Vibration Suppression of Electromagnetic-Force-Restoration Weighing Cell Using Wave Control

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(Manuscript received June 30, 2018, revised Oct. 9, 2018)

High-speed and high-precision weighing is required in industrial and medical fields. Recently, medical services have improved rapidly and the demand for medicines have increased. Because improper medication can lead to life-threatening symptoms, precise filling and weighing all medicines are vital processes. On the other hand, there are requirements from suppliers to increase production volume. For this problem, reduction the time by applying vibration-suppression control for a weighing cell is presented. The weighing cell exhibits oscillating responses as it has vibration mechanism inside. This vibration increases the weighing time and then reflected-wave rejection is introduced to suppress the vibration. Because reflected-wave rejection is a phase-stabilizing method, the controller attains a robustness against modeling errors. As a result, the weighing cell did not vibrate regardless of existence of a load. In addition, lift-up operations of vial bottles with water by the weighing cell were also conducted. The water surface did not fluctuate and immediately settled. Reflected-wave rejection certainly contributes to reducing the time for weighing.

Keywords: sensorless force control, disturbance observer, nominal parameter

1. Introduction

Precise servoing enhances a capability of industrial machines and production quality, while precise measurement establishes retention of high quality and reliability for products (1). Pursuit of precision is essential for the high-quality production. On the other hands, speeding up of manufacturing process is important to increase production volume and meet demands from consumers. This dilemma is one of the problems in the industry, and this problem is addressed by modifying the whole system, including a mechanical system, a process sequence, and a servo control system.

In the medical field, there is demands for increasing a production volume (2)–(4). The medical service improves rapidly and users accordingly increases with demanding various medicines. Owing to development and diversification of the service, the medicines are certainly good remedies, but it has possibility to become a life-threatening material. Advanced medical service, such as anti-cancer therapy, require tight restriction for dosage and product management becomes high responsible. To maintain the high-quality, manufacturers inspect whole products, called as full-inspection. Though this inspection process certainly ensures the quality and reliability, it takes a lot of time. To shorten the process time, there are many approaches; using the fast-response measurement, modifying a feeding system under mechanical constraints, suppressing vibrations which occur in process.

Manufacturing process is composed of three stages; feeding containers, filling medicines to the containers, weighing the containers filled with the medicines. To shorten a weighing time, electromotive-force-restoration (EMFR) weighing cells are widely used (5)–(8). In weighing process, there is subprocess that transporting the containers upon the weighing cells. The containers are transported upon the weighing cells in conventional design, but it takes a lot of time due to mechanical constraints of transporters and a manufacturing line. Here, we could find that this time can be reduced by transporting the weighing cells to a place of the containers because of less mechanical constraints. This architecture also reduces complexity of the transporter and production cost. However, this causes the other problem that the transporting the weighing cells induces their natural vibrations. This causes damages to a mechanical system of the weighing cell and the weighing system is forced to wait for vibration to subside. Here, we also found that there is room to shorten the weighing process by applying vibration suppression control.

