Real-time Sensorless Estimation of Position and Force for Solenoid Actuators

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In various fields, small actuation systems are required to aid human activities in a narrow space and to realize fine motions. Although some miniature actuators have been developed, the size of the sensors attached to them prevents the miniaturization of the systems. In this paper, a sensorless actuation system with a compact solenoid actuator is proposed. An input signal including the AC and DC components is used. The inductance and position are estimated from the AC component. The DC component is applied in order to drive the solenoid actuator. Simulations and experiments concerning frequency characteristics and simultaneous estimation are conducted to verify the validity of the proposal. From the results of the frequency characteristics, the position and force estimation are accurately achieved up to 10 Hz frequency. As a result of the simultaneous estimation, the position and force are simultaneously estimated in real time. This proposal is useful, because a small solenoid actuation system whose position and force are simultaneously estimated in real time without the need for any position and force sensors is realized.

Keywords: sensorless, solenoid actuator, real-time estimation, position and force simultaneous estimation

1. Introduction

Recently, emphasis has been put on communication techniques based on haptic information. Two examples are illustrative: in robotics and in the medical field. In robotics, to realize interactions between humans and their environment, robots that are used in the human environment have been developed. Environmental information is necessary for the robots to be able to adapt to the environment. By employing haptic information, human motion and environmental information can be analyzed⁽¹⁾. In the medical field, haptic information is required for surgical teleoperation systems⁽²⁾. In conventional surgical teleoperation systems, operators cannot feel the softness of an organ, as only visual information is used. Therefore, operators often injure the organ. Haptic information enables the operators to receive tactile feedback regarding the organ, which reduces the number of the accidents. As stated above, haptic communication techniques are useful in various fields.

A tactile display is a haptic application that enables us to receive tactile feedback regarding the surface of a remote object⁽³⁾⁽⁴⁾. This display typically consists of many actuators such as dielectric elastomer actuators⁽³⁾ and pneumatic actuators⁽⁴⁾. To represent the object's surface accurately, high-density integration of actuators, provided by miniaturization, is required.

Small actuators are required not only for the development of a tactile display but also for supporting human activities

in narrow spaces and for the realization of fine motion. For example, a globular magnetic actuator with four shape-memory-alloy coils that can scan a complex pipe has been developed⁽⁵⁾. An XYZ stage consisting of piezo actuators has been researched to realize microscopic manipulation⁽⁶⁾.

Both position information and force information for an actuator are important to control its motion. For example, a bilateral control, which can communicate haptic information, requires simultaneous position information and applied force information⁽⁷⁾; by obtaining this information, various fine motion control methods can be applied to the actuator.

Because certain sensors are typically attached to the actuator to obtain this information, the system is large. Therefore, a sensorless actuation is an effective method for miniaturizing the system. As examples of sensorless actuation, position-sensorless controls on a linear tubular motor⁽⁸⁾ and an interior permanent magnet synchronous motor⁽⁹⁾ were studied. Removing the sensors provides other advantages such as reducing costs and frequency of maintenance.

A solenoid actuator is used to realize a small actuation system in this paper. Solenoid actuators are used in valves and switches. A braking system consisting of a solenoid actuator has been introduced⁽¹⁰⁾. The main features of the solenoid actuators are reduced cost, simple structure, and high output with a small volume.

The methods used to estimate the position (without using position sensors) for a solenoid actuator are divided into two main groups. The first estimation method uses a pulse width modulation (PWM) voltage input⁽¹¹⁾⁽¹²⁾. The inductance of the solenoid is calculated based on the voltage and current responses. The inductance has a relation with the mover position; the position can be estimated using the relation. However, the PWM waveform includes harmonics, which cause

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undesirable effects such as unpleasant sounds and damage to the electronic circuits. I. Dülk *et al.* studied simultaneous estimation of the position and applied force, although the values of the position and force were discrete⁽¹²⁾. It is desirable that the estimated values be continuous when controlling the position and force.

