Comparison of Methods for Solving the Singular Configuration of a Wheel-Legged Mobile Robot

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In this paper, the singular configuration problem of a wheel-legged mobile robot due to the rank deficiency in inverse kinematics is discussed. Rank deficiency occurs when multiple kinematic constraints conflict with each other or when the steering joint becomes perpendicular to the ground surface. A method was proposed for avoiding a singular configuration while simultaneously solving the constraints that uses inverse kinematics that considers priority and solves the singular configuration of the steering joint using kinematic constraints on acceleration. Consequently, a new singular configuration occurs in low-speed wheeled locomotion. The Levenberg-Marquardt method is used for the singular configuration in low-speed wheeled locomotion. This method is necessary for determining a suitable damping factor, because it affects the solution of the inverse kinematics. Therefore, a comparison of different damping factor determination methods and a proposed method that considers the steering motion of each leg are highlighted.

Keywords: wheel-legged mobile robot, inverse kinematics, singular configuration, Levenberg-Marquardt method

1. Introduction

The singular configuration of robot is a problem in motion generation, because the robot cannot move in a specific direction. Moreover, an inverse kinematics calculation cannot give a solution. Methods have been proposed for solving this problem\(^1\)\(^–\)\(^11\). In this paper, the singular configuration of the wheel-legged mobile robot is discussed. Such a robot is a hybrid of wheel and leg mechanisms. The wheel-legged mobile robot is expected to move using wheel and leg mechanisms in miscellaneous environments. Therefore, there are related studies aiming at describing motion in various environments\(^1\)\(^2\)\(^–\)\(^\text{15}\). The wheel-legged mobile robot has two types of singular configuration. First, the singular configuration occurs due to the rank deficiency when multiple kinematic constraints conflict each other. Second, the singular configuration occurs when the steering joint is perpendicular to the wheel contact point. A method using a special mechanism was proposed for the singular configuration of the steering joint\(^1\)\(^6\). However, this method requires a design that incorporates special mechanisms in advance. In addition, another method that obtains the inverse kinematics solution of the steering joint separately was proposed\(^1\)\(^7\). This method affects the entire body motion, because the steering joint obtains the inverse kinematics solution separately. Thereafter, another method for the singular configuration was proposed when the inverse kinematics calculation of multiple kinematic constraints uses a selection matrix\(^1\)\(^8\). However, this method degrades motion performance as a result of not controlling the specific direction. To respond to these singular configuration problems, a method that uses kinematic constraints on acceleration for generating steering motion in the singular configuration of the steering joint was adopted\(^1\)\(^9\). In addition, it uses inverse kinematics considering priority\(^2\)\(^0\) for the singular configuration with the inverse kinematics calculation of multiple kinematic constraints, and realizes motion generation in the singular configuration of the steering joint. Consequently, a new singular configuration occurs in low-speed wheeled locomotion, because the wheel rotational velocity includes the steering joint term in the Jacobian matrix.

In this paper, kinematic constraints on acceleration and inverse kinematics considering priority for the singular configuration of the wheel-legged mobile robot is applied. Then, we solve the singular configuration in low-speed wheeled locomotion by application of the Levenberg-Marquardt (LM) method\(^2\)\(^1\)\(^–\)\(^2\)\(^2\). The general least squares calculation such as the Newton-Raphson method, the steepest descent method, and the quasi-Newton method were often used in the past works. However, the reduction of the computational efficiency near the singularity and the poor convergence become a problem. In contrast, the LM method prevents rank deficiency of the Jacobian matrix by adding a damping term to its pseudo-inverse matrix in the least squares calculations. Thus, the LM method prevents the solution of a least square calculation from being diverging. The damping factor in the LM method affects the solution of the least squares calculations. Therefore, an effective damping factor determination method for the singular configuration on the wheel-legged mobile robot is discussed and the conventional determination method of the damping factor is applied. In addition, a determination method based on manipulability\(^2\)\(^3\), which is an index that quantifies the operability of a robot from the kinematic viewpoint, is proposed. In previous research, a method for determining the damping factor based on manipulability of each leg in the wheel-legged mobile robot was proposed\(^2\)\(^4\). In
Methods for Singular Configuration of Wheel-Legged Mobile Robot

Kenta Nagano et al.

In this paper, the previously proposed our method is improved so that it considers, in each leg, the singular value of the target joint and the manipulability of other joints. As the result, it suppresses the vibration of the target joint, and reduces the tracking error of the base link. Then, it is compared with the effectiveness of the conventional method using three-dimensional simulations and experiments.

This paper is organized as follows. Section 2 presents a modeling and the singular configuration of the wheel-legged mobile robot. Section 3 applies kinematic constraints on acceleration and inverse kinematics considering priority for the singular configuration of the wheel-legged mobile robot. Section 4 describes the application of the LM method to the wheel-legged mobile robot and a determination method of the damping factor. Section 5 shows results of three-dimensional simulations and experiments. Finally, Sect. 6 draws conclusions and targets.

2. Robot Modeling and Singular Configuration

In this section, the robot modeling, kinematic constraints at velocity level, and the singular configuration at velocity level for describing the singular configuration in low-speed wheeled locomotion are detailed.

2.1 Coordinate System and Model of the Robot

An overview of the wheel-legged mobile robot is shown in Fig. 1. The structure of one leg is shown in Fig. 2 and the coordinate system of the robot is defined. The relationship of each coordinate system is shown in Fig. 3, and in addition, a planar model of a robot is shown in Fig. 4.