Vibration-suppression control has been widely researching for speeding-up of manufacturing process. Input shaping methods which is able to be introduced with few resource and does not affect to a stability are widely applied to industrial motion-controllers (9)–(10). Although applying such methods is very simple, these are not robust against model error and then feedback controllers are introduced. In practical use, tolerance against uncertainties such as noise, disturbance and model errors, called as robustness, is important and vibration suppressions while attaining the robustness have widely
The vibration suppression of the Electromagnetic-Force-Restoration (EMFR) weighing cell is discussed in this paper. The EMFR weighing cell is used in the process of measuring the weight of vial bottles. The process is composed as follows: (a) a bottle feeder on the conveyor line transports the vial bottles to predefined positions, (b) the dual shaft-motors move up the EMFR weighing cell, (c) the EMFR weighing cell lifts up the vial bottle and measures its weight, (d) the dual shaft-motors move down the EMFR weighing cell, (e) the EMFR weighing cell measures the weight without the container for zero-point adjustment. Thus, the weighing process has measurements operations twice a cycle. For this system, the paper describes how to introduce the vibration suppression control as follows. In section 3, the robust control technique of the shaft motors and cooperative control of them are introduced to control the motion of the transporter precisely. Since the dual shaft-motors have possibility to cause a torsional vibration of the transporter, the cooperation controller is designed to suppress the torsion. In section 4, characteristics of the EMFR weighing cell is examined in order to introduce the vibration-suppression controller. In section 5, a method how to introduce reflected-wave rejection is presented. In this work, the reflected-wave rejection is installed on the controller for the cooperation controller. In Fig. 2, here, \( s \), \( x_m \), \( I \), \( d \), \( K_t \), \( M_m \), and \( Q \) are Laplace operator, motor position, armature current, disturbance acting on the motor, thrust force constant, motor mass, and Q filter of the DOB, respectively. The Q...
filter determines a bandwidth of the acceleration control and should be set at high value as possible. Superscripts $\bar{\omega}^{\text{ref}}$ and $\bar{\omega}^{\text{mp}}$ stand for a reference signal and compensation signal calculated by the DOB, and a subscript $\bar{\omega}$ denotes a nominal value. An input-output transfer function and disturbance-suppression characteristics is expressed as

$$\frac{x_m}{\bar{x}_m} = \frac{1}{s^2 \alpha Q + (1 - Q)} \tag{1}$$

$$\frac{x_m}{d} = \frac{1}{s M_m s^2 \alpha Q + (1 - Q)} \tag{2}$$

$$\alpha = \frac{K_t}{M_t} \tag{3}$$

where $\alpha$ shows an index of modeling error. This equation denotes that the acceleration control is able to be realized under existence of the modeling error\(^{(15)}\). In this study, the paper designs the interaction mode control on an assumption that the acceleration control is attained within the control bandwidth.

### 3.2 Interaction Mode Control

To control the COG position and the torsion of the motors, the interaction mode control is introduced. A block diagram of the interaction mode control is shown in Fig. 3. Here, $x_m$, $m$, $C_p$, and $T$ denote vectors of motor positions and modal space positions, a controller matrix and a modal-transformation matrix. In this time, COG position and the torsion is set as modal space positions and abstract from $x_m$ as

$$m = Tx_m \tag{4}$$

$$T = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{5}$$

It should be noted that the acceleration control is also established on the modal space owing to the DOB. A transfer function from modal-acceleration reference to modal position is derived as

$$m = Ts^{-2}IT^{-1}\bar{m}^{\text{ref}} = s^{-2}\bar{m}^{\text{ref}}, \tag{6}$$

where $I$ is a $2 \times 2$ unit matrix. It also denotes that the modal space is decoupled in the control bandwidth, namely the COG position and torsion are able to be controlled individually. Then, controller design become quite simple\(^{(14)}\).

### 3.3 Verification of Positioning Accuracy

The transporter should be settled with little fluctuation to make the weighing cell be settled in steady state. The acceleration control and the interaction mode control is applied to the transporter and the paper checks the positioning performance. Parameters used in this verification is listed in Table 1. The control period is set at 80 $\mu$s. In this time, commands to the transporter is set as

$$m^{\text{cmd}} = \begin{bmatrix} 3.8 \times 10^{-3} & 0 \end{bmatrix}^T \tag{7}$$

and the $Q$ filter and the controller matrix are designed as

$$Q = \begin{bmatrix} 1000 \\ s + 1000 \end{bmatrix} \tag{8}$$

$$C_p = \text{diag} [10000 + 200s, 4900 + 140s] \tag{9}$$

A positioning response is shown in Fig. 4. The upper one is whole result and the lower is enlarged view focusing on the steady-state response. This figure shows that the there is little torsion between two motors and the COG position is settled with little fluctuation. This result denotes that the interaction mode control correctly operate and robust control is attained.

