Quick Reaction Force Control for Three-Inertia Resonant System

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This paper proposes a new three-inertia model that considers new transmission factors to achieve quick reaction force control. New transmission factors are confirmed by the analysis results of load-side dynamic characteristics. The load-side dynamic characteristics are measured via input impact torque using a hammer and motor-side velocity response. The experimental results of the input impact torque using a hammer confirm a new anti-resonance frequency with regard to the load-side dynamic characteristics. Therefore, new transmission factors are obtained based on the new anti-resonance frequency. However, the conventional three-inertia model has no new anti-resonance frequency. The consistency of the proposed three-inertia model that considers new transmission factors against plant system is confirmed by using a state observer based on the proposed three-inertia model. Additionally, the effectiveness of the force control system based on the proposed three-inertia model is confirmed by numerical simulations and experiments.

Keywords: industrial robot, three-inertia system, reaction force control

1. Introduction

In the industrial field, force control is focused to increase the application of industrial robots. In general, the joint of an industrial robot is modeled based on a two-inertia system that considers the torsional stiffness of a gear. Therefore, force control systems have been proposed based on the two-inertia model (1)–(5). However, based on the payload condition, the dynamic characteristics of the joint in an industrial robot indicate the characteristics of a three-inertia resonant system. When the dynamic characteristics of the joint indicate a three-inertia resonant system, it is difficult to achieve a quick reaction force control based on two-inertia model for three-inertia resonant system because an interaction of two-inertia model with environment is differed behavior of a joint model.

To achieve a quick reaction force control against a three-inertia resonant system, a force controller should be designed based on three-inertia model. However, conventional controllers based on three-inertia model is focused on velocity control (6)–(13), and position control (14)–(17). Hence, these papers do not sufficiently discuss the load-side dynamic characteristics which has a behavior to an environment. In these papers, a disturbance torque in conventional three-inertia model is defined as an external torque which has a behavior to a torsional torque of the topside (τ_{e2}). Therefore, when a three-inertia system interacts with the environment, the load-side dynamics characteristics of the conventional three-inertia model are defined as the behavior between only the τ_{e2} and the an environment. Using a force controller based on the conventional three-inertia resonant system, a quick reaction force control does not achieve because vibrations of reaction force response occur. Hence, the conventional three-inertia model is insufficient to consider the interaction with environment of the plant system. To achieve a quick reaction force control, a new three-inertia model considering the interaction of plant system with environment is required.

A characteristic of the interaction between the plant system and the environment is that the motor-side velocity of the robot is vibrated by the effect of the reaction force of the environment. To analyze the vibration phenomenon, a physical model, such as mass and spring, has been analyzed using the modal transformation method (18) that focused on resonance frequencies. The transformed model is similar to that used to analyze the nonlinear response spectrum in the seismic design of a building against an earthquake (19). During an earthquake, the resonance frequencies of the building are vibrated by the earthquake, and building is vibrated by the total vibration of resonance frequencies. Therefore, in the building, the presence of some signal pathways against earthquake is assumed. Similarly, in the robot arm, the presence of some signal pathways against reaction force is also assumed.

This paper proposes a new three-inertia model considering the interaction of plant system with environment to achieve a quick reaction force control for a three-inertia resonant system. A new three-inertia model is based on the analysis results of the load-side dynamic characteristic when the impact torque is input using a hammer to the end-effector. To analyze the load-side dynamic characteristic, the frequency characteristic is measured using the results of motor-side velocity response when a hammer strike on the end-effector of an industrial robot. As shown by the measurement results of the frequency characteristic, new transmission factors of reaction force is confirmed that differs from the conventional three-inertia model. The proposed three-inertia model is modeled considering new transmission factors.