The second estimation method uses an AC input⁽¹³⁾. The AC phase is measured to estimate the inductance. The position is determined by the relation with the inductance. By inserting a capacitor, resonance is utilized to realize accurate position estimation. However, two wires are required to estimate and drive the mover. To miniaturize the system, the number of the signal wires should be reduced. In addition, the resonant frequency was 125 Hz in the experiment conducted by S.-T. Wu *et al.*; therefore, the time required for the estimation was too long. A short estimation period is required to control the motion.

In this paper, simultaneous estimation of the mover position and applied force for a compact solenoid actuator is proposed. The input signal, which includes an AC component and a DC component, is applied to the solenoid actuator using one wire. The AC component is used for position estimation and the DC component is used for force generation. The characteristics of this proposal are as follows:

- Small solenoid actuation system.
- Lower harmonics.
- Real-time estimation.
- Simultaneous estimation of both position and force.

Simulations and experiments are conducted to verify the validity of this proposal.

This paper is organized as follows. Section 2 describes the model of the solenoid actuator. Section 3 describes the principles of sensorless position estimation. Section 4 describes the principles of sensorless force estimation. The characteristics of the solenoid actuator under test are presented in Sect. 5. The simulation results are presented in Sect. 6. The experimental results are presented in Sect. 7. Finally, the paper is concluded in Sect. 8.

2. Modeling of Solenoid Actuators

This section describes the model of the solenoid actuator. A solenoid actuator consists of a mover, stator, coil, and spring, as shown in Fig. 1. The stator and mover are made of metal that has a high magnetic permeability, such as steel.

When current flows through the solenoid actuator, a magnetic field is generated. The magnetic coenergy W_{mag} is

Fig. 1. Structure of solenoid actuator

expressed as

$$W_{\text{mag}} = \frac{1}{2} L(x, i) i^2, \dots \dots \dots (1)$$

where L , x , and i denote the coil inductance, mover position, and current, respectively. By differentiating Eq. (1), the magnetic force F_{mag} is calculated as

$$F_{\text{mag}} = \frac{\alpha}{2} \frac{\partial L(x, i)}{\partial x} i^2, \dots \dots \dots (2)$$

where α denotes the adjustment coefficient. The details of this effect are described in Appendix 1. The current i consists of AC and DC components. The AC component i_{ac} is much smaller than the DC component I_{dc} . Therefore, F_{mag} is controlled by modulating I_{dc} . Figure 2 shows the force applied to the mover: F_{mag} , F_{spr} , and F_{ext} . F_{spr} is the elastic force generated by the spring. It is calculated as

$$F_{\text{spr}} = k(x_0 - x), \dots \dots \dots (3)$$

where k denotes the elastic coefficient of the spring. The origin of the x axis is set at the lowest point. The point x_0 is the equilibrium point at which gravity balances the elastic force when the DC current I_{dc} does not flow. The length of the gap between the mover and the stator is equal to the mover position. F_{ext} is an external force such as a friction force and a force applied by a human. The motion equation is derived as

$$m\ddot{x} = F_{\text{spr}} - F_{\text{mag}} - F_{\text{ext}}, \dots \dots \dots (4)$$

where m denotes the mass of the mover.

3. Principle of Sensorless Position Estimation

The circuit equation is derived as

$$v = Ri + \frac{d\Psi}{dt}, \dots \dots \dots (5)$$

where v , R , Ψ , and t denote the voltage, internal resistance, linkage flux, and time, respectively. On the right-hand side of Eq. (5), the first term describes the voltage drop due to the internal resistance. The second term represents the induced electromotive voltage. The linkage flux Ψ is expressed as

$$\Psi = Li, \dots \dots \dots (6)$$

Thus, Eq. (5) can be rewritten as

Fig. 2. Force applied to mover

$$v = Ri + \frac{dL}{dt}i + L\frac{di}{dt} \dots (7)$$

When a signal that includes AC and DC components is applied to the solenoid actuator, the voltage and current are represented by

$$v = v_{ac} + V_{dc}, \dots (8)$$

$$i = i_{ac} + I_{dc}, \dots (9)$$

where the subscripts “ac” and “dc” denote the AC and DC components. v_{ac} and V_{dc} are the voltage. Putting Eqs. (8) and (9) into Eq. (7) yields

$$v_{ac} + V_{dc} = R(i_{ac} + I_{dc}) + \frac{dL}{dt}(i_{ac} + I_{dc}) + L\frac{di_{ac}}{dt} \dots (10)$$