### 4. Dynamics of the EMFR Weighing Cell

The output of the EMFR weight cell oscillates when it is transported. This is because the EMFR weight cell has a PID controller and phase compensators to restore a lever position exists in its body to a pre-defined point\(^{(15)}\). An operating principle of the EMFR weight cell is to measure required force to restore the lever position pushed by a load mass. This system picks up inertial force excited by acceleration/deceleration in the transportation and then natural vibration is induced. To check this phenomena, the paper shows the verification using an EMFR weighing cell. In this verification, a sweep signal is set as the COG-position command.
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and the output of the EMFR weighing cell is collected. Here, no load is installed on the weighing cell excepting a weighing pan. A position response of the transporter and an output of the EMFR weighing cell are shown in Fig. 5. Due to the bandwidth of the position control, the transporter does not track to the sweep signal. However, it is not a problem because general form of a frequency response from the transporter position to the output of the EMFR weighing cell can be obtained regardless of the waveform of the transporter position. From this result, their power spectrums and frequency response of the EMFR weighing cell are obtained as Fig. 6. As for the gain characteristics, it shows a slope of 40 dB/dec within 50 Hz. It means that the EMFR weighing cell picks up the inertial force. This inertial force is excited by acceleration/deceleration of the weighing pan. Therefore, vibration suppression of the weighing pan is required to speeding up the settling speed of the EMFR weighing cell. As for the phase response, it started from 180 deg and denoted that the output is correlated to acceleration. A phase lag is caused by a dissipation term in the EMFR weighing cell. The paper handled this system as a minimum-phase system, because the EMFR weighing cell outputs no undershoot against moving of the transporter (it can be confirmed in section 6). Because the non-minimum-phase system is expressed as combination of a corresponding minimum-phase system and an all-pass filter which has zeros on the right-half plane, it causes the undershoot. Otherwise, when the undershoot is not confirmed, zeros do not exist in the right-half plane excepting near the imaginary axis and then the system can be considered as the minimum-phase system. If there are not zeros significantly deteriorating the phase responses, the phase-stabilization methods can be implemented. Then, the paper assumed this plant as the minimum-phase system.

Here, three important characteristics were found. (i) The position of the weighing pan is observed by using the output of the EMFR weighing cell. This is a major advantage because load-side information can be obtained without additional sensors. (ii) The EMFR weighing cell behaves like a linear system. The output frequency of the EMFR weighing is same with the drive-frequency of the transporter and harmonic distortion was not confirmed. (iii) The vibration persists while exchanging a kinetic energy between restoring force and inertial force. This is a basic phenomenon of the vibration. Since the EMFR weighing cell is almost a linear plant, the vibration suppression control can be applied. It should be noted that weight of the weighing pan changes the frequency response and the vibration-suppression controller should be designed for the finished system.

5. Transportation Based on Wave Control

The flexible structure in the EMFR weighing cell causes the vibration which lengthen the weighing process. Reflected-wave rejection is introduced to suppress the vibration of the weighing pan. First, the paper introduces the mathematical model to apply reflected-wave rejection. Next, reflected-wave rejection is introduced and the control architecture of the transportation with reflected-wave rejection is shown.

5.1 Wave Model

Wave model is one of the representation methods for multi-mass resonant systems. This model is derived from the distributed-constant model governed by the wave equation and express motion with using a traveling wave and a reflected wave. A wave model is shown in Fig. 7, where $x_{tp}$, $x_{wp}$, and $T_w$ denote positions of the transporter and the weighing pan, and the propagation time of the traveling wave and the reflected wave. This figure shows that the wave model is described with using a time-delay element $e^{-sT_w}$. A relation between transporter position and weighing-pan position is expressed as

$$x_{wp} = \frac{2e^{-sT_w}}{1 + 2e^{-sT_w}} x_{tp}.$$

The wave model has infinite number of poles and these poles are arranged at equal intervals on the imaginary axis. In this model, a resonance is represented as superposition of the traveling wave and reflected wave. Here, the vibration does not occur when the reflected wave is correctly rejected.