The coordinate systems of Fig. 3 are as follows:

- $\Sigma_W$: Coordinates of the world
- $\Sigma_B$: Coordinates of the base link
- $\Sigma_{ci}$: Coordinates of the contact point

Next, the parameters in Fig. 4 are defined as follows:

- $p_B \in \mathbb{R}^{3 \times 1}$ is the base position vector with respect to $\Sigma_W$.
- $p_{ci} \in \mathbb{R}^{3 \times 1}$ is the tip position vector of each leg with respect to $\Sigma_B$.
- $\Phi = [\phi \theta \psi]^T$ is the Euler attitude angle of the base link.
- $\theta_{leg} = [\theta_1, \theta_2, \theta_3]^T \in \mathbb{R}^3$ is the joint angle vector of each leg. Here, $\theta_1$, $\theta_2$, and $\theta_3$ are the crotch yaw, crotch roll, and knee roll joint angle, respectively.
- $\theta_{wheel} = [\gamma_i, \phi_i]^T$ is the rotation angle related to wheeled locomotion. Here, $\gamma_i$ and $\phi_i$ are the steering and rolling

![Fig. 1. Overview of robot](image1)

<table>
<thead>
<tr>
<th>Table 1. Link Parameters</th>
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<tbody>
<tr>
<td><strong>Mass [kg]</strong></td>
</tr>
<tr>
<td>Base</td>
</tr>
<tr>
<td>Crotch</td>
</tr>
<tr>
<td>Thigh</td>
</tr>
<tr>
<td>Calf</td>
</tr>
<tr>
<td>Shin</td>
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<tr>
<td>Wheel</td>
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![Fig. 2. Structure of one leg](image2)

![Fig. 3. Coordinate system of robot](image3)

![Fig. 4. Model of robot](image4)
angles of the wheel, respectively.

where \( N \) is the number of joint angles in the leg. Here, \( N = 3 \) in our robot. Finally, the respective rotation vector of each leg is summarized by \( \theta_i = [\theta_{\text{legi}}^T \theta_{\text{wheeli}}]^T \).

### 2.2 Kinematic Constraints at Velocity Level

Because motion generation of the wheel-legged mobile robot requires considering mechanical constraints, and defining kinematic constraints of the robot, which then moves considering the mechanical constraints. Since the robot has wheel and leg mechanisms, kinematic constraints for each are defined.

The constraint of the wheel is that the slippage of the contact point becomes zero. The constraint of the wheel slippage in the three-dimensional model is expressed as follows:

\[
v_{ci} = w \dot{p}_c - \dot{w} R_c [R \phi] 0 0
\]

where \( v_{ci} \) is the slippage of the contact point with respect to \( \Sigma_c \), and \( R \) is the radius of the wheel. In addition, \( w R_c \) is a rotation matrix from \( \Sigma_w \) to \( \Sigma_c \). Next, the tip velocity vector of each leg with respect to \( \Sigma_w \) is expressed by the following equation obtained by differentiating \( w \dot{p}_c = w \dot{p}_b + w R_b \dot{p}_c \). \( w \dot{p}_c \) is the tip position vector of each leg with respect to \( \Sigma_w \).

\[
w \dot{p}_c = w \dot{p}_b + w R_b \dot{p}_c + w R_b R_b \dot{p}_c
\]

where \( w R_b \) is expressed by \( w R_b = w \omega_B w R_b \). In addition, \( \Lambda \) is a transformation operator that transforms a \( 3 \times 1 \) vector into a \( 3 \times 3 \) skew-symmetric matrix. Thus, the cross product can be transformed into various forms using \( \Lambda \), i.e.,

\[
\mathbf{a}(\in \mathbb{R}^3) \times \mathbf{b}(\in \mathbb{R}^3) = -\mathbf{b} \times \mathbf{a} = -\mathbf{a} \times \mathbf{b}
\]

Therefore, \( w R_b \dot{p}_c \) is expressed as follows:

\[
w R_b R_b \dot{p}_c = w \dot{w} \omega_B w R_b \dot{p}_c
\]

\[
=(w R_b \dot{p}_c)^T \omega_B
\]

\[
= (w R_b \dot{p}_c)^T \omega_B
\]

In addition, the tip velocity vector of each leg with respect to \( \Sigma_B \) is expressed as follows:

\[
\dot{B} \dot{p}_c = J_{\text{legi}} \dot{\theta}_{\text{legi}} + J_{\gamma_i} \gamma_i
\]

where \( J_{\text{legi}} \in \mathbb{R}^{3 \times 3} \) is the Jacobian matrix of the rotational joint, and \( J_{\gamma_i} \in \mathbb{R}^{3 \times 1} \) is the Jacobian matrix of the steering joint. Each Jacobian matrix can be obtained as follows:

\[
J_{\text{legi}} = \frac{\partial \dot{p}_c}{\partial \theta_{\text{legi}}} = [z_{\theta_i} \times a_{\theta_i} z_{\theta_i} \times a_{\theta_i} z_{\theta_i} \times a_{\theta_i}]
\]

\[
J_{\gamma_i} = \frac{\partial \dot{p}_c}{\partial \gamma_i} = [z_{\gamma_i} \times a_{\gamma_i}]
\]

where \( z_{\gamma_i} \) is the rotation axis vector and \( a_{\gamma_i}(= \vec{a}_{\gamma_i}) \) is the vector connecting each joint center to the corresponding leg contact point. Here, \( v_{ci} \) in (1) is set to zero to prevent slippage of the wheel. (1) is multiplied by \( w R_b \), on both sides to simplify. Therefore, (1) is transformed as follows:

\[
0 = w R_b \dot{p}_b - \dot{w} \dot{p}_c + w R_b R_b \dot{p}_c
\]

\[
+ J_{\text{legi}} \dot{\theta}_{\text{legi}} + J_{\gamma_i} \gamma_i - w R_b \gamma_i
\]

From (9), the entire body motion satisfying the constraint of the wheel is generated for slippage not to occur at the ground contact point. Then, (9) is transformed into matrix form as follows:

\[
0 = \begin{bmatrix} w R_b^T & -B \dot{p}_c \end{bmatrix} \begin{bmatrix} w \dot{p}_b \\ w \omega_B \end{bmatrix}
\]

\[
+ \begin{bmatrix} J_{\text{legi}} & J_{\gamma_i} \end{bmatrix} \begin{bmatrix} \dot{R} \\ \dot{\gamma}_i \\ 0 \end{bmatrix}
\]

\[
= J_{\text{le}} \dot{\theta}_{\text{le}} + J_{\gamma_i} \gamma_i
\]

where \( J_{\text{le}} \dot{\theta}_{\text{le}} \in \mathbb{R}^{3 \times (N+2)} \). Therefore, the position of each leg becomes controllable from the constraint of the leg position.