If the AC and DC components are assumed to be separated, Eq. (10) can be divided into two equations as

$$v_{ac} = Ri_{ac} + \frac{dL}{dt}i_{ac} + L\frac{di_{ac}}{dt}, \dots (11)$$

$$V_{dc} = RI_{dc} + \frac{dL}{dt}I_{dc}. \dots (12)$$

From Eq. (12), dividing both sides by I_{dc} yields

$$R + \frac{dL}{dt} = \frac{V_{dc}}{I_{dc}}. \dots (13)$$

Putting Eq. (13) into Eq. (11) yields

$$v_{ac} = \frac{V_{dc}}{I_{dc}}i_{ac} + L\frac{di_{ac}}{dt}. \dots (14)$$

When $i_{ac} = \bar{i}_{ac} \cos(\omega t + \phi)$, the circuit equation can be solved as

$$v_{ac} = \bar{v}_{ac} \cos(\omega t + \phi + \theta), \dots (15)$$

where $\bar{i}_{ac}$, ω , ϕ , $\bar{v}_{ac}$, and θ denote the AC current amplitude, angular velocity, initial phase of the AC current, AC voltage amplitude, and the phase difference between AC voltage and current, respectively. $\bar{v}_{ac}$ and θ are calculated as

$$\bar{v}_{ac} = \bar{i}_{ac} \sqrt{\left(\frac{V_{dc}}{I_{dc}}\right)^2 + (\omega L)^2}, \dots (16)$$

$$\theta = \tan^{-1} \left(\frac{\omega L}{\left(\frac{V_{dc}}{I_{dc}}\right)} \right). \dots (17)$$

Solving L in Eq. (16) yields

$$L = \frac{1}{\omega} \sqrt{\left(\frac{\bar{v}_{ac}}{\bar{i}_{ac}}\right)^2 - \left(\frac{V_{dc}}{I_{dc}}\right)^2}. \dots (18)$$

This equation indicates that the inductance of the solenoid can be obtained from the AC and DC voltage and current. The inductance has a relation with the mover position. By identifying the relation in advance, the position can be estimated from the inductance.

4. Principle of Sensorless Force Estimation

The motion equation is given in Eq. (4). The elastic force F_{spr} in Eq. (3) and the inertial force $m\ddot{x}$ can be estimated from the estimated position. The magnetic force F_{mag} in Eq. (2) can be derived from the DC current. Therefore, the external force F_{ext} can be estimated as

$$F_{ext} = F_{spr} - F_{mag} - m\ddot{x}. \dots (19)$$

5. Characteristics of Solenoid Actuator under Test

This section presents the characteristics of the solenoid actuator that is used in the simulations and experiments. Figure 3 shows the compact solenoid actuator, DS-05A, J100-001, made by CKD Corporation. The size is H: 43.5 mm × W: 15 mm × D: 10 mm. The mass of the mover is 4.80 g. The internal resistance is 20 Ω. The elastic coefficient of the spring is 50 N/m. A laser sensor, CD22-15A, made by OPTEX FA Corporation, was used to measure the actual mover position.

The relation between the mover position and the inductance was measured by an LCR meter. The input voltage and frequency were set at 3 V and 1 kHz, respectively. We assume that the relation can be described by a quadratic polynomial of the following form:

$$x = aL^2 + bL + c, \dots (20)$$

where a , b , and c are constants. Figure 4 shows the measured relation. The symbol “♦” shows the measured points. The solid line shows a quadratic polynomial approximation. The parameters obtained are shown in Table 1.

