL) and it is convenient to compensate a large set of disturbances. However, it is not possible to extract the disturbances individual components, unless modified versions⁽²⁾ are employed, which can result in fictitious terms without physical meanings. On the other hand, Instantaneous Conservative Power theory⁽³⁾ proposes the decomposition in a number of terms with physical meaning. However, even a reactive compensation will need a high bandwidth converter to inject current harmonics that

appear when the grid voltage is distorted. This phenomenon also happens for the *pq* theory.

Techniques based on the Fourier series are shown in⁽⁴⁾, and methods based on the *abc/dq* transformation are also widely used⁽⁵⁾. The use of multiple *dq* frames rotating at different speeds and directions⁽⁶⁾⁽⁷⁾ enables the extraction of the individual positive- and negative-sequence harmonics. However, the strategies presented in^{(4)–(7)} require coordinate transformations.

Methods based on the Fortescue Transformation⁽⁸⁾ usually do not allow the separation of different types of disturbances. Authors of⁽⁹⁾ propose a new definition of instantaneous sequence components, which cannot separate the individual harmonics. Reference⁽¹⁰⁾ proposes a method based on the inner (dot) product to obtain the instantaneous fundamental positive-sequence.

This paper proposes a novel strategy to extract the instantaneous positive-, negative- and zero-sequences for each harmonic, based on the dot product (DP) of three-dimensional *abc* stationary frame space vectors. Additionally, a graphical explanation based on *abc* and $\alpha\beta 0$ coordinate systems is presented to clarify the comprehension of this method. Three cases for the minimization of load current disturbances are simulated and experimentally tested, including the individual compensation of fundamental reactive power, individual current harmonics and current unbalances.

Compared to existing methods, the proposed algorithm has a simple and alternative interpretation, requiring no coordinate transformations. Moreover, it can also be applied to

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 Fig. 1. Space vector in the abc system ($\mathbb{R}^3$)

 Fig. 2. abc and $\alpha\beta 0$ coordinate systems ($\mathbb{R}^3$)

compensate voltage disturbances such as specific harmonics, unbalances, voltage sags and voltage swells. This strategy can also be used in three-phase four-wire systems.

2. Disturbance Calculation based on Space Vector Dot Product

Figure 1 shows an orthonormal basis abc formed by the unit length vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ of $\mathbb{R}^3$.

A set of time varying coordinates $\phi_a(t)$, $\phi_b(t)$, $\phi_c(t)$ defines the instantaneous space vector $\vec{\Phi}$ (Fig. 1):

$$\vec{\Phi} = \phi_a(t)\vec{a} + \phi_b(t)\vec{b} + \phi_c(t)\vec{c} = [\phi_a \ \phi_b \ \phi_c]^T \dots \dots \dots (1)$$

where T is the matrix transpose operator. The coordinates ϕ_a , ϕ_b , ϕ_c can be the line to neutral voltages v_a , v_b , v_c defining the vector $\vec{V}$, or the line currents i_a , i_b , i_c defining the vector $\vec{I}$. The proposed algorithm uses only variables in the abc coordinate system, but its explanation is more conveniently done in the $\alpha\beta 0$ system defined in ⁽¹¹⁾⁽¹³⁾ and shown in Fig. 2.

Considering $\vec{\Phi}$ given by (1), the $\alpha\beta$ plane is the locus of the vectors with null instantaneous zero-sequence, defined by

$$\phi_a(t) + \phi_b(t) + \phi_c(t) = 0. \dots \dots \dots (2)$$

According to Fig. 3, an arbitrary space vector $\vec{\Phi}$ can be decomposed in a zero-sequence vector $\vec{\Phi}^0$ and in the vector $\vec{\Phi}_{\alpha\beta}$.

The vector $\vec{\Phi}_{\alpha\beta}$ remains on the $\alpha\beta$ plane and contains, respectively, the vectors corresponding to the h^{th} order positive- and negative-sequence harmonic components $\vec{\Phi}_h^+$ and $\vec{\Phi}_h^-$, as it is shown in Fig. 3. The vector $\vec{\Phi}_h^+$ rotates in counterclockwise direction with $h\omega$ speed and constant amplitude $\sqrt{3}/2\Phi_h^+$, where Φ_h^+ is the peak amplitude of the h^{th} order positive-sequence harmonic. The vector $\vec{\Phi}_h^-$ rotates in the clockwise direction with $h\omega$ speed and constant amplitude $\sqrt{3}/2\Phi_h^-$, where Φ_h^- is the peak amplitude of the h^{th} order negative-sequence harmonic.