### 2.3 Singular Configuration of the Robot

The singular configuration of the wheel-legged mobile robot in relation to velocity is described. There are two types of singular configuration problem at the velocity level. First, \( J_\gamma \) defined by (8) becomes \( 0 \) in (10) and (12) when \( a_{\gamma_i}(= \vec{a}_{\gamma_i}) \) becomes parallel to \( z_{\gamma_i} \), i.e., the steering rotation axis being perpendicular to the ground surface. The inverse kinematics solution w.r.t. \( \dot{\gamma}_i \) diverges in this posture. Therefore, the steering joint can not generate suitable motion at the turning motion. This posture is the singular configuration of steering joint (Fig. 5). Second, rank deficiency occurs when solving the simultaneous Eqs. (11) and (13). This case is the singular configuration when solving the constraints simultaneously (Fig. 6). Then, these singular configuration problems are necessary to be solved because they affect the motion generation. The solution of these singular configuration problems is described in the Sect. 3.
3. Using Inverse Kinematics Considering Priority at the Acceleration Level

The singular configuration at the velocity level becomes problem in the motion generation as described in Sect. 2.3. Therefore, we describe the extension of kinematic constraints to the acceleration level for the singular configuration of the steering joint, and the inverse kinematics considering priority for the singular configuration when solving the constraints simultaneously in this section.

There are following problems which relate to the singular configuration at the velocity level. First, the motion of the steering joint is not obtained in the velocity level. Second, the avoidance of the rank deficiency in the velocity level is possible by the numerical calculation method as the LM method. For these problems, the reasons for using the extension of kinematic constraints to the acceleration level and the inverse kinematics considering priority are as follows. First, kinematic constraints are extended to the acceleration level for the singular configuration of steering joint. The inverse kinematics solution can be obtained even in the singular configuration of steering joint because the column of the corresponding Jacobian matrix is modified using acceleration information. Therefore, the extension to kinematic constraint of the acceleration level is an effective technique to solve the singular configuration of the steering joint. Second, task priority is considered for solving the inverse kinematics problem with multiple constraints. This approach avoids the problem of singular configuration as shown in Fig. 6. Then, the application of the inverse kinematics considering priority is known to be effective for motion generation of the redundant robot with multiple constraints.

In addition, the kinematic constraints are necessary to be extended to the acceleration level in order to obtain steering motion. Thus, we apply the inverse kinematics considering priority at the acceleration level. The details of the extension of kinematic constraints to the acceleration level and the inverse kinematics considering priority are described in the Sects. 3.1 and 3.2.

3.1 Kinematic Constraints at the Acceleration Level

The constraint of wheel acceleration is obtained by differentiating (9) with respect to time as follows:

$$\begin{align*}
0 &= w^T_R \ddot{p}_B + w^T_R \dddot{p}_B - B \dot{p}_c \times w^T \omega_B - B \dot{p}_c \times w^T \dot{\omega}_B \\
&+ J_{\text{leg}} \ddot{\theta}_{\text{leg}} + J_y \dddot{y}_i + J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_y \dot{y}_i
\end{align*}$$

(14)

Here, each parameter can be rewritten as follows:

$$\begin{align*}
w^T_R \ddot{p}_B &= (w^T \omega_B \cdot R_B) \dddot{p}_B \\
B \dot{R}_c &= B \dot{w} \cdot R_B \\
B \omega_c &= B \omega_{h_B} + \cdots + B \omega_{h_w} + B \omega_{p_r} \\
= z_{h_i} \dot{\theta}_{h_i} + \cdots + z_{h_B} \dot{\theta}_{h_B} + z_{\gamma_i} \dot{y}_i \\
J_{\text{leg}} \dot{\theta}_{\text{leg}} &= J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_{\gamma} \dot{y}_i \\
B \omega_c &= (J_{\text{leg}} \dot{\theta}_{\text{leg}})^T + J_{\gamma} \dot{y}_i \\
B \omega_{\gamma_i} &= (J_{\text{leg}} \dot{\theta}_{\text{leg}})^T + J_{\gamma} \dot{y}_i
\end{align*}$$

(15-21)

where $J_{\text{leg}} \dot{\theta}_{\text{leg}} \in \mathbb{R}^{3x(N+1)}$, $B \omega_c \in \mathbb{R}^3$. Then, (14) can be transformed into matrix form as follows:

$$\begin{align*}
0 &= \left[\begin{array}{c}
w^T_R \ddot{p}_B \\
J_{\text{leg}} \ddot{\theta}_{\text{leg}} + J_y \dddot{y}_i + J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_y \dot{y}_i
\end{array}\right] \\
&+ \left[\begin{array}{c}
B \dot{R}_c \\
J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_y \dot{y}_i
\end{array}\right] \\
&+ \left[\begin{array}{c}
B \omega_{\gamma_i} + J_{\text{leg}} \dot{\theta}_{\text{leg}} \\
B \omega_{\gamma_i} + J_{\gamma} \dot{y}_i
\end{array}\right]
\end{align*}$$

(22-23)

where $J_{\text{leg}} \dot{\theta}_{\text{leg}} \in \mathbb{R}^{3x(N+1)}$, $B \omega_{\gamma_i} \in \mathbb{R}^3$. In addition, $\eta_i$ is a vector that consists of the angular acceleration of joints and wheel and the angular velocity of the steering joint.

Next, the constraint of the leg position with regard to acceleration is obtained by differentiating (6) with respect to time as follows:

$$\begin{align*}
B \dot{p}_c &= J_{\text{leg}} \ddot{\theta}_{\text{leg}} + J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_y \dddot{y}_i + J_y \dot{y}_i \\
B \dot{p}_c &= \left[\begin{array}{c}
\dot{\theta}_{\text{leg}} \\
\dot{y}_i
\end{array}\right] \\
&+ \left[\begin{array}{c}
J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_y \dot{y}_i
\end{array}\right] \\
&+ \left[\begin{array}{c}
J_{\text{leg}} \dot{\theta}_{\text{leg}} + J_y \dot{y}_i
\end{array}\right]
\end{align*}$$

(24-25)

where $J_{\text{leg}} \dot{\theta}_{\text{leg}} \in \mathbb{R}^{3x(N+2)}$. In addition, $\eta_i$ is a vector that consists of the angular acceleration of joints and wheel and the angular velocity of the steering joint.