Figure 5 shows the relation between DC current and elastic force in Eq. (3) on the non-contact condition. The symbol “♦” represents the measured elastic force. The magnetic attractive force is represented in Eq. (2). Because the DC current is small, the inductance L can be assumed to be a function of only the mover position x . $\partial L(x)/\partial x$ is calculated from Eq. (20) as

Fig. 3. Solenoid actuator

Fig. 4. Relation between mover position and inductance

Table 1. Parameters of solenoid actuator

coefficients of relation	a	0.0335
between mover position	b	-1.56
and inductance	c	17.8
adjustment coefficient	α	4.55

Fig. 5. Magnetic force characteristics (no load)

$$\frac{\partial L(x)}{\partial x} = -\frac{1}{\sqrt{b^2 - 4 \cdot a \cdot (c - x)}} \dots \dots \dots (21)$$

The solid line in Fig. 5 shows the approximation using the same equation form as in Eq. (2). The parameter α was determined as shown in Table 1.

6. Simulations

This section describes our simulation results. To confirm the validity of the proposal, several simulations were conducted.

6.1 Position Estimation The frequency characteristics of the position estimation method were confirmed by simulations. The DC current was used to displace the mover and create a sinusoidal motion. The AC component (whose voltage and frequency were 3 V and 1 kHz, respectively) was constantly input. Figure 6 shows the simulation result of the position estimation at various frequencies. The vertical axis represents the error ratio of the position estimation, which is calculated as

$$E_p = 10 \log_{10} \frac{\bar{x}}{\hat{x}}, \dots \dots \dots (22)$$

where $\bar{x}$ and $\hat{x}$ denote the amplitude of the actual position and the estimated position, respectively. The symbol “♦” represents the calculated points. Up to a frequency of roughly 10 Hz, the position can be estimated accurately. The position estimation accuracy worsens at higher frequencies than the natural frequency of $\sqrt{k/m}/2\pi = 16.2$ Hz.

6.2 Real-time Characteristics The estimation period is confirmed using a simulation. The position oscillates by modulating the DC component at 1 Hz. The estimated position is compared to the actual position. The root-mean-square (RMS) error is calculated from the actual and estimated position over one period by shifting the estimated position as

$$\text{RMS}_{\text{error}} = \sqrt{\frac{1}{N} \sum_{k=0}^N \{x(k) - \hat{x}(k + \Delta t)\}^2}, \dots \dots \dots (23)$$

where N and Δt denote the number of the data points in a period and the shifting time, respectively. Δt being at the minimum point of the RMS error indicates that the actual position $x(k)$ is similar to the estimated position $\hat{x}(k)$, with a shift of Δt . In short, Δt represents the time required for position estimation. Figure 7 shows the calculated RMS error when changing Δt . The symbol “♦” shows the points calculated using Eq. (23). The smallest RMS error is $0.12 \mu\text{m}$ at

Fig. 6. Frequency characteristics of position estimation (simulation)

Fig. 7. RMS error calculation for real-time characteristics (simulation)

$\Delta t = 0.328$ ms. This result confirms that the position can be estimated within 0.328 ms.

6.3 Force Estimation The frequency characteristics of the force estimation were confirmed by simulations. It is assumed that the mover makes contact with an object, which is represented as a spring-damper model. The external force is calculated as

$$F_{\text{ext}} = K_e(x_e - x) - D_e \dot{x}, \dots \dots \dots (24)$$

where K_e , D_e , and x_e represent the elastic and viscous damping coefficient of the contact object, and the contact point with the object, respectively. In this simulation, the contact environment consists of a hard object whose coefficients K_e and D_e are set at 1000 N/m and 0.1 Ns/m. The DC current oscillates under the condition which the mover consistently comes into contact with the object. Figure 8 shows the simulation result of the force estimation at various frequencies. The vertical axis represents the error ratio of the force estimation, which is calculated as