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When controller directly related to the performance of the vibration admittance controller and the position of the load-side vibration can be suppressed. The 1st-order resonant frequency of the wave model is

\[ \omega_{w1} = \frac{\pi}{2T_w} \]  

Assuming the 1st-order resonant frequency of the actual system is \( \omega_{w1} \), the following relation is derived:

\[ \omega_{w1} = \frac{\pi}{2T_w} \]  

From (16), the nominal value of delay time \( T_{wn} \) in (13) is expressed as

\[ T_{wn} = \frac{\tau}{2\omega_{an}} \]  

Introducing this delay into the feedback system, reflected-wave rejection is attained. Such delay can be easily implemented with using a block random-access-memory on a processor.

5.4 Reflected Wave Rejection Using Load-side Acceleration

The target system in this study does not have equipment to sensing the load-side position, even though reflected-wave rejection requires that information. Then, an observation technique should be established. In the section 4, the paper showed that the EMFR weighing cell picks up the inertial force and pseudo-acceleration. The paper shows the method to construct reflected-wave rejection with using the load-side acceleration.

In general, a second integral value of the acceleration is equal to the position. However, the sensors have offset or drift, namely direct-current (DC) component, and then genuine integral process cannot be applied. Then, the load-side acceleration provides the position for high-frequency domain. The low-frequency component is observable from a drive-side position. The torsion between the drive-side and the load-side does not occur in low-frequency domain. The low-frequency component is observable from a drive-side position. The torsion between the drive-side and the load-side does not occur in low-frequency domain. The low-frequency component is observable from a drive-side position. The torsion between the drive-side and the load-side does not occur in low-frequency domain.

To integrate the load-side acceleration, DC component should be eliminated. For this purpose, the paper introduces a high-pass filter \( H(s) \) expressed as

\[ H(s) = \frac{s}{s + g_{hpf}} \]  

where \( g_{hpf} \) denotes a stop-band frequency of the filter \( H(s) \). To cut a DC component of the second integral value, triple filters are applied as

\[ \dot{x}_{wp} = \frac{1}{s^2}H^3(s)x_{wp} \]  

where a superscript \( \cap \) stands for the high-frequency component. On the other hand, the low-frequency component is estimated by using a complementary filter of \( H^3(s) \) shown as

\[ L(s) = 1 - H^3(s) \]  

Here, \( L(s) \) is a low-pass filter. By using this filter, the low-frequency component is estimated as

\[ \dot{x}_{wp} = L(s)x_{wp} \]  

where a superscript \( \cap \) stands for the low-frequency component. Then, the load-side position is obtained as

\[ \dot{x}_{wp} = \dot{x}_{wp} + \dot{x}_{wp} \]  

Then, reflected-wave rejection can be constructed.
6. Experiments

This section shows experiments of the vibration suppression control. Here, the paper confirmed the performance of the vibration suppression in 2 cases, when a load is installed on the weighing pan or not. The paper checks the vibration-suppression effect from view points of the peak-amplitude of vibration and the power spectrum of the output weight.

6.1 Vibration Suppression without Load

6.1.1 Setup In this experiment, the EMFR weighing cell is transported by dual rod-type shaft motors (S180Q; GMC Hillstone) and they are driven by servo amps (ADAX3-01ML2; Hitachi Industrial Equipment Systems Co., Ltd.). Positions of the rod-type shaft motors are detected by position encoders (RGH24H15A30A; Renishaw). To implement reflected-wave rejection, the paper checked a frequency response of the EMFR weighing cell and measures a 1st-order resonant frequency. Figure 10 shows the frequency response of the EMFR weighing cell when it is transported following to a step signal. Here, we found the 1st-order resonant frequency around 74 Hz and the 2nd-order resonant frequency around 142 Hz. From this result, propagation time of the traveling wave was set at 3.38 ms. Parameters used in the experiments is listed in Table 1. The control period was set at 80 μs. Moving up and moving down commands for the transporter were set as