The kinematic constraints on acceleration considering all legs are expressed as follows:

$$\begin{align*}
\begin{bmatrix}
0 \\
J_{\text{leg}_1} \\
\vdots \\
J_{\text{leg}_\alpha}
\end{bmatrix} &+ \begin{bmatrix}
J_{\text{acc}_{\text{L}}^\alpha} \\
\ddot{\xi}_B \\
\dddot{\xi}_B
\end{bmatrix} + \begin{bmatrix}
0 \\
J_{\text{acc}_{\text{L}}^\alpha} \\
\dddot{\xi}_B
\end{bmatrix} \eta_i + \begin{bmatrix}
\dddot{\xi}_B
\end{bmatrix}
\end{align*}$$

(27)
Then, (27) and (28) are combined as follows:

\[
\begin{align*}
\begin{bmatrix} \dot{\hat{p}}_w \\ \dot{\hat{p}}_3 \\ \vdots \\ \dot{\hat{p}}_2 \\ \dot{\hat{p}}_1 \end{bmatrix} &= \begin{bmatrix} J_{\text{acc}} & 0 & \cdots & 0 \\ 0 & J_{\text{acc}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_{\text{acc}} \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_k \end{bmatrix} \\
\end{align*}
\]

(28)

Therefore, the constraint of the wheel is the high-priority task, and the position of the leg position follow the target trajectory. The reference value of the torque for each joint is obtained by the acceleration control and the disturbance observer. The reference value of the torque is given as follows:

\[
\tau_{\text{ref}} = M_n \ddot{\eta} + \tau_{\text{dis}},
\]

(36)

where \(M_n\) is the diagonal matrix of the nominal inertia. \(\ddot{\eta}\) is the estimated disturbance torque by the disturbance observer. Then, \(\dot{\eta}_{\text{ref}}\) is given as follows:

\[
\dot{\eta}_{\text{ref}} = \dot{\eta}_{\text{ref}} + K_p (\eta_{\text{ref}} - \eta_{\text{ref}}) + K_d (\dot{\eta}_{\text{ref}} - \dot{\eta}_{\text{ref}})
\]

(37)

The reference value of the torque for the steering joint is obtained by the velocity control as follows:

\[
\tau_{\gamma} = K_{\gamma} (\dot{\gamma} - \dot{\gamma}) + \tau_{\text{dis}},
\]

(38)

where \(K_{\gamma}\) is the proportional gain of the steering joint.

4. Singular Configuration Problem in Low-speed Wheeled Locomotion

In the previous section, the method that uses inverse kinematics that considers priority in regard to acceleration for the singular configuration of the wheel-legged mobile robot is described. However, a new singular configuration occurs in low-speed wheeled locomotion.

The coefficient vector of \(\gamma\) becomes the zero vector in (22), when the vector \(a_\gamma\) is coaxial to the rotation axis vector \(z_\gamma\) and the wheel angular velocity \(\phi_\gamma = 0\). This condition is the singular configuration of low-speed wheeled locomotion. Therefore, \(\dot{\gamma}_{\text{cmd}}\) becomes zero when the wheel is rotated at low-speed and stopped. Then, the steering joint undergoes vibration or divergence.

In this paper, the LM method for the singular configuration of low-speed wheeled locomotion is applied. In particular, the damping factor in the LM method affects the calculation results. Therefore, the conventional determination method of the damping factor is described, and the determination method of the damping factor for the wheel-legged mobile robot is proposed.

4.1 Application of the LM Method

The LM method to the pseudo-inverse matrix of the inverse kinematics considering priority with regard to acceleration is applied. Therefore, the pseudo-inverse matrix in (30) and (31) are extended as follows:

\[
\begin{align*}
J_{\text{acc}}^+ &= J_{\text{acc}}^T (J_{\text{acc}} J_{\text{acc}}^T + \lambda I)^{-1} \\
J_{L}^+ &= J_{L}^T (J_{L} J_{L}^T + \lambda L)^{-1}
\end{align*}
\]

(39)

(40)

From (39) and (40), the LM method prevents rank deficiency of the Jacobian matrix in the singular configuration by adding a damping factor in the pseudo-inverse matrix. The inverse
kinematics solution is not maximized, because the pseudo-inverse matrix has full rank in the singular configuration. Therefore, the LM method can suppress excessive output in the singular configuration.

4.2 Determination Methods of the Damping Factor

The damping factor of the LM method greatly affects the solution of the inverse kinematics calculation. Therefore, determination of the damping factor in accord with the motion of the robot is necessary and related studies related are classified into two groups, as follows.

- Methods based on error minimization (25–27)
- Methods based on robot manipulability (28–31)

In this paper, an effective method for the singular configuration in low-speed wheeled locomotion of the wheel-legged mobile robot is considered. Therefore, Sugihara’s method (25) and Deo’s method (28) both based on error minimization, and Ford’s method (29) based on robot manipulability are applied.

In addition, a new method based on manipulability is also proposed.

4.2.1 Method Based on the Error Minimization Problem

Sugihara proposed a method using a damping factor that added a small bias to the square norm of the error (25) and method based on the error minimization problem for the wheel-legged mobile robot is applied.