The proposed three-inertia model is confirmed a consistency
with the plant system of a joint of industrial robot by using a state observer of a multi-input type. In this paper, a torque sensor is attached to measure the reaction force response. Therefore, a state observer is designed using observable outputs of motor-side velocity $\omega_m$ and reaction force $\tau_L$ to enhance the estimation of a state observer. Using a multi-input type state observer based on the proposed three-inertia model, a reduced estimation error is confirmed when the industrial robot pushes against an environmental object. Hence, the consistency of the proposed three-inertia model against the plant system is confirmed, and the force control system based on the proposed three-inertia model is expected to improve the reaction force response. The effectiveness of a force controller based on the proposed model is confirmed by numerical simulations and experiments.

2. Reaction Force Control System Based on Conventional Three-Inertia Model

In general, the joint of an industrial robot is modeled as a two-inertia system. However, the force control based on a two-inertia model does not achieve a quick reaction force control because an interaction of two-inertia model with environment is different behavior of joint model. Therefore, to achieve a quick reaction force control, a force controller based on a three-inertia model is required. Moreover, the analysis results of using a force controller based on a two-inertia model is represented in the Appendix.

2.1 Conventional Modeling

Figure 1 shows an overview of an industrial robot. In Fig. 2, the black line shows the frequency characteristic of the joint of the industrial robot in Fig. 1, and the red line shows the frequency characteristic of the three-inertia model. The block diagram of the conventional three-inertia model is represented in Fig. 3.

Based on the block diagram shown in Fig. 3, the state equation of the conventional three-inertia model is expressed in Eqs. (1) and (2).

\[
x = \begin{bmatrix} \omega_1 \\ \theta_1 \\ \omega_2 \\ \theta_2 \\ \omega_3 \\ \theta_3 \\ \end{bmatrix}, \quad y = \begin{bmatrix} \omega_m \\ \theta_m \\ \omega_r \\ \theta_r \\ \end{bmatrix}
\]

\[
x = Ax + B_m I_{cmd} + B_L \tau_L, \quad y = Cx
\]

\[
A = \begin{bmatrix} -\frac{D_1}{J_1} & -\frac{K_1}{J_1} & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & \frac{K_2}{J_2} & -\frac{D_2}{J_2} & -\frac{K_2}{J_2} & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -\frac{K_3}{J_3} & -\frac{D_3}{J_3} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \end{bmatrix}, \quad B_m = [K_t R_G \theta_{cmd} \tau_{cmd} I_{cmd} I_{cmd}] \\
C = [R_G 0 0 0 0]
\]

Using Eqs. (1) and (2), the frequency characteristic of the input reaction force $\tau_L$ to the motor-side velocity response $\omega_m$ is as depicted in Fig. 4. The transfer function of the frequency characteristic shown in Fig. 4 is expressed in Eq. (3).

\[
\frac{\omega_1}{\tau_L(s)} = \frac{K_t K_c}{J_3 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad \cdots (3)
\]

The force controller shown in Fig. 5 is designed based on the conventional three-inertia model which has the behaviors shown in Figs. 2 and 4.

2.2 Design of Force Controller

To design the state feedback system shown in Fig. 5, the transfer functions of each state variable are required. The transfer functions from a current command to each state variable are derived as follows.
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The state feedback system and integral controller is as follows.

\[ \frac{\omega_m}{I_{cmd}} = \frac{\sum_{i=0}^{5} b_{0i} s^i}{s^5 + \sum_{i=0}^{5} a_{1i} s^i} \]  \hspace{1cm} (4)

\[ \frac{\omega_2}{I_{cmd}} = \frac{\sum_{i=0}^{3} b_{2i} s^i}{s^3 + \sum_{i=0}^{3} a_{2i} s^i} \] \hspace{1cm} (5)

\[ \frac{\omega_3}{I_{cmd}} = \frac{\sum_{i=0}^{1} b_{3i} s^i}{s + \sum_{i=0}^{1} a_{3i} s^i} \] \hspace{1cm} (6)

\[ \frac{\theta_1}{I_{cmd}} = \frac{4 \sum_{i=0}^{4} b_{1i} s^i}{s^4 + \sum_{i=0}^{4} a_{1i} s^i} \] \hspace{1cm} (7)

\[ \frac{\theta_2}{I_{cmd}} = \frac{2 \sum_{i=0}^{2} b_{2i} s^i}{s^2 + \sum_{i=0}^{2} a_{2i} s^i} \] \hspace{1cm} (8)

\[ \frac{\tau_L}{I_{cmd}} = \frac{\sum_{i=0}^{5} b_{3i} s^i}{s^5 + \sum_{i=0}^{5} a_{3i} s^i} \] \hspace{1cm} (9)

The state feedback system is applied using Eqs. (4)–(9).