\[
m_{\text{up}} = \begin{bmatrix} 12.0 \times 10^{-3} & 0 \end{bmatrix}^T \tag{23}
\]

\[
m_{\text{down}} = \begin{bmatrix} 7.6 \times 10^{-3} & 0 \end{bmatrix}^T \tag{24}
\]

Here, the Q filter, the controller matrix and the stopband frequency for the load-side position estimation filter were set as

\[
Q = \frac{1000}{s + 1000} \tag{25}
\]

\[
C_p = \text{diag}[10000 + 200x, 4900 + 140x] \tag{26}
\]

\[
g_{\text{hpf}} = 10.0 \tag{27}
\]

In this time, no load excepting the weighing pan was not set. The moving commands are alternately input to the system, then a stroke of the up-down is 4.5 mm. In this experiment, comparison of the responses with and without reflected-wave rejection was conducted.

6.1.2 Results The responses of the moving up and moving down are shown in Figs. 11 and 12. In both figures, the upper one shows the position responses of the transporter and the lower one shows the weight response of the EMFR weighing cell. Although there is little difference between the position responses, significant difference can be found in the weight responses. A persistent vibration is not found when using reflected-wave rejection, while the other one excites a large sustained oscillation. It should be noted that single surge of weight responses when using reflected-wave rejection is the inertial force along with the acceleration and deceleration. Furthermore, saturation of the output is confirmed when the controller does not have reflected-wave rejection. This means that the inertial force exceeds the allowable load and hence there is a risk to damage the weighing cell. On the other hands, the controller with reflected-wave rejection certainly reduce the vibration, including a peak amplitude. To check the vibration suppression effect, the paper prepares power spectrums of the weight responses shown in Fig. 13. The 1st and 2nd resonances were well suppressed. The figure shows that reflected-wave rejection certainly suppressed the resonances.

6.2 Vibration Suppression with Vial Bottles

6.2.1 Setup In this experiment, a vibration-suppression performance when a load was set is validated.
Hardware setup and control parameters are the same with that of the previous experiment. Because the system has 2 characteristics depending on whether the EMFR weighing cell contacts to the vial bottle or not, the paper set the control parameters to the same ones. In addition to this, the paper prepared liquid samples filled in vial bottles as the load shown in Fig. 14. Along with the transportation, water surfaces start to oscillate. The left and right bottles are lifted up by the transporter with/without reflected-wave rejection, respectively. Here, the paper focused on the middle points of water surfaces and checked its vibration. The water surfaces were caught by a camera which takes 60 frames per second. That is, a sampling period of the deviation of the water surfaces is 16.67 ms.

6.2.2 Results Figure 15 shows transitions of water surfaces with/without reflected-wave rejection. The left one was transported with reflected-wave rejection control while the right one was done without that. When applying reflected-wave rejection control, a ripple on the water surface is rapidly suppressed. On the other hands, a ripple persisted when not applying reflected-wave rejection despite the transporter started to move down. Because a residual vibration in previous process has possibility to influence next process, the system without vibration suppression should set long waiting-time for each process. In addition, the right one had a large amplitude of the ripple compared with the left one. This problem is because the transporter without vibration-suppression control is very stiff against the load. Concomitantly, we found that the vial bottle on the right side was flipped by robust-controlled transporter and the load position was slipped to left as shown in Fig. 16. In a product-transportation system, mis-registration of products is a fatal problem. Since reflected-wave rejection adds the compliance against the load, the load position was not slipped and the ripple on the water surface was not excited. This is obvious because the structure shown in Fig. 9 is similar to the admittance controller adding the damping effect to the system. Reflected-wave rejection is one of the phase-stabilization methods and hence it has robustness against load variation. In the other words, the system has a capability to suppress the vibration with/without the load. In this time, the load inertia was comparatively light compared with the weighing pan and the load variation was sufficiently small. As a result, the water surface of the left bottle converges in 0.3 s and that of the right one continues to oscillate over 0.8 s. Then, the system is able to start weighing earlier by introducing reflected-wave rejection with meeting the requirements for the product-transportation system. In conventional system, the weighing process takes around 2.5 s and this reduction is significant. Since the filling process contains many processes, it can not be said that this time-shortening directly improves a production volume, due to existence of a comparatively long data-acquisition time by precise A/D converters such as delta-sigma type A/D converter. However, this technique certainly shortens the waiting time for vibration to suppress and contributes to the mass production.