The error vector is defined as follows in the kinematic constraints on acceleration of all legs in (29).

\[ e_W = J_W \dot{s}_{\text{cmd}}^w - J_W \dot{s}_{\text{ref}}^w \]  \hspace{1cm} (41)
\[ e_L = \dot{\varphi}_L \dot{L} - \ddot{\varphi}_L \] \hspace{1cm} (42)

where \( e_W \in \mathbb{R}^3 \) is an error vector of the base link acceleration, and \( e_L \in \mathbb{R}^3 \) is an error vector of the leg acceleration. In addition, \( e_W \) is converted into the same space as \( e_L \) using the Jacobian matrix of the base link. Then, the minimization problem of the error vector is as follows:

\[ E_W(\bar{\theta}) = \frac{1}{2} e_W^T e_W \rightarrow \text{min}. \] \hspace{1cm} (43)
\[ E_L(\bar{\theta}) = \frac{1}{2} e_L^T e_L \rightarrow \text{min}. \] \hspace{1cm} (44)

Thus, \( \lambda_W I, \lambda_L I \in \mathbb{R}^{3 \times 3} \) are terms of the damping factor in each constraint given as follows:

\[ \lambda_W I = E_W(\bar{\theta}) I + \overline{W}_W \] \hspace{1cm} (45)
\[ \lambda_L I = E_L(\bar{\theta}) I + \overline{W}_L \] \hspace{1cm} (46)

where \( \overline{W}_W, \overline{W}_L \in \mathbb{R}^{3 \times 3} \) are amounts of small bias. Here, \( \overline{W}_W = \text{diag}(\overline{\omega}_W) \), and \( \overline{W}_L = \text{diag}(\overline{\omega}_L) \) are given.

4.2.2 Method Based on the Trust Region Method

Deo et al. proposed a damping factor (28) based on the trust region method (32) and the same is applied to the wheel-legged mobile robot, and the minimization problem of the error norm for each constraint of the wheel-legged mobile robot is solved. Thereby, the penalty function is obtained as follows:

\[ ||\omega_W|| = C_W \] \hspace{1cm} (47)
\[ ||\omega_L|| = C_L \] \hspace{1cm} (48)

where

\[ \omega_W = J_W^{\text{acc}} (J_W^{\text{acc}} J_W^{\text{acc}} + \lambda_W I)^{-1} (J_W \dot{\varphi}_B - b_W) \]

Next, obtain the \( \lambda_W, \lambda_L \) by solving (47) and (48) using Newton’s method (33). Here, (47) and (48) are transformed as follows:

\[ \frac{1}{||\omega_W||} - C_W = 0 \] \hspace{1cm} (51)
\[ \frac{1}{||\omega_L||} - C_L = 0 \] \hspace{1cm} (52)

Thus, the updated equation of Newton’s method for the constraint of the wheel is obtained as follows:

\[ \lambda_W(k + 1) = \lambda_W(k) + \frac{f_{W1}(k)}{f_{W2}(k)} \] \hspace{1cm} (53)

\[ f_{W1}(k) = ||\omega_W(k)||^2 (||\omega_W(k)|| - C_W) \] \hspace{1cm} (54)

\[ f_{W2}(k) = C_W \omega_W(k) (J_W^{\text{acc}}(k)J_W^{\text{acc}}(k) + \lambda_W(k)I)^{-1} (J_W^{\text{acc}}(k)J_W^{\text{acc}}(k) + \lambda_W(k)I)^{-1} \]

\[ \frac{1}{||\omega_L||} C_L = 0 \] \hspace{1cm} (52)

Similarly, the updated equation of Newton’s method for the constraint of the leg position is obtained as follows:

\[ \lambda_L(k + 1) = \lambda_L(k) + \frac{f_{L1}(k)}{f_{L2}(k)} \] \hspace{1cm} (56)

\[ f_{L1}(k) = ||\omega_L(k)||^2 (||\omega_L(k)|| - C_L) \] \hspace{1cm} (57)

\[ f_{L2}(k) = C_L \omega_L(k) (J_L^{\text{acc}}(k)J_L^{\text{acc}}(k) + \lambda_L(k)I)^{-1} (J_L^{\text{acc}}(k)J_L^{\text{acc}}(k) + \lambda_L(k)I)^{-1} \]

Thus, normal inverse kinematics in the trust region is realized and the damping factor suppression is computed to output the trust region when excessive output is generated in the singular configuration. Deo’s method discriminates every calculation whose output is within the trust region. Then, the damping factor is calculated when output is outside of the trust region. In addition, the damping factor is set to zero when output is in the trust region, and in this paper, the calculations are performed in a similar manner.

4.2.3 Method Based on Robot Manipulability

Ford et al. proposed a method using the value of the determinant of the Jacobian matrix to the negative exponential of the Napier number as the damping factor (29), and it is applied to the wheel-legged mobile robot.

Given the damping factor for the constraint of the wheel \( \lambda_W \) and the damping factor for the constraint of the leg position \( \lambda_L \) as follows:

\[ \lambda_W = \alpha_W e^{-\det(J_W^{\text{acc}} J_W^{\text{acc}})} \] \hspace{1cm} (59)
\[ \lambda_L = \alpha_L e^{-\det(J_L^{\text{acc}} J_L^{\text{acc}})} \] \hspace{1cm} (60)

where \( \alpha_W, \alpha_L \) are values obtained empirically.
4.2.4 Proposed LM Method

Ford’s method cannot perform the suppression in accordance with the posture of each leg for the inverse kinematic calculation of the plurality of legs for the wheel-legged mobile robot, because the same damping must be provided for all the legs. In addition, the damping effect affects motion performance, because the damping factor is not reduced too far from the singular configuration. Therefore, a damping factor considering the posture of each leg and the effect of damping after leaving the singular configuration is proposed. Moreover, the wheel-legged mobile robot must effectively reflect steering motion, because the singular configuration of the steering joint becomes a problem. Thus, an evaluation index of effectively reflected motion of the steering joint by decomposing the manipulability of the steering joint from the manipulability of each leg is proposed, and the term on the damping factor of each constraint \(\lambda W I, \lambda L I \in \mathbb{R}^{3 \times 3}\) are given as follows:

\[
\begin{align*}
\lambda W I &= \begin{bmatrix}
\lambda W_1 & 0 & \cdots & 0 \\
0 & \lambda W_2 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \lambda W_k
\end{bmatrix}, \\
\lambda L I &= \begin{bmatrix}
\lambda L_1 & 0 & \cdots & 0 \\
0 & \lambda L_2 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \lambda L_k
\end{bmatrix}
\end{align*}
\]

where

\[
\begin{align*}
\lambda W_i &= \alpha W_i e^{-w W \beta W I_{dli}} \quad (63) \\
\lambda L_i &= \alpha L_i e^{-w L \beta L I_{dli}} \quad (64)
\end{align*}
\]

