Power Control Optimization of an Energy Storage System in DC Electric Railways

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The “optimal” control of a stationary energy storage system in a DC electric railway network is achieved by minimizing the total energy supplied from all the related traction substations. Given a timetable and the speed profiles of all the trains in the network, this paper addresses the problem of finding an optimal charging and discharging power control of the storage system. In this paper, we model the problem as a mixed integer programming problem and show the solutions for several parameter values. We then discuss the validity of our model and estimate the advantages of the energy storage system in terms of energy saving. These processes are useful for designing and installing an energy storage system in a DC electric railway network.

Keywords: DC feeding system, electric railway network, regenerative energy, mixed integer programming

1. Introduction

In several DC electric railway networks, energy storage systems have recently been installed by the railway operators who own the infrastructure. There are several reasons for this, e.g., voltage stabilization supplied to each train, reserving minimum tractive energy for an emergency evacuation from underground tunnels, and energy saving. This paper focuses on the last role and we try to find a way to optimally charge and discharge the energy storage systems in terms of maximizing the amount of saved energy.

Before tackling this problem, it is crucial to understand features of power consumption in train operation. Driving operation of a train which consists of less powering and more coasting is said to achieve energy saving, but it leads to longer running time. In a railway system, each train is required to run between adjacent stations in a planned running time. Hence, energy saving based on this principle is limited. Instead, we focus on the recent trend that modern rolling stock has regenerative brakes. They are able to supply regenerative energy in a DC feeding railway network. Imagine a situation that a train braking is located close to a train powering. In this situation, the regenerative energy from the train braking is supplied to the train powering. Imagine another situation that a train braking exists but there is no train powering nearby. Then “regenerative energy squeezing” under light load condition happens, i.e., the regenerative energy is wasted. Since technologies such as silicon-carbide semiconductor modules and/or high-power motors have improved regenerative performance of rolling stock, to avoid the regenerative energy squeezing is now more and more crucial. By optimally controlling the energy storage system in the network, we try to avoid the regenerative energy squeezing and save the total energy consumption.

Suppose that speed profiles and driving operation of all the trains are predetermined. On this assumption, if we decide charging/discharging power of the energy storage system at every time instant, then the power supplied from each traction substation is calculated by solving feeding circuit equations. Thus, by optimally deciding the transition of charging/discharging power of the energy storage system, we can minimize the total energy supplied from the substations. We call this transition “the optimal control” of the energy storage system.

In this paper, we develop an optimal control method by modelling the problem as an mixed integer programming problem on several additional assumptions, and solving it by using a mixed integer programming solver. For several parameter values we solve the problem and compare the solutions. We then discuss the validity of our model by using the Train Operation Power Simulator [1], and estimate the positive effect of the energy storage system in terms of energy saving when it is installed and optimally controlled. These processes are useful when we plan to design and install an energy storage system in a DC electric railway network.

2. Energy Storage System Constraints, Assumptions and Validation Procedures

Before presenting the optimal control method, we briefly describe constraints on the energy storage system, additional assumptions on modeling the problem and a validation procedure by using the Train Operation Power Simulator.

2.1 Energy Storage System Constraints

To model
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a DC electric railway network with an energy storage system more accurately, we should consider constraints on the performance of the energy storage system. One among them is the “SOE constraint”, i.e., the state-of-energy of the energy storage system should be kept within a certain range, since deviating from this range reduces the life of the energy storage system. The energy storage system should be charged/discharged with a proper timing to keep the SOE constraints. Since the energy storage system cannot know how the trains in the network are operated in general, the energy storage system is controlled based on the voltage measured in real time while we keep the SOE constraint. Another control method is based on the real-time train position data.

2.2 Assumptions

In the previous section we have introduced the assumption that speed profiles and driving operation of all the trains are predetermined. To model the optimal control problem as a mixed integer programming problem, we need additional assumptions shown below.

Assumption (1): A discrete time model is adopted.

Each train’s powering/regenerative power depends on the interaction among all the trains in operation. It also depends on the characteristics of substation rectifiers and rolling stock as well as the resistance of electric wires and rails. Although the train operation power varies continuously, it is enough to identify charge/discharge power of the energy storage system for each short time interval $\Delta t$ (we refer to it as “each time” hereafter). We therefore adopt a discrete time model to approximate the situation. Specifically, the time and the energy are divided into length of time unit $\Delta t$.

Assumption (2): Charging energy of the energy storage system is lost at a fixed rate.

In reality, not all of the stored energy can be used for the train traction energy. Hence, we introduce a fixed efficiency rate $\eta$ to express the effectiveness of the transformation of the energy storage system. The meaning of this parameter is that the discharging energy used for train operation is equivalent to $\eta$ times as much as the charging energy.