7. Conclusions

The paper applied reflected-wave rejection for the EMFR weighing cell to achieve the high-speed weighing. When
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transporting the EMFR weighing cell, it picks up the inertial force. Simultaneously, it works to make the lever position restored and then the kinetic energy circulates between restoring force and inertial force. Then, the vibration of the internal structure is induced. Because the EMFR weighing cell behaves like the linear system, reflected-wave rejection can be applied. The paper introduced the wave model and derive the model which fit to the actual system. To implement reflected-wave rejection, no additional sensor for load-side detection was equipped but only the output of the weighing cell was used. Due to introduction of reflected-wave rejection, no additional sensor for load-side detection was equipped but only the output of the weighing cell was used. The paper introduced the wave model and derive the model which fit to the actual system. To implement reflected-wave rejection, no additional sensor for load-side detection was equipped but only the output of the weighing cell was used. This work like a phase compensator since it is an all-pass filter. A bode diagram of this filter when \( T_w = 20 \) ms is shown in app. Fig. 1. Considering this structure, the resonant system can be stabilized by a generalized feedback compensator \( H(s) \) whose transfer function is

\[
P_d = \left( \frac{2e^{-T_w s}}{1 + e^{-2T_w s}} \right) \text{ (A1)}
\]

are first conducted and accordingly for the lumped-parameter system are conducted.

The control scheme of reflected-wave rejection shown in Fig. 8 can be transformed to a simplified feedback controller as shown in app. Fig. 1. Considering this structure, the resonant system can be stabilized by a generalized feedback compensator \( H(s) \) whose transfer function is

\[
H(s) = \left( \frac{1}{2} e^{-T_w s} \right) \text{ (A2)}
\]

This works like a phase compensator since it is an all-pass filter. A bode diagram of this filter when \( T_w = 20 \) ms is shown in app. Fig. 2. This result denotes that the filter \( H(s) \) is a set of the phase-lead compensators and the phase-lag compensators. To verify this phase-compensation effect, let us describe \( P_d \) by a form of infinite series

\[
P_d(s) = K \sum_{i=1}^{\infty} \frac{\kappa_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \text{ (A3)}
\]

where \( N \in \mathbb{N} \), \( i \in \mathbb{N} \), \( K, \kappa_i, \zeta_i, \) and \( \omega_i \) denote the number of

\[
q^{\text{mL}}(s, 0) = \begin{cases} 1 & \text{Position controller} \\ e^{-T_w s} & \text{Reflected-wave rejection} \end{cases}
\]

app. Fig. 1. Equivalent structure of reflected-wave rejection

Acknowledgment

This work was partially supported by JSPS KAKENHI Grant Number 18H03784.