and \(I_{dli} \in \mathbb{R}^{3 \times 3}\) is the identity matrix. \(\beta W_i, \beta L_i\) are scalar parameters. Here, the evaluation index \(w W_i, w L_i\) are given as follows:

\[
\begin{align*}
w W_i &= K_{WW} \sqrt{\det(J_{legi}^T J_{legi}) + K_{Wyi} \det(k_{Wy i} k_{Wy i})} \\
w L_i &= K_{L} \sqrt{\det(J_{legi}^T J_{legi}) + K_{Li} \det(k_{Lyi} k_{Lyi})}
\end{align*}
\]

\[
\begin{align*}
\kappa_{Wyi} &= \frac{\partial \dot{u}_i}{\partial \gamma_i} \\
\kappa_{Lyi} &= \frac{\partial \dot{u}_i}{\partial \gamma_i}
\end{align*}
\]

where \(\dot{u}_i\) and \(\dot{u}_i\) are kinematic constraints at the acceleration level. The first term in (65) and (66) is the manipulability of the third joint from the first joint. The second term in (65) and (66) is the singular value of the joint on which we focus. The singular value of the target joint represents the size of the rotational motion that can be removed from the target joint. Normally, each index is not suitable for being combined, because the order is different. Therefore, each index is deformed to become the same order (64).

In the case of the developed robot, we focus on the steering joint as the target joint. The singular values of the steering joint in (67) and (68) are given as follows:

\[
\begin{align*}
\kappa_{Wyi} &= J_{yi} + \omega_{i}^2 J_{yi} - J_{i}^{\omega \times} R_{i} \begin{bmatrix}
\dot{R}_{i} & 0 \\
0 & 0
\end{bmatrix} \\
\kappa_{Lyi} &= J_{yi}
\end{align*}
\]

The singular value of the steering joint represents the size of the rotational motion that can be extracted from the steering joint.

From the above, the LM method considering the manipulability of each leg is realized. The LM method is expected to influence the steering joint to be reflected effectively. Therefore, the effective suppression of the divergence and the vibration is realized when the wheel is rotating in the stopped or low-speed positions. The pseudo-inverse matrices in (39) and (40) become full rank when the damping factors have a positive value in the LM method. In this case, the inverse matrix calculation is possible. The proposed method determines the damping factor using the manipulability. Then, the damping factor is increased by the effect of the manipulability in the singular configuration by the rank deficiency of the pseudo-inverse matrix. Therefore, the inverse kinematics calculation is always stable even in the singular configuration.

5. Simulation and Experimental Results

The verification is made by using three-dimensional simulation of motion generation in consideration of the singular configuration in low-speed wheeled locomotion. The simulation uses the robot control simulator (ROCOS) (35). Here, the steering behavior in the case of providing a low-speed constant velocity trajectory to \(\Phi, \Phi B\), and \(\Phi p_{b}\) is confirmed. As simple case, straight motion as simple motion is provided, and slalom motion as complicated case. Next, the effect of the proposed method through experiment is verified, and a comparison of the method based on robot manipulability and the proposed method is given. Here, the behavior when giving the target trajectory to \(\Phi, \Phi B\), and \(\Phi p_{b}\), as in the simulation, is confirmed. The target trajectory provides straight motion and slalom motion. Finally, the condition of the simulation and the experiment is shown in Table 2. The overall block diagram of the control system is shown in Fig. 7.

5.1 Simulation Results of Straight Motion

In this subsection, a target trajectory that moves 0.1 [m/s] in the direction of the base link is provided, thus, confirming the steering behavior in the case of straight motion at constant speed.

The simulation results of the steering joint angle of the first leg and the tracking trajectory of the base position in the x-direction are shown in Fig. 8. First, the steering joint angle of a single kinematic constraint on acceleration in Sect. 3 is shown in Fig. 8(a). The steering joint angle with the methods

**Table 2. Simulation and experimental condition**

<table>
<thead>
<tr>
<th>Contents</th>
<th>Simulation</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling time [sec]</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Initial state [m/s]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Proportional gain of base position (k_{u})</td>
<td>diag[0.0, 0.0, 0.0]</td>
<td>diag[0.0, 0.0, 0.0]</td>
</tr>
<tr>
<td>Derivative gain of base position (k_{u})</td>
<td>diag[0.0, 0.0, 0.0]</td>
<td>diag[0.0, 0.0, 0.0]</td>
</tr>
<tr>
<td>Proportional gain of base orientation (k_{\gamma})</td>
<td>diag[0.0, 10.0, 10.0]</td>
<td>diag[0.0, 10.0, 10.0]</td>
</tr>
<tr>
<td>Derivative gain of base orientation (k_{\gamma})</td>
<td>diag[0.0, 10.0, 10.0]</td>
<td>diag[0.0, 10.0, 10.0]</td>
</tr>
<tr>
<td>Proportional gain of tip position (k_{\gamma})</td>
<td>diag[0.0, 10.0, 10.0]</td>
<td>diag[0.0, 10.0, 10.0]</td>
</tr>
<tr>
<td>Derivative gain of tip position (k_{\gamma})</td>
<td>diag[0.0, 10.0, 10.0]</td>
<td>diag[0.0, 10.0, 10.0]</td>
</tr>
</tbody>
</table>
Methods for Singular Configuration of Wheel-Legged Mobile Robot (Kenta Nagano et al.)