Assumption (3): The number and positions of energy storage systems are restricted.

To simplify the problem, we assume that only one energy storage system is installed in a DC electric railway network. We also assume that it is inside a substation.

Since we assume that speed profiles and driving operation of all the trains are predetermined (Fig. 1(a)), the power that each train consumes/generates at each time is also fixed (Fig. 1(b)). Hence, once charging/discharging power of the energy storage system is identified (Fig. 2(a)), the total energy supplied from all the substations is determined by solving feeding circuit equations (Fig. 2(b)). We note that charging/discharging power at time $t$ changes according to the operation energy of the trains. When all the trains are braking at time $t$, for instance, the energy storage system cannot be discharged because there are no train which consumes acceleration energy.

We summarize below the input and the output of the problem. At each time, many “sample control points” where each of them has some charging/discharging power are given. For each sample control point, the total energy supplied from all the substations is calculated. These data are the input data of the mixed integer programming problem (Fig. 3(a)). Its objective function is to minimize the total energy supplied from all the substations in a period of time from the departure of the first train to the arrival of the final train (Fig. 3(b)). Major constraints of the problem are that only one sample control point is selected and that the SOE constraint is kept at each time. An example of a series of optimal control points is
3. Mixed Integer Programming Formulation

We formulate the optimal energy storage system control problem as a mixed integer programming problem. We introduce below some notation. Before that, we note that the set of nonnegative integers is denoted by \( \mathbb{N} \). Also, the set of (nonnegative) real values is denoted by \( \mathbb{R} \).

**Notation on sets and elements**
- \( \Delta \in \mathbb{Z}^+ \): The time unit [s].
- \( T: = [0, \Delta, 2\Delta, \cdots, t_{\text{max}}] \): The time set.
  - Its element is often represented by \( t \).
  - Time 0 represents the departure time of the first train.
  - Time \( t_{\text{max}} \) represents the arrival time of the final train.
- \( J \): The set of substations.
- \( g \): The energy storage system.
- \( N: = \{1, \cdots, n_{\text{max}}\} \): The set of sample control points of the energy storage system.
  - Its element is often represented by \( n \).

**Notation on constants**
- \( P^{\text{SS}}(t,n) \in \mathbb{R}_+ \): The total power supplied from all the substations when a sample control point \( n \in N \) is selected at time \( t \in T \setminus \{t_{\text{max}}\} \) [W].
  - They are ordered as \( P^{\text{SS}}(t,j) \geq \cdots \geq P^{\text{SS}}(t,n_{\text{max}}) \).
- \( P^{\text{E}}(t,n) \in \mathbb{R} \): The charging/discharging power of the energy storage system when a sample control point \( n \in N \) is selected at time \( t \in T \setminus \{t_{\text{max}}\} \) [W].
  - If \( P^{\text{E}}(t,n) \geq 0 \), the sample point \( n \) represents charging.
  - If \( P^{\text{E}}(t,n) < 0 \), the sample point \( n \) represents discharging.
- \( P^{\text{SS}}(t,n) \) is called “charging power” whether \( P^{\text{E}}(t,n) \) is positive or negative.
  - They are ordered as \( P^{\text{SS}}(t,j) \geq \cdots \geq P^{\text{SS}}(t,n_{\text{max}}) \).
  - \( P^{\text{SS}}(t,1) \): The maximum instantaneous charging power.
  - \( P^{\text{SS}}(t,n_{\text{max}}) \): The maximum instantaneous discharging power.
- \( E^{\text{E}} \in \mathbb{R}_+ \): The lower limit of the charging capacity of the energy storage system \( J \).
- \( E^{\text{E}} \in \mathbb{R}_+ \): The upper limit of the charging capacity of the energy storage system \( J \).
Notation of parameters

• $\alpha \in R$: A discharging cost per 1 [J] of the energy storage system between the time $t_{\text{max}}$ and the next day’s time 0.
• $\beta \in R$: A charging cost per 1 [J] of the energy storage system between the time $t_{\text{max}}$ and the next day’s time 0.
• $\eta \in [0, 1]$: An energy translation efficiency rate from charging energy to discharging energy of the energy storage system.

Each term of the objective function (1) as well as the constraints (2)–(5) is explained below:

1. The storage energy at the time $t_{\text{max}}$ is the sum of the storage energy at the time $t$ and the efficiency multiplier $f(t, n, \eta)$ times the charging power of the selected sample control point.
2. The difference of stored energy between the operation of the final train and that of the next day’s first train is calculated.
3. At each time, only one sample control point is chosen.
4. The storage energy is kept in the range from $E_{\text{est}}(0)$ to $E_{\text{est}}(t_{\text{max}})$.

This represents the SOE constraint.