$$E_f = 10 \log_{10} \frac{\bar{F}_{\text{ext}}}{\hat{F}_{\text{ext}}}, \dots \dots \dots (25)$$

where $\bar{F}_{\text{ext}}$ and $\hat{F}_{\text{ext}}$ denote the amplitude of the actual external force and the estimated force, respectively. The symbol “♦” represents the calculated points. Force estimation is possible up to a frequency of 10 Hz. At higher frequencies, the position estimation worsens, as shown in Fig. 6. Therefore, the accuracy of the elastic force calculation in Eq. (3) deteriorates. In addition, the magnetic force calculation depends on the position, in terms of $\partial L/\partial x$; therefore, the accuracy of the force estimation worsens.

6.4 Simultaneous Estimation Figures 9 and 10 show the simulation results when the position and force are simultaneously estimated. The solid line and broken line in Fig. 9

Fig. 8. Frequency characteristics of force estimation (simulation)

Fig. 9. Position response of simultaneous position and force estimation with periodic external force (simulation)

Fig. 10. Force response of simultaneous position and force estimation with periodic external force (simulation)

represent the actual position and estimated position, respectively. The solid line and broken line in Fig. 10 represent the actual force and estimated force, respectively. Here, we assume that the external force is 4.00 g in weight (39.2 mN) and that the DC component is constantly input. The position and force are accurately estimated in real-time. The small error in the force estimation is due to the position estimation error.

7. Experiments

This section describes our experimental results. Real-time estimation experiments for the mover position and force were conducted to verify the validity of the proposal.

7.1 Experimental Setup The experimental system is shown in Fig. 11. The controller generated the input signal. The D/A converter output a staircase wave. The signal was input to the solenoid actuator via a low-pass filter and an operational amplifier. The voltage and current were detected by the A/D converter. A high speed digital control system called

Fig. 11. Experimental system

Fig. 12. Frequency characteristics of position estimation (experiment)

Fig. 13. RMS error calculation for real-time characteristics (experiment)

PE-Expert4, made by Myway Plus Corporation, was used. The main CPU is a digital signal processor. The A/D converter sampled the data every $5\ \mu\text{s}$. The D/A converter output a signal every $20\ \mu\text{s}$.

7.2 Position Estimation The position estimation experiment was conducted under the non-contact condition. Figure 12 shows the experimental results of the position estimation. The vertical axis represents the error ratio of the position estimation calculated using Eq. (22). The symbol “◆” shows the measured points. The symbol “■” represents the simulation result shown in Fig. 6. Position estimation could be achieved at more than $-2\ \text{dB}$ accuracy up to 10 Hz.

7.3 Real-time Characteristic The real-time characteristics were measured by changing the DC component. The RMS error was calculated using Eq. (23); the result is shown in Fig. 13. The symbol “◆” represents the calculated points. The minimum value of the RMS error is 0.08815 mm at $\Delta t = 2\ \text{ms}$. In short, real-time estimation of the position can be achieved within 2 ms. The difference between the simulation and experimental results is due to the difference of the sampling time. The experimental data was derived by using software called PE-View X, made by Myway Plus Corporation. The sampling time was 1 ms, which was longer than the

Fig. 14. Position response of simultaneous position and force estimation with periodic external force (experiment)

Fig. 15. Force response of simultaneous position and force estimation with periodic external force (experiment)

sampling time used in the simulations.

7.4 Simultaneous Estimation The position and force were simultaneously estimated. The external force was periodically applied by putting a 4.00 g weight (39.2 mN) on the mover. As the mass of the mover m was very small, the inertial force $m\ddot{x}$ was assumed to be zero. Figures 14 and 15 shows the experimental results of the simultaneous estimation of the mover position and the external force. The solid line and broken line in Fig. 14 represent the position measured by the laser sensor and the estimated position, respectively. The solid line in Fig. 15 represents the estimated force. The position was accurately estimated in real-time. The force was estimated at 45 mN, which is slightly larger than the force of gravity on the weight (i.e., 39.2 mN). This error comes from the error in the position estimation and the static friction.