Fig. 7. Overall block diagram of the control system

Fig. 8. Simulation results of straight motion

Table 3. Standard deviation of the steering angle in simulation results of straight motion

<table>
<thead>
<tr>
<th>Contents</th>
<th>SD \times 10^{-3} [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A single kinematic constraint on acceleration</td>
<td>72.09568</td>
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<tr>
<td>Method based on error minimization problem</td>
<td>5.961853</td>
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<tr>
<td>Method based on the trust region method</td>
<td>22.66684</td>
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<tr>
<td>Method based on robot manipulability</td>
<td>5.218641</td>
</tr>
<tr>
<td>Proposed LM method</td>
<td>5.675313</td>
</tr>
</tbody>
</table>

Table 4. RMSE of base link in simulation results of straight motion

<table>
<thead>
<tr>
<th>Contents</th>
<th>RMS [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A single kinematic constraint on acceleration</td>
<td>2.436655</td>
</tr>
<tr>
<td>Method based on error minimization problem</td>
<td>2.591461</td>
</tr>
<tr>
<td>Method based on the trust region method</td>
<td>5.218641</td>
</tr>
<tr>
<td>Method based on robot manipulability</td>
<td>2.690212</td>
</tr>
<tr>
<td>Proposed LM method</td>
<td>2.690212</td>
</tr>
</tbody>
</table>

Based on the error minimization in Sect. 4.2.1 and the trust region method in Sect. 4.2.2 are shown in Fig. 8(b). The steering joint angle with the methods based on robot manipulability in Sect. 4.2.3 and the proposed method in Sect. 4.2.4 are shown in Fig. 8(c). Second, the base position with a single kinematic constraint on acceleration in Sect. 3 is shown in Fig. 8(d). The base position with the methods based on the error minimization in Sect. 4.2.1 and the trust region method in Sect. 4.2.2 are shown in Fig. 8(e). The base position with the methods based on robot manipulability in Sect. 4.2.3 and the proposed method in Sect. 4.2.4 are shown in Fig. 8(f). Finally, the standard deviation of the steering angle and the root mean square error (RMSE) of the tracking trajectory of the base position are shown in Tables 3 and 4, respectively.

From the simulation results, it can be seen that steering joint vibration occurs in the case of a single kinematic constraint on acceleration for the starting motion in Fig. 8(a). In addition, a tracking error occurs in the base link for the starting motion in Fig. 8(d), as a result of the vibration of the steering joint. The method based on the error minimization problem is realized for vibration suppression of the steering joint in Fig. 8(b). However, a tracking error occurs in the base link in Fig. 8(e), because the damping effect is increased by the accumulation of errors. The method based on the trust region method is realized in the vibration suppression of the
steering joint in Fig. 8(b). However, the vibration suppression effect is small, because the trust region cannot be reduced to realize any motion. In addition, base position tracking is realized in Fig. 8(e). The method based on robot manipulability is realized in the vibration suppression of the steering joint in Fig. 8(c). However, a tracking error of the base position occurs in Fig. 8(f). The manipulability changes little, when the steering joint becomes the singular configuration, because this method determines the damping factor based on full robot manipulability. Therefore, a constant error occurs, because the damping factor changes little. The proposed method effectively realizes steering joint vibration suppression in Fig. 8(c), because the damping factor is determined by the posture of each leg. In addition, the tracking error of the base position is reduced in Fig. 8(f), because the effect of the steering joint is reflected. Then, the method based on the error minimization problem, the method based on robot manipulability, and the proposed method realize effective vibration suppression from the standard deviation of the steering angle in Table 3. In addition, the single kinematic constraint on acceleration, the method based on the trust region method, and the proposed method realize the tracking trajectory of the base position from the RMSE of the tracking trajectory of the base position in Table 4. From the above, the proposed method is seen to be the best realization method of trajectory tracking of the base position and motion generation of the steering joint in the singular configuration.

5.2 Simulation Results of Slalom Motion  In this subsection, a target trajectory of the base link is provided as follows:

\[ w_{x_B} = at \]  \hspace{1cm} \text{(71)}

\[ w_{y_B} = b \left(1 - \cos \left(\frac{2\pi t}{T}\right)\right) \]  \hspace{1cm} \text{(72)}

where \( a = 0.25, b = 0.05, T = 10.0 \). Thus, the steering behavior in the case of slalom motion is confirmed.

The simulation results of the steering joint angle of the first leg and the tracking trajectory of the base position in the x-direction and the y-direction are shown in Fig. 9. First, Fig. 9(a) shows the steering joint angle of a single kinematic constraint on acceleration in Sect. 3. Figure 9(b) shows the steering joint angle with the methods based on the error minimization problem with a scaling factor. The damping factors (45) and (46) are modified as \( \lambda_{W1} = k_{W1}E_{W1}(\dot{\theta}l) + \mathbf{W}_{1W} \) and \( \lambda_{L1} = k_{L1}E_{L1}(\ddot{\theta}l) + \mathbf{W}_{1L} \), where \( k_{W1} \) and \( k_{L1} \) are the scaling factors. The results are improved as compared with the original method based on error minimization problem in Fig. 9(b), but the tracking error is still large. Figure 9(c) shows the steering joint angle with the methods based on robot manipulability in Sect. 4.2.3 and the proposed method in Sect. 4.2.4. Second, Fig. 9(d) shows the base position of a single kinematic constraint on acceleration in Sect. 3. Figure 9(e) shows the base position with the methods based on the error minimization problem in Sect. 4.2.1 and the trust region method in Sect. 4.2.2, and showing the method based on error minimization problem with a scaling factor. The damping factors (45) and (46) are modified as \( \lambda_{W1} = k_{W1}E_{W1}(\dot{\theta}l) + \mathbf{W}_{1W} \) and \( \lambda_{L1} = k_{L1}E_{L1}(\ddot{\theta}l) + \mathbf{W}_{1L} \), where \( k_{W1} \) and \( k_{L1} \) are the scaling factors. The results are improved as compared with the original method based on error minimization problem in Fig. 9(b), but the tracking error is still large. Figure 9(f) shows the base position with the methods based on robot manipulability in Sect. 4.2.3 and the proposed method in Sect. 4.2.4. Finally, the RMSE of the steering joint and the tracking trajectory of the base position are shown in Tables 5 and 6, respectively. Here, the steering angle error is the error between the ideal steering angle that is calculated from the geometric relationships.

From the simulation results, the same effect of vibration suppression and the trajectory tracking as in the straight motion can be confirmed in slalom motion. Then, the method
based on robot manipulability and the proposed method realize effective vibration suppression from the RMSE of the steering angle in Table 5. In addition, a single kinematic constraint on acceleration, the method based on the trust region method, and the proposed method realize the tracking trajectory of the base position from the RMSE of the tracking trajectory of the base position in Table 6. From the above results, the proposed method is seen to be the best realization method of tracking trajectory of the base position and motion generation of the steering joint in the singular configuration of complicated motion.