4. Case Study

This section presents some calculation results, validation of our mixed integer programming model and an estimation of the effect on energy saving when our optimal control method is applied.

4.1 Input Data and Computational Environment

We prepare a sample railway line which has neither gradients nor curves for the case study. There are twelve stations. The distance between stations is randomly chosen in the range of 1.5 to 2.5 [km]. In every 4.5 [km] there are six substations, and the energy storage system is installed in the substation that is nine [km] away from the line left end (Fig. 7). The train diagram is periodic at a headway of five or ten minutes and all of the trains stop at each station. A diagram of a headway of ten minutes is presented in Fig. 8. Table 1 lists rolling stock specifications. Regarding the auxiliary machine power, two patterns are set according to the outside temperature. One is the smallest power which corresponds to outside temperature 15°C and the other is higher power which corresponds to 35°C.

**Fig. 7.** Positions of the substations and the energy storage system

**Fig. 8.** A train diagram of a headway of ten minutes

**Table 1.** Rolling stock specifications

<table>
<thead>
<tr>
<th>Train</th>
<th>Total mass of each train [ton]</th>
<th>Starting acceleration [km/h/min]</th>
<th>Deceleration of braking [km/h/min]</th>
<th>The ratio of motored cars to trailer cars</th>
<th>Traction circuit efficiency</th>
<th>Auxiliary machine power [kW] corresponding to temperature 15°C</th>
<th>Auxiliary machine power [kW] corresponding to temperature 35°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train 1</td>
<td>286</td>
<td>3.3</td>
<td>2.5</td>
<td>1:1</td>
<td>0.855</td>
<td>39</td>
<td>284</td>
</tr>
</tbody>
</table>
4.2 Interpretations of the Optimal Control Method

As an example, we set the input values as follows and solve the mixed integer programming problem: the train diagram is a headway of ten minutes (Fig. 8), the auxiliary machine power is correspondent to 15°C, the lower limit of the charging capacity of the energy storage system $E_{stg}^g$ is set at 0[J] and the upper limit $E_{stg}^g$ is set at 30[kWh], and the absolute value of the maximum instantaneous charging power $P_{stg}^g(t, 1)$ and discharging power $P_{stg}^g(t, n_{max})$ are set at 2000[kW]. Figure 9 shows the transition of the power and the charging energy of the optimal solution at each time. The storage energy 0[%] corresponds to 0[kWh] and 100[%] corresponds to 30[kWh]. The third and fourth charts of this figure show two calculation results. The bold dashed lines represent no energy storage systems installed, while the thick solid lines represent a case of the energy storage system installed and the optimal control applied. Since the train diagram is a headway of ten minutes, the optimal control method is iterated by ten minutes from 7:20:00 to 7:50:00. In Fig. 9, the time axis is therefore from 7:30:00 to 7:40:00. We present some interpretations of the optimal solutions below:

(a) At the time when the total power consumed by the trains is negative (around 7:31:15, 7:33:30, 7:34:00 and 7:38:30).

- Figure 10 shows the train positions, the power consumed by the trains, the output power of the substations and the charging power of the energy storage system at the time 7:31:15.

- Trains 1, 2, 5 and 6 are braking near the energy storage system, whereas Train 4 is powering and Train 3 is coasting.

- To decrease the regenerative energy squeezing, the energy storage system is charged with the regenerative energy of the trains breaking.

(b) At the time except (a) when the energy storage system is charged (around 7:30:45, 7:33:00 and 7:35:30).

- Figure 11 shows the train positions, the power consumed by the trains, the output power of the substations and the charging power of the energy storage system at the time 7:30:45.

- Trains 3, 4, 5 and 6 are braking, whereas Trains 1 and 2 are powering and they are positioned away from Trains 5 and 6.

- The optimal solution indicates that there is a case where it is more efficient for the trains breaking to charge the energy storage system with their regenerative energy than to provide Train 1 or 2 that is positioned far away with their regenerative energy.

(c) At the time when the energy storage system is discharged (around 7:30:00, 7:32:00, 7:34:30 and 7:37:00).

- Figure 12 shows the train positions, the power consumed by the trains, the output power of the substations and the charging power of the energy storage system at the time 7:32:10.

- The energy storage system is discharged when the amount of the total energy consumed by the trains is large, in order to be charged as much as possible at the time of (a) and (b).

- The energy storage system is discharged when the difference indicates the amount of decreasing regenerative energy squeezing.

4.3 Comparison of the Calculation Results Obtained by Varying Energy Storage System Performance

We present the following three cases: the first case where a diagram of a headway of ten minutes is given and the auxiliary...
is varied according to twelve patterns: from 500 [kW] to 6000 [kW] at 500 [kW] intervals. The total number of calculation patterns is $3 \times 12 = 36$ for each case.