8. Conclusions

This paper proposed real-time sensorless estimation for solenoid actuators. There are four advantages to using this technique: 1) compact size, 2) lower harmonics, 3) simultaneous estimation of position and force, and 4) real-time estimation. Simulations and experiments were conducted to confirm the frequency characteristics and real-time characteristics, and that simultaneous estimation was possible. In the experimental results for the frequency characteristics, the accuracy of the position estimation was confirmed to be more than -2 dB at frequencies under 10 Hz. As the experimental results of the real-time characteristics show, the

proposed technique was able to successfully estimate the position within 2 ms. In the experimental results for the simultaneous estimation, both the position and force were estimated at the same time; however, there were about 5 mN force errors.

As mentioned above, the novelty of this proposal is the real-time simultaneous estimation of position and force for a compact solenoid actuator by inputting a signal with lower harmonics. This proposal is useful for fields that require compact actuation systems, such as tactile displays and pipe inspection devices.

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Appendix

1. Explanation of adjustment coefficient α based on magnetized mover and stator

The measured force is several times larger than the theoretically predicted electromagnetic force, thus the authors introduce an adjustment coefficient α based on the properties of the magnetized mover and stator. The magnetic force F_{mag} is divided into two parts as follows:

$$F_{\text{mag}} = \frac{1}{2} \frac{\partial L}{\partial x} i^2 - \frac{m_1 m_2}{x^2}, \dots \dots \dots (A1)$$

where m_1 and m_2 denote the magnetization intensity of the mover and the stator, respectively. On the right-hand side of Eq. (A1), the first term is the force generated by the magnetic energy W_{mag} ; the second term is the elastic force generated by magnetizing the mover and the stator. Using the Taylor series expansion around the initial position x_0 , Eq. (A1) can be rewritten as

$$F_{\text{mag}} \approx \frac{1}{2} \frac{\partial L}{\partial x} i^2 - \frac{m_1 m_2}{x_0^2} + 2 \frac{m_1 m_2}{x_0^3} (x_0 - x). \quad \dots\dots\dots (A2)$$

On the right-hand side of Eq. (A2), the second term is constant. This term can be ignored by setting the initial position x_0 at the equilibrium position at which the mover weight, elastic force, and this term are balanced. Therefore, the magnetic force can be expressed as

$$F_{\text{mag}} \approx \frac{1}{2} \frac{\partial L}{\partial x} i^2 + 2 \frac{m_1 m_2}{x_0^3} (x_0 - x). \quad \dots\dots\dots (A3)$$

In Sect. 5, the magnetic force characteristics were measured using the balance between the elastic force and the magnetic force, as shown in Fig. 5, where the mover was not fixed. The relation between the current and the position can be calculated as

$$\begin{aligned} F_{\text{spr}} &= k(x_0 - x) \\ &= F_{\text{mag}} = \frac{1}{2} \frac{\partial L}{\partial x} i^2 + 2 \frac{m_1 m_2}{x_0^3} (x_0 - x). \quad \dots\dots\dots (A4) \end{aligned}$$

Therefore,

$$x_0 - x = \frac{1}{2} \left(\frac{1}{k - 2 \frac{m_1 m_2}{x_0^3}} \right) \frac{\partial L}{\partial x} i^2. \quad \dots\dots\dots (A5)$$

Putting Eq. (A5) into Eq. (A3) yields

$$F_{\text{mag}} = \frac{\alpha}{2} \frac{\partial L}{\partial x} i^2, \quad \dots\dots\dots (A6)$$

where

$$\alpha = 1 + \frac{2 \frac{m_1 m_2}{x_0^3}}{k - 2 \frac{m_1 m_2}{x_0^3}}. \quad \dots\dots\dots (A7)$$

This value α was experimentally determined.

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