 Fig. 3. Vectors $\vec{V}_{PLL}$ and $\vec{\Phi}$ (decomposed into $\vec{\Phi}_{\alpha\beta}$, $\vec{\Phi}_1^+$, $\vec{\Phi}_1^-$, $\vec{\Phi}_h^+$, $\vec{\Phi}_h^-$, $\vec{\Phi}^0$)

 Fig. 4. Vectors $\vec{\Phi}_1^+$, $\vec{\Phi}_h^+$, $\vec{\Phi}_{\alpha\beta}$, $\vec{V}_{PLL}^+$, $\vec{V}_{PLL}^+h_{||}$ and $\vec{V}_{PLL}^+h_{\perp}$ in the $\alpha\beta$ plane

2.1 Calculation of h^{th} order Positive-sequence Harmonic components $\vec{\Phi}_h^+$ The dot product of two vectors rotating at the same speed and direction is constant. Otherwise the DP will be an oscillating signal with zero mean value. This property can be used to calculate the h^{th} order positive-sequence harmonic components $\vec{\Phi}_h^+ = [\phi_{ah}^+ \ \phi_{bh}^+ \ \phi_{ch}^+]^T$. Thus, a set of orthogonal vectors $\vec{V}_{PLLh\perp}^+$ and $\vec{V}_{PLLh||}^+$, described by (3) and presented in Fig. 4 on the $\alpha\beta$ plane, which rotate in the counterclockwise direction with speed $h\omega$, and whose coordinates have unitary amplitude is necessary to calculate $\vec{\Phi}_h^+$.

$$\begin{aligned} \vec{V}_{PLLh\perp}^+ &= \begin{bmatrix} v_{PLLah\perp} \\ v_{PLLbh\perp} \\ v_{PLLch\perp} \end{bmatrix} = \begin{bmatrix} \sin(h\theta_{PLL}) \\ \sin(h\theta_{PLL} - 2\pi/3) \\ \sin(h\theta_{PLL} + 2\pi/3) \end{bmatrix} \dots \dots \dots (3) \\ \vec{V}_{PLLh||}^+ &= \begin{bmatrix} v_{PLLah||} \\ v_{PLLbh||} \\ v_{PLLch||} \end{bmatrix} = \begin{bmatrix} \cos(h\theta_{PLL}) \\ \cos(h\theta_{PLL} - 2\pi/3) \\ \cos(h\theta_{PLL} + 2\pi/3) \end{bmatrix} \end{aligned}$$

For $h = 1$, the vectors $\vec{V}_{PLLh\perp}^+$ and $\vec{V}_{PLLh||}^+$ are synchronized to the positive-sequence fundamental frequency component $\vec{V}_1^+$ of the mains vector $\vec{V}$. This is done by obtaining the angle θ_{PLL} , which is generated by a three-phase PLL[†] that forces the vector $\vec{V}_{PLL1||}^+$, calculated by imposing $h = 1$ in (3),

[†] Any three phase PLL that tracks the fundamental positive sequence of

 Fig. 5. Positive-sequence ($\vec{\Phi}_h^+$) extraction algorithm—single-wire block diagram

to be instantaneously parallel to the positive-sequence vector of the mains fundamental frequency ($\vec{V}_1^+$). The PLL strategy used in this paper is shown in Appendix.

The desired vector $\vec{\Phi}_h^+$ can be decomposed in two parts, that are parallel to $\vec{V}_{PLL h||}^+$ and $\vec{V}_{PLL h\perp}^+$ (Fig. 4), according to

$$\vec{\Phi}_h^+ = [\phi_{ah}^+ \ \phi_{bh}^+ \ \phi_{ch}^+]^T = \Phi_{h||}^+ \vec{V}_{PLL h||}^+ + \Phi_{h\perp}^+ \vec{V}_{PLL h\perp}^+ \dots (4)$$

If the amplitudes $\Phi_{h||}^+$ and $\Phi_{h\perp}^+$ are known, $\vec{\Phi}_h^+$ can be calculated by (4), as it is shown in the right side of the block diagram of Fig. 5. The evaluation of $\Phi_{h||}^+$ is described as follows.