5.3 Experimental Results of Straight motion

The steering behavior in the case of straight motion at constant speed is confirmed, and the target trajectory to the base link that moved 0.40 [m] in 4.0 [s] in the x-direction is obtained. In addition, the target trajectory to the base link in order to make the same initial posture with the simulation that rose 0.104 [m] in 4.0 [s] in the z-direction is also obtained.

The experimental results of the position and the tracking error of the base link, the angle of the base link, and the joint angle of the first leg are shown in Figs. 10 and 11, respectively. Next, the experimental result of the method based on robot manipulability in Sect. 4.2.3 is shown in Fig. 10. The experimental result of the proposed method in Sect. 4.2.4 is shown in Fig. 11. From the above results, the proposed method is seen to be the best realization method of tracking trajectory of the base position and motion generation of the steering joint in the singular configuration of complicated motion.

5.4 Experimental Results of Slalom Motion

The steering behavior in the case of slalom motion, and the target trajectory to the base link that moved 1.25 [m] in 5.0 [s] in the x-direction and 0.10 [m] in 5.0 [s] in the y-direction is obtained. In addition, the target trajectory to the base link in order to make the same initial posture with the simulation that rose 0.104 [m] in 4.0 [s] in the z-direction is also obtained.

From the experimental results, it can be seen that both methods realize motion with regard to the kinematic constraints on acceleration in Figs. 10 and 11. The method based on robot manipulability does not change all the damping factor in Fig. 12. In contrast, the proposed method changes the damping factor of the constraint of the wheel in Fig. 13(a). The damping factor of the constraint of the leg position is not changed in Fig. 13(b), because the effect of the steering motion is small in the constraint of the leg position. The proposed method of tracking errors has a lower result than the conventional method, when comparing the RMSE of the tracking trajectory of the base position in Table 7, because the damping factor changes.

### Table 5. RMSE of the steering angle in simulation results of slalom motion

<table>
<thead>
<tr>
<th>Contents</th>
<th>RMS x10^{-2} [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A single kinematic constraint on acceleration</td>
<td>12.04838</td>
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<tr>
<td>Method based on error minimization problem</td>
<td>9.200365</td>
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<tr>
<td>Method based on error minimization problem with scaling</td>
<td>6.228316</td>
</tr>
<tr>
<td>Method based on the trust region method</td>
<td>7.850311</td>
</tr>
<tr>
<td>Method based on robot manipulability</td>
<td>5.021265</td>
</tr>
<tr>
<td>Proposed LM method</td>
<td>3.395431</td>
</tr>
</tbody>
</table>

### Table 6. RMSE of base link in simulation results of slalom motion

<table>
<thead>
<tr>
<th>Contents</th>
<th>RMS [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A single kinematic constraint on acceleration</td>
<td>7.208521</td>
</tr>
<tr>
<td>Method based on error minimization problem</td>
<td>248.8571</td>
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<tr>
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<td>Method based on robot manipulability</td>
<td>106.3476</td>
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<td>Proposed LM method</td>
<td>6.466350</td>
</tr>
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</table>
Methods for Singular Configuration of Wheel-Legged Mobile Robot (Kenta Nagano et al.)

Fig. 11. Experimental results of straight motion in the case of the proposed LM method

respectively. Next, the experimental result of the method based on robot manipulability in Sect. 4.2.3 is shown in Fig. 14. The experimental result of the proposed method in Sect. 4.2.4 is shown in Fig. 15. In addition, the damping factor of the method based on robot manipulability and the proposed method are shown in Figs. 16 and 17, respectively. Finally, the RMSE of the tracking trajectory of the base position is shown in Table 8.

From the experimental results, the tracking error of the base link in the method based on the manipulability occurs at slalom motion in Fig. 14. In contrast, the proposed method realizes the smaller tracking error of the base link than the conventional method as shown in Fig. 15 and Table 8. In

### Table 7. RMSE of base link in experimental results of straight motion

<table>
<thead>
<tr>
<th>Contents</th>
<th>RMS [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method based on robot manipulability</td>
<td>2.380860</td>
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<tr>
<td>Proposed LM method</td>
<td>2.083389</td>
</tr>
</tbody>
</table>
Methods for Singular Configuration of Wheel-Legged Mobile Robot (Kenta Nagano et al.)

Fig. 14. Experimental results of slalom motion in the case of the method based on robot manipulability

Fig. 15. Experimental results of slalom motion in the case of the proposed LM method

Table 8. RMSE of base link in experimental results of slalom motion

<table>
<thead>
<tr>
<th>Contents</th>
<th>RMS [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method based on robot manipulability</td>
<td>6.207844</td>
</tr>
<tr>
<td>Proposed LM method</td>
<td>4.629907</td>
</tr>
</tbody>
</table>

addition, the damping factor of the method based on the manipulability does not change in Fig. 16 because the change of the manipulability is small. The damping factor of the proposed method changes corresponding to the motion of the robot in Fig. 17. Therefore, the proposed method reduces the tracking error of the base link because the damping factor changes according to the motion of the robot. From the above, stable motion in the singular configuration can be realized by the proposed method, which suppresses the error of the base position.
Methods for Singular Configuration of Wheel-Legged Mobile Robot (Kenta Nagano et al.)

In this paper, the inverse kinematics considering priority in regard to acceleration for the singular configuration of the wheel-legged mobile robot is applied. Three types of determination methods for the damping factor in the LM method were also applied to the wheel-legged mobile robot. In addition, a determination method of the damping factor was proposed, comparison between the effects through three-dimensional simulation for each method was done. From the simulation results, the proposed method has produced effective vibration suppression in the singular configuration of low-speed wheeled locomotion. Finally, experiments with the method based on robot manipulability and the proposed method were performed, and from the experimental results, it was noticed that the proposed method reduced the effect in the base position.

References

Methods for Singular Configuration of Wheel-Legged Mobile Robot (Kenta Nagano et al.)


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