References

modes used in modeling, a designation number for mode, a constant gain, a modal constant of the i-th mode, a damping coefficient of the i-th mode, and a natural frequency of the i-th mode, expressed as

\[ k_i = (−1)^{i−1} \] \hspace{1cm} \text{(A4)}

\[ \zeta_i = 0 \] \hspace{1cm} \text{(A5)}

\[ \omega_n = \frac{(2i−1)\pi}{2T_w} \] \hspace{1cm} \text{(A6)}

\[ K = \omega_n^2 \sum_{k=1}^{\infty} \frac{(−1)^{k−1}}{(2k−1)^2} \approx 0.8225\omega_n^2 \] \hspace{1cm} \text{(A7)}

Modes which have positive modal-constants, odd-number modes, require the phase leading whereas modes with negative modal constants, even-number modes, require the phase lagging. Here, the i-th mode shows dominant dynamics within a bandwidth \[ [(i−1)\pi/T_w, (i−1)\pi/T_w] \]. These frequencies are the same with that of the changing point of the phase-compensation effect by the feedback compensator \( H(s) \). app. Fig. 2 shows that phases of the odd-number modes lead and phases of the even-number modes lag by the feedback-compensation filter \( H(s) \). Then, the stability margins of all the modes are increased. An open-loop transfer function \( L(s) \) is expressed as

\[ L(s) = \frac{e^{-2T_w s}}{1 + e^{-2T_w s}} \] \hspace{1cm} \text{(A8)}

and gain and phase characteristics are expressed as

\[ |L(s)| = \frac{1}{2\cos(T_w \omega)} \] \hspace{1cm} \text{(A9)}

\[ \angle L(s) = \pi - T_w \omega - \left[ \frac{\omega + \omega_n}{2\omega_n} \right] \pi \] \hspace{1cm} \text{(A10)}

Therefore, the Nyquist plot draws a line perpendicular to the imaginary axis while passing through a point \((-0.5, j0)\). The nyquist plot of \( L(s) \) is shown in app. Fig. 3. All the modes draw same line. The phase margins are 60 degree for all modes.

This phase compensation is also effective for the lumped-parameter systems. Let us consider a distributed-parameter system and a lumped-parameter system described by a form of infinite products

\[ P_{d}(s) = \prod_{i=1}^\infty \frac{\omega_n^2}{s^2 + \omega_n^2} \] \hspace{1cm} \text{(A11)}

\[ P_{l}(s) = \prod_{i=1}^{\infty} \frac{\omega_n^2}{s^2 + \omega_n^2} \] \hspace{1cm} \text{(A12)}

where \( P_{l} \) is a plant system of the lumped-parameter system. These models have the same phase-characteristics from the 1st to the N-th mode shown as

\[ \forall \omega < (2N+1)\omega_1, \angle P_{d}(s) = \angle P_{l}(s) = -\left[ \frac{\omega + \omega_1}{2\omega_1} \right] \pi \] \hspace{1cm} \text{(A13)}

Then, the phase stabilization by reflected-wave rejection is effective where \( \omega < (2N+1)\omega_1 \). Compared with the distributed-parameter system, the gain characteristics of the lumped-parameter system is affected by low-order modes. For any \( j \in \mathbb{N} \), following equation

\[ \forall \omega \sqrt{\omega_1^2 + \sum_{i=1}^{j} \frac{\omega_n^2}{s^2 + \omega_n^2}} < 1 \] \hspace{1cm} \text{(A14)}

is satisfied. Furthermore, the gain of the \( P_{l} \) decreases where \( \omega > \omega_n \) because there is no peak over this frequency and the gain never reaches to 1 where \( \omega > N\pi/T_w \). Hence, the stability where \( \omega > (2N+1)\omega_1 \) is guaranteed by the small gain theorem. Nyquist plot of the \( P_{l} \) when \( N = 1 \) is shown in app. Fig. 4. Since the gain attenuation shrinks Nyquist plot, the plot goes away from the point \((-1, j0)\) and
the stability margin increases. Furthermore, the gain of the system is smaller than 1 where \( \omega > \frac{N}{r/T_w} \) and the plot draws a whirlpool centering on the origin. Then, the system has larger phase margin than 60 degree. These results show that reflected-wave rejection is effective for low-order systems.

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