The dot product ($\bullet$) of $\vec{\Phi}$ and $\vec{V}_{PLL h||}^+$ is

$$\vec{\Phi} \cdot \vec{V}_{PLL h||}^+ = (\vec{\Phi}_1^+ + \vec{\Phi}_1^- + \vec{\Phi}_h^+ + \vec{\Phi}_h^- + \vec{\Phi}^0) \cdot \vec{V}_{PLL h||}^+ \dots (5)$$

Since the zero-sequence component is instantaneously orthogonal to $\vec{V}_{PLL h\perp}^+$ and $\vec{V}_{PLL h||}^+$,

$$\vec{\Phi}^0 \cdot \vec{V}_{PLL h||}^+ = 0 \dots (6)$$

The other partial dot products in (5) have zero mean values, except for $\vec{\Phi}_h^+ \cdot \vec{V}_{PLL h||}^+$ which is constant. The evaluation of the mean value of the dot product $\vec{\Phi} \cdot \vec{V}_{PLL h||}^+$ results in

$$\begin{aligned} \vec{\Phi} \cdot \vec{V}_{PLL h||}^+ &= \frac{1}{T_P} \int_0^{T_P} \vec{\Phi} \cdot \vec{V}_{PLL h||}^+ dt = \vec{\Phi}_h^+ \cdot \vec{V}_{PLL h||}^+ \\ &= \sqrt{\frac{3}{2}} \Phi_{h||}^+ \sqrt{\frac{3}{2}} = \frac{3}{2} \Phi_{h||}^+ \dots (7) \end{aligned}$$

where T_P is the period of the signal generated by the dot product[†]. Based on (7), $\Phi_{h||}^+$ is easily calculated by (8), by using only *abc* coordinates.

$$\begin{aligned} \Phi_{h||}^+ &= \frac{2}{3} \vec{\Phi} \cdot \vec{V}_{PLL h||}^+ \\ &= \frac{2}{3} \frac{1}{T_P} \int_0^{T_P} ([\phi_a \ \phi_b \ \phi_c]^T \cdot [v_{PLL a h||} \ v_{PLL b h||} \ v_{PLL c h||}]^T) dt \dots (8) \end{aligned}$$

Using the same reasoning, $\Phi_{h\perp}^+$ can be evaluated by

the input signal can be used⁽¹⁰⁾⁽¹³⁾⁽¹⁴⁾. This paper uses the strategies implemented in the *abc* coordinate system (See Appendix).

[†] For usual applications the lowest frequency of the dot product occurs for unbalances, which is twice the mains frequency. This results in $T_P = T_w/2$, where T_w is the grid voltage period.

 Fig. 6. Vectors $\vec{V}_1^+$, $\vec{\Phi}_h^-$, $\vec{\Phi}_{\alpha\beta}^+$, $\vec{V}_{PLL h||}^+$, $\vec{V}_{PLL h\perp}^+$ and $\vec{V}_{PLL h||}^-$ in the $\alpha\beta$ plane

 Fig. 7. Negative-sequence ($\vec{\Phi}_h^-$) extraction algorithm—single-wire block diagram

$$\begin{aligned} \Phi_{h\perp}^+ &= \frac{2}{3} \vec{\Phi} \cdot \vec{V}_{PLL h\perp}^+ \\ &= \frac{2}{3} \frac{1}{T_P} \int_0^{T_P} ([\phi_a \ \phi_b \ \phi_c]^T \cdot [v_{PLL a h\perp} \ v_{PLL b h\perp} \ v_{PLL c h\perp}]^T) dt \dots (9) \end{aligned}$$

The mean value can be obtained by using a low pass filter. Figure 5 presents the positive-sequence extraction algorithm in a single-wire block diagram, where $\vec{\Phi} \cdot \vec{V}_{PLL h||}^+$ is low pass filtered, originating $\vec{\Phi}_h^+ \cdot \vec{V}_{PLL h||}^+$. This signal, followed by a gain 2/3 results in the magnitude of the projection of $\vec{\Phi}_h^+$ on the vector $\vec{V}_{PLL h||}^+$ (see Fig. 4), named as $\Phi_{h||}^+$. The instantaneous values of the coordinates of $\vec{\Phi}_h^+$ are finally obtained by calculating $\Phi_{h||}^+ \vec{V}_{PLL h||}^+$. The same reasoning is done for the orthogonal component $\vec{\Phi} \cdot \vec{V}_{PLL h\perp}^+$, resulting in the extracted positive-sequence components of the h^{th} harmonics, given by (4).