The lower limit of the charging capacity of the energy storage system ($E_{stg}^{\min}$) is set at 0 [J] and the upper limit ($E_{stg}^{\max}$) is varied according to three patterns: 10 [kWh], 20 [kWh] and 30 [kWh]. For each upper limit, the absolute value of the maximum instantaneous charging power ($|P_{stg}^{\text{chg}}(t,1)|$) and discharging power ($|P_{stg}^{\text{disch}}(t,n_{\text{max}})|$) are set at the same value (we call it “the maximum instantaneous power”) and is varied according to twelve patterns: from 500 [kW] to 6000 [kW] at 500 [kW] intervals. The total number of calculation patterns is $3 \times 12 = 36$ for each case.
We adopt the same implementation framework as in Subsection 4.1, and solve the problem with a PC which has Core i7-5960X (eight cores 16 threads, 3.0–3.5 GHz) and 64 GB RAM. The time to get the optimal solution are around from 3 [s] to 15 [s].

The comparison between the total power supplied from all the substations for the cases is shown in Figs. 14–16. We confirm the inflection points beyond which the effect on energy saving is less increased even though the maximum instantaneous power is increased. Table 2 shows these inflection points for each case according to the charging capacities. In this table, the inflection point is defined as the point where the improvement rate is less than 0.2% when the maximum instantaneous power is increased.

When installing the energy storage system, railway operators should take care of the balance between the charging capacity and the maximum instantaneous power. From this case study, we suggest that $|P_{stg}^g(t, 1)| = |P_{stg}^g(t, n_{max})| = 1500$ [kW] when $E_{stg}^g$ is 10 [kWh], $|P_{stg}^g(t, 1)| = |P_{stg}^g(t, n_{max})| = 2500$ [kW] or 3000 [kW] when $E_{stg}^g$ is 20 [kWh], and $|P_{stg}^g(t, 1)| = |P_{stg}^g(t, n_{max})| = 4000$ [kW] when $E_{stg}^g$ is 30 [kWh].

Therefore, we indicate that our mixed integer programming model can calculate the maximum effect on energy saving and the solution is is useful to design the energy storage system.

### 4.4 Confirmation of the Validity of the Mathematical Optimization Model

To confirm the validity of the mixed integer programming model, we designate the optimal sample control method of the energy storage system for the cases shown in Table 2, and calculate the total energy supplied from all the substations using the Train Operation Power Simulator. As a result, the difference between the objective function value and the simulation result is from 0.1% to 1.8% in terms of relative error. For example, Fig. 17 shows the comparison of the total energy supplied from all the substations between the simulation result and the objective function value for the basic case. We can say that our mixed integer programming model has high accuracy.

### 4.5 Estimation of the Effect of Energy Saving

Using the Train Operation Power Simulator, we calculate the total energy supplied from all the substations in the absence of the energy storage system for each case. We compare these results with the simulation results of Subsection 4.4 and estimate the effect on energy saving when the energy storage system is installed and the optimal control method is applied. Figure 18 shows the effect on energy saving.

When the energy storage system is installed with 30 [kWh] and 4000 [kW] in this sample line, the effect on energy saving is 12.2% under the basic case, as against 8.3% under the case No.1. There is a large difference between them, so that
we should consider the auxiliary machine power to evaluate the effect on energy saving. Under the case No.2, the high-
density train diagram causes less regenerative energy can-
celed, so that the effects on energy saving are about half as compared with the basic case.

5. Conclusion

The optimal control method of the energy storage system from an energy-saving point of view is to minimize the to-
total energy supplied from all the substations. Given the plan of driving operation of all the trains, we has developed the mixed integer programming model to optimally control the energy storage system. We have obtained the results shown below.

- We show some calculation results obtained by varying parameter values. We confirm the inflection points be-
yond which the effect on energy saving is less increased even though the performance of the energy storage sys-
tem is increased. This mixed integer programming model can calculate the maximum effect on energy saving. Our model can be useful to design the energy storage system.
- We validate our mixed integer programming model by using the Train Operation Power Simulator. The difference between the objective function value of the mixed inte-
ger programming model and the simulation result is from 0.1% to 1.8% in terms of relative error. Thus, this model has high accuracy.
- We calculate the effect on energy saving under several cases using the Train Operation Power Simulator. The effects on energy saving are within the range of 2.5% to 12.2%. They vary according to not only the train density but also the auxiliary machine power.

Our mixed integer programming model enables us to es-
timate the maximum effect on energy saving. In our future work, we will focus on real-time control of the energy storage system. We will try to develop another control method which is comparable with the optimal control method presented in this paper, by using the real-time train operation and rolling stock data.

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