2.2 Calculation of h^{th} order Negative-sequence Harmonic Components $\vec{\Phi}_h^-$

The negative-sequence harmonics $\vec{\Phi}_h^- = [\phi_{ah}^- \ \phi_{bh}^- \ \phi_{ch}^-]^T$ can be obtained by substituting the pair $\vec{V}_{PLL h||}^+$, $\vec{V}_{PLL h\perp}^+$ by a pair of negative-sequence orthogonal vectors $\vec{V}_{PLL h||}^-$ and $\vec{V}_{PLL h\perp}^-$, which rotates with speed $h\omega$ in the clockwise direction, as it is shown in Fig. 6. These two vectors are easily obtained by swapping the signals corresponding to phases *b* and *c* in (3). This is done by the “phase swap” block in the single-wire diagram of Fig. 7.

2.3 Calculation of h^{th} order Zero-sequence Harmonic Components $\vec{\Phi}_h^0$

The calculation of $\vec{\Phi}_h^0 = [\phi_{ah}^0 \ \phi_{bh}^0 \ \phi_{ch}^0]^T$ is based on the dot products $\vec{\Phi} \cdot \vec{V}_{PLL h||}^0$ and $\vec{\Phi} \cdot \vec{V}_{PLL h\perp}^0$, where

Fig. 8. Zero-sequence ($\vec{\Phi}_h^0$) extraction algorithm—single-wire block diagram

Fig. 9. Simplified zero-sequence ($\vec{\Phi}_h^0$) extraction algorithm single-wire block diagram

$\vec{V}_{PLLh\parallel}^0$ and $\vec{V}_{PLLh\perp}^0$ are zero-sequence vectors with $h\omega$ frequency, given by

$$\begin{bmatrix} \vec{V}_{PLLh||}^0 = v_{PLLah||}[1 \ 1 \ 1]^T = \cos(h(\theta_{PLL}))[1 \ 1 \ 1]^T \\ \vec{V}_{PLLh\perp}^0 = v_{PLLah\perp}[1 \ 1 \ 1]^T = \cos(h(\theta_{PLL} + \pi/2))[1 \ 1 \ 1]^T. \\ \dots\dots\dots \end{bmatrix} \quad (10)$$

In order to avoid unnecessary calculation, the block diagram in Fig. 8 can be simplified into the one shown in Fig. 9. The instantaneous phase and amplitudes of a zero-sequence component signal are equal. Therefore, the product of only one term of the vector $\vec{\Phi}^0 = [\phi^0 \phi^0 \phi^0]^T$ is needed to calculate the zero- h^{th} order sequence component by means of

$$\phi^0 = \frac{\phi_a + \phi_b + \phi_c}{3} \dots\dots\dots (11)$$

3. Application to Parallel Disturbance Compensators—Simulation Results

Figure 10 shows the grid voltages (220V_{RMS}, 60 Hz), a delta connected RL switched load (Load 1: $R_1 = 33 \Omega$, $L_1 = 400$ mH), a non linear load composed by a three-phase diode rectifier (Load 2: $R_2 = 370 \Omega$, $L_2 = 1.7$ mH, $C_2 = 47 \mu\text{F}$) and a three-phase voltage source converter which compensates a set of chosen load disturbances. The converter is rated for 220 V line to line RMS voltage, 3.5 A RMS line current, 350 V DC-link voltage, for line inductor filters of 10 mH and 800 μF DC side capacitor.

Closing only switch S_1 of Load 1 results in a single-phase RL load connected between phases a and b (unbalanced load with reactive power). Simultaneous closure of S_1 and S_2 results in a balanced three-phase load with reactive power. Switch S_3 of Load 2 connects to the grid a diode rectifier circuit, with LC filter operating in discontinuous mode, providing distorted currents to the tests.

Fig. 10. Simulation and Experimental setup: VSC converter and loads

Fig. 11. Block Diagram of the control loops

Figure 11 presents the control diagram for the converter of Fig. 10, including the reference current calculator, the DC voltage control loop, the current tracking loop, the PLL and the PWM block. To maintain coherence with the proposed algorithms for disturbance calculation, all the blocks in Fig. 11 are implemented in *abc* system, with no coordinate transformation.

Current loop employs three digital proportional integral (PI) controllers (one for each phase), with the anti-windup algorithm presented in⁽¹¹⁾. To avoid the well known loss of controllability of three-phase, three-wire converters using three independent controllers, this paper subtracts the instantaneous zero-sequence component, given by $(i_{fa} + i_{fb} + i_{fc})/3$, from the measured filter currents i_{fa} , i_{fb} , i_{fc} . The current loop PI controller parameters were calculated to set the damping factor of the closed loop poles to 0.4, resulting in $K_p = 40.4$ and $T_i = 402 \mu s$. The transfer function of the PI controller is described in (12) and its discrete form is shown in (13) using backward Euler method. Besides that, a feed forward of the grid voltage is also included, to improve the disturbance and transient response.

$$G_{PI}(s) = K_P \left(1 + \frac{1}{T_{iS}} \right) \dots\dots\dots (12)$$

$$G_{PI}(z) = K_P \left(1 + \frac{T}{T_i(1 - z^{-1})} \right) \dots\dots\dots (13)$$

Furthermore, three single update sampled PWM are employed. Instantaneous zero-sequence is added to the PWM reference signal to emulate Space Vector PWM behavior⁽¹²⁾. Carrier and sampling frequency were set to 12 kHz and grid frequency is 60 Hz.

Fig. 12. Reference Calculation Block of Fig. 11 for the Fundamental Reactive Current Compensator

 Fig. 13. Simulated results for fundamental reactive power compensator: top - line to neutral voltages (v_a, v_b, v_c), middle: line currents (i_a, i_b, i_c), bottom- load currents (i_{la}, i_{lb}, i_{lc})

The PLL block is based on dot product PLL⁽¹³⁾, also known as p-PLL, and it is presented in Appendix. The PLL PI controller gains were adjusted as described in⁽¹⁴⁾, resulting in $K_{PPLL} = 139.426$ and $K_{IPLL} = 0.574$. The DC voltage control loop employs a PI compensator and regulates v_{dc}^2 instead of v_{dc} for exact linearization of the DC loop transfer function: The controller parameters are set to $K_{pv} = 5.94 \times 10^{-5}$ and $T_{iv} = 49$ ms, which results in a settling time equal to 9 cycles of mains frequency.

The proposed method to calculate positive- and negative-sequences of voltage and currents is applied to the three different disturbance compensators described below:

- Fundamental Reactive Currents Compensation;
- Fundamental Negative-Sequence Compensation;
- Selective Current Harmonics Compensation.

3.1 Fundamental Reactive Currents Compensation

The calculation of the reference signals for the fundamental reactive current compensator, described in the block diagram of Fig. 12, can be derived from Fig. 5 by:

- imposing the load currents as input vector $\vec{\Phi} = \vec{I}_l = [i_{la} \ i_{lb} \ i_{lc}]^T$;
- imposing $h = 1$;
- using only the upper signal path corresponding to the current components orthogonal to the positive-sequence of the fundamental grid voltages.

Moving average low pass filter with a sampling frequency of 12 kHz and a window width of $T_w/2$ (where T_w is the grid voltage period) was employed for the three compensators, resulting in a filter settling time of $T_w/2$. Simulation results were obtained by using the PSIM software. Figure 13 shows line to neutral voltages, grid currents and load currents for

Table 1. Calculated values of fundamental positive-sequence of grid and load currents—reactive power compensation—Simulation results

	Grid (A - peak)	Load (A - peak)
$t < 0.2$ s	$0.936 \angle -0.7^\circ$	$0.936 \angle 0.1^\circ$
$t \gg 0.2$ s	$1.712 \angle 2.0^\circ$	$3.711 \angle -63.4^\circ$

Fig. 14. Reference Calculation Block of Fig. 11 for the Fundamental Negative-Sequence Current Compensator

the fundamental reactive power compensation. Steady state operation with non linear Load 2 (S_3 On) is presented until 0.2 s. At 0.2 s switches S_1 and S_2 are turned on, including Load 1 (linear balanced RL load).

Table 1 summarizes the calculated values of the positive-sequence of the load and the grid fundamental currents (peak values) for $t < 0.2$ s and $t \gg 0.2$ s. The peak value of the grid voltage positive-sequence is $179.63 \angle 0^\circ$ V.

Simulations show that reactive fundamental currents are adequately compensated. After 0.2 s the current amplitude was reduced from 3.711 A to 1.712 A and the phase angle was reduced from -63.4° to 2.0° . Time response is expected to be equal to the moving average filter width ($T_w/2$), and thus the longer transient times in Fig. 13 are explained by the RL load large time constant ($\tau = L_1/R_1 \sim 12$ ms), where the steady state is reached in about 2 cycles. Moreover, current harmonics do not disturb the operation of the reactive power compensator. This is especially important for high power converters, which will be able to inject only mains frequency currents, and operate with lower switching frequencies.

3.2 Fundamental Negative-sequence Compensation

The calculation of the reference signals for the fundamental negative-sequence current compensator, described in the block diagram of Fig. 14, can be obtained from Fig. 7 by:

- imposing the load currents as input vector $\vec{\Phi} = \vec{I}_l = [i_{la} \ i_{lb} \ i_{lc}]^T$;
- imposing $h = 1$.

Figure 15 shows line to neutral voltages, grid currents and load currents for the Fundamental Negative-Sequence Current Compensator. Steady state operation with unbalanced linear load (Load 1 with only S_1 On) is presented until 0.2 s. At 0.2 s the load becomes a balanced inductive load (S_1 and S_2 are turned on).

Table 2 presents the calculated values of the positive- and negative-sequences of the load and the grid fundamental currents (peak values) for $t < 0.2$ s and $t \gg 2$ s. The peak of the grid voltage positive-sequence is $179.63 \angle 0^\circ$ V.

For $t < 0.2$ s the negative-sequence is adequately compensated (reduced from 1.707 A to 0.040 A). The long settling time for $t > 0.2$ s is again explained by the RL load transient.

Fig. 15. Simulated results for fundamental negative-sequence compensator: top - line to neutral voltages (v_a, v_b, v_c), middle: line currents (i_a, i_b, i_c), bottom- load currents (i_{la}, i_{lb}, i_{lc})

Table 2. Calculated values of fundamental symmetrical components for grid and load currents - negative component compensation- Simulation results

		I_{grid}	I_{load}
$t < 0.2s$	positive	$1.709\angle -77.2^\circ$	$1.707\angle -77.7^\circ$
	negative	$0.040\angle -172.3^\circ$	$1.707\angle -17.7^\circ$
$t \gg 0.2s$	positive	$3.427\angle -77.5^\circ$	$3.415\angle -77.7^\circ$
	negative	0.000	0.000

Fig. 16. Reference Calculation Block of Fig. 11 for the compensation of 5th and 7th harmonics

3.3 Selective Current Harmonics Compensation

The calculation of the reference signals for specific current harmonics compensation described in the block diagram of Fig. 16, considering only the 5th and 7th harmonics, can be obtained from Fig. 5 and Fig. 7 by:

- imposing the load currents as input vector $\vec{\Phi} = \vec{I}_l = [i_{la} \ i_{lb} \ i_{lc}]^T$;

Fig. 17. Simulated results for current harmonics: top - line current (i_a), bottom- load current (i_{la})

Table 3. Calculated 1st, 5th and 7th current components (A peak) for specific current harmonics compensation—Simulation results

	I_{grid}	I_{load}
1	0.952	0.947
5	0.002	0.840
7	0.003	0.742
11	0.501	0.500
13	0.376	0.376

- considering that the three-phase load currents have the same waveform and are equally displaced. This results in a negative-sequence 5th harmonic and a positive-sequence 7th harmonic. As a consequence, Fig. 16 simultaneously uses the algorithms shown in Figs. 7 and 5, with $h=5$ and $h=7$ respectively.

Since the PI controller with sampling frequency of 12 kHz presents excessive tracking error for the 5th and 7th harmonics, the harmonics compensator replaces this controller by three proportional plus resonant (PR) controllers tuned at 60 Hz, 300 Hz and 420 Hz respectively, according to (14).

$$G_{PR}(s) = 40.4 \left(1 + \frac{(2 \cdot 2\pi 60)s}{s^2 + (2\pi 60)^2} + \frac{0.1 \cdot (2 \cdot 5 \cdot 2\pi 60)s}{s^2 + (5 \cdot 2\pi 60)^2} + \frac{0.1 \cdot (2 \cdot 7 \cdot 2\pi 60)s}{s^2 + (7 \cdot 2\pi 60)^2} \right) \dots \dots \dots (14)$$

Using Tustin with pre-warping⁽¹⁵⁾ method in (14) results in the following discrete transfer function.

$$G_{PR}(z) = 40.4 + \frac{1.269(1-z^2)}{1-1.999z^{-1}+z^{-2}} + \frac{6.320(1-z^2)}{1-1.975z^{-1}+z^{-2}} + \frac{8.884(1-z^2)}{1-1.952z^{-1}+z^{-2}} \dots \dots \dots (15)$$

Figure 17 presents simulated line and load current waveforms for the compensation case of Fig. 16. For $t < 0.2s$ S_3 is on, and the non linear load is connected to the grid. For $t > 0.2s$ S_3 is off.

Table 3 shows calculated values of the 1st, 5th and 7th harmonics for the grid and load phase a current, for $t < 0.2s$.

The 5th and 7th harmonics are adequately attenuated. Figure 17 shows that the settling time is around half of mains period, which corresponds to the employed moving average filter window size. For $t > 0.2s$ grid harmonic components presented in Table 3 are null, since there is no load connected to the grid. The distorted grid current waveform of Fig. 17 is due to the high amplitude of 11th and 13th components, which were not expected to be compensated.

Fig. 18. Experimental results for fundamental reactive power compensator: top - line to neutral voltages (v_a, v_b, v_c), middle: line currents (i_a, i_b, i_c), bottom- load currents (i_{la}, i_{lb}, i_{lc})

Table 4. Calculated Symmetrical components for reactive power compensation- Experimental results

	$V_{grid} (V_{peak})$	$I_{grid} (A_{peak})$	$I_{load} (A_{peak})$
positive	$176.15 \angle -0.5^\circ$	$1.62 \angle -3.5^\circ$	$3.42 \angle -62.6^\circ$
negative	$0.82 \angle -149.6^\circ$	$0.02 \angle -40.8^\circ$	$0.04 \angle -41.1^\circ$
zero	$1.25 \angle -106.8^\circ$	0.00	$0.01 \angle -62.6^\circ$

If the complete harmonics compensation is desired, $h = 1$ must be used in Fig. 5 to calculate the fundamental positive-sequence and, $h = 1$ must also be used in Fig. 7 to calculate the fundamental negative-sequence. The total harmonic current is calculated by $\sum \vec{I}_h = \vec{I} - \vec{I}_1^+ - \vec{I}_1^-$.

4. Experimental Results

Control loops and PWM blocks of the experimental setup were implemented in a TMS320F28335 DSP. Waveforms were obtained using Agilent DSO6014 oscilloscope, Tektronix A6302 current probes, and then data were post-processed by using MATLAB software (waveform plotting, reactive currents spectra, positive and negative-sequence calculations). The performed experiments used the same loads and the same controllers described in Sect. 3 and the setup of Fig. 10. Only steady state operation is shown, since the objective of the experimental results is mainly to validate the reference current calculation block.

Figure 18 shows the line to neutral voltages corresponding to the fundamental reactive current compensation using Load 1 (S_1 and S_2 On) and Load 2 (S_3 On).

Load currents, shown at the bottom of Fig. 18, presents high amplitude and are delayed from their corresponding line to neutral voltages. Compensated currents have lower amplitude and are visually in phase with the grid voltages.

Table 4 shows the fundamental symmetrical components of the grid voltage, the compensated grid current and the load current.

Reactive currents were compensated as expected (grid current amplitude is lower and its phase angle was reduced from 62.6° to -3.5°), and the results are similar to the ones obtained in Table 1 for the simulated case.

Figure 19 shows the fundamental negative-sequence compensation using Load 1 (S_1 On and S_2 Off) for the simulated

Fig. 19. Experimental results for fundamental negative-sequence compensator waveforms: top - line to neutral voltages (v_a, v_b, v_c), middle: line currents (i_a, i_b, i_c), bottom- load currents (i_{la}, i_{lb}, i_{lc})

Table 5. Calculated symmetrical components for fundamental negative component compensation- Experimental results

sequence	V_{grid}	I_{grid}	I_{load}
positive	$175.82 \angle 0.3^\circ$	$1.57 \angle -73.0^\circ$	$1.59 \angle -76.8^\circ$
negative	$0.63 \angle -80.4^\circ$	$0.03 \angle -3.1^\circ$	$1.59 \angle -16.7^\circ$
zero	$0.29 \angle -137.8^\circ$	$0.01 \angle -109.7^\circ$	$0.02 \angle -127.9^\circ$

Fig. 20. Experimental results for current harmonics: top - line current (i_a), bottom- load currents (i_{la})

case of Fig. 15. High amplitude load currents exist only at phases a and b . Grid currents are balanced, with lower amplitude and reactive components, as expected. Table 5 summarizes the calculated symmetric components of the measured values, showing good agreement with the results presented in Table 2 for the simulated case. The negative-sequence was reduced from 1.59 A to 0.03 A.

Figure 20 shows the compensation of 5th and 7th order harmonics using Load 2 (S_3 on) for the simulated case of Fig. 15. Table 6 shows the calculated values of the 1st, 5th, 7th, 11th and 13th harmonics amplitudes for phase a of grid and load currents.

Table 6 shows that the 5th and 7th harmonics were adequately compensated by the proposed method and that the results are similar to the simulated case shown in Table 3.

5. Conclusion

This paper has presented a method to individually calculate

Table 6. Calculated 1st, 5th and 7th current components (peak value A) for specific current harmonics compensation—Experimental results

	I_{grid} (peak value A)	I_{load} (peak value A)
1	1.089	0.934
5	0.036	0.841
7	0.028	0.794
11	0.563	0.569
13	0.521	0.496

the instantaneous positive, negative and zero-sequences of any current or voltage harmonic component, which is based on the dot product of three-dimensional space vectors in the stationary frame. The method can be applied to calculate the reference signals for voltage and current disturbance compensators. Simulated data have shown the compensation of reactive power, load unbalance and current harmonics, and all these three cases were experimentally verified, validating the algorithm. The proposed method is similar to the modified *pq* theory method. Both algorithms present different approaches for explaining the same phenomena. Moreover, the proposed and the *dq* methods require twelve multiplications and evaluation of six trigonometric functions. Therefore, they have the same computational effort.

Considering the complete converter control system, the advantages of this approach over the *dq* implementation are:

- All measurements are performed in the original *abc* system. In *dq* frame control, all measurement must be transformed to synchronous frame, increasing computational effort;
- Since the actuation must occur in the original *abc* reference frame, computational effort is also decreased;
- All the control loops are performed in *abc* coordinates, as it is commonly found in literature^{(11)(13)–(15)} and⁽¹⁶⁾.

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Appendix

PLL

The implemented PLL block is shown in app. Fig. 1.

The Voltage Controlled Oscillator (VCO) block generates two sets of unitary amplitude, positive-sequence signals, with frequency ω_{PLL} , that are the coordinates of vectors $\vec{V}_{PLL1\perp}^+$ and $\vec{V}_{PLL1\parallel}^+$. The phase detector block calculates the dot product between $\vec{V}$ and $\vec{V}_{PLL1\perp}^+$, and attenuates the oscillating signals with zero mean value by means of a low pass filter. The PI controller changes ω_{PLL} in a way to force $\vec{V}_{PLL1\perp}^+$ to be in quadrature with the fundamental positive-sequence of $\vec{V}$, and $\vec{V}_{PLL1\parallel}^+$ to be in phase with the fundamental positive-sequence of $\vec{V}$. The PI controller is adjusted according to⁽¹⁴⁾. This paper uses a moving average filter with $T_w/2$ window as a low pass filter.

app. Fig. 1. PLL block diagram

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