The transfer function of the force controller including the state feedback system and integral controller is as follows.

\[ \frac{\tau_L}{\tau_L} = \frac{b_{10}}{s^5 + \sum_{i=0}^{5} a_{1i} s^i} \] \hspace{1cm} (10)

\[ a_{16} = a_6 + f_1 b_{16} \]

\[ a_{17} = a_6 + f_1 b_{17} + f_1 b_{14} \]

\[ a_{14} = f_1 b_{14} + f_1 b_{11} + f_1 b_{13} + f_2 b_{12} \]

\[ a_{12} = a_1 + f_1 b_{12} + f_1 b_{11} + f_2 b_{13} + f_2 b_{22} \]

\[ a_{11} = a_1 + f_1 b_{11} + f_1 b_{14} + f_2 b_{13} + f_2 b_{21} \]

\[ a_{10} = a_0 + f_1 b_{10} + f_2 b_{10} + f_2 b_{12} \]

\[ a_{0} = K_f b_0 K_e \]

To design the controller gains, the denominator polynomial \( D_3(s) \) of the force controller is defined as follows.

\[ D_3(s) = (s + p_1) \prod_{i=1}^{3} (s^2 + 2 \zeta_i s + \omega_i^2) \] \hspace{1cm} (11)

\[ = s^3 + a_{p6} s^2 + a_{p5} s + a_{p4} s^4 + a_{p3} s^3 + a_{p2} s^2 + a_{p1} s + a_{p0} \] \hspace{1cm} (12)

Based on the denominator polynomial, the controller gains are obtained as expressed in Eqs. (13)–(19).

\[ f_{u1} = \frac{1}{b_{u1}} (a_{p6} - a_3) \] \hspace{1cm} (13)

\[ f_{u2} = \frac{1}{b_{u4}} (a_{p5} - a_4 - f_{u1} b_{u4}) \] \hspace{1cm} (14)

\[ f_{u2} = \frac{1}{b_{u23}} (a_{p4} - a_3 - f_{u1} b_{u13} - f_{u1} b_{u23}) \] \hspace{1cm} (15)

\[ f_{u2} = \frac{1}{b_{u22}} (a_{p3} - a_2 - f_{u1} b_{u12} - f_{u1} b_{u22} - f_{u2} b_{u22}) \] \hspace{1cm} (16)

\[ f_{u3} = \frac{1}{b_{u23}} (a_{p2} - a_1 - f_{u1} b_{u11} - f_{u1} b_{u21} - f_{u2} b_{u21}) \] \hspace{1cm} (17)

\[ f_{1} = \frac{1}{b_{q30}} (a_{p1} - a_0 - f_{u1} b_{u10} - f_{u1} b_{u20} - f_{u2} b_{u20}) \] \hspace{1cm} (18)

\[ K_f = \frac{a_{p0}}{b_{r2}} \] \hspace{1cm} (19)

In addition, the feedforward system shown in Fig. 5 is designed to improve the reaction force response. The transfer function of the feedforward system is expressed in Eq. (20).

\[ FF(s) = K \prod_{i=1}^{3} (s^2 + 2 \zeta_i \omega_i s + \omega_i^2) \prod_{i=1}^{3} (s^2 + 2 \zeta_i \omega_i s + \omega_i^2) \] \hspace{1cm} (20)

Finally, the force controller shown in Fig. 5 is designed such that the bandwidth is that represented in Fig. 6. The pole placement of the force controller based on a conventional three-inertia model is shown in Fig. 7.

A state observer is required for the state feedback system. However, the accurate estimation value of the three-inertia resonant system is difficult to realize when using only the motor-side information \((I_{cmd}, \omega_m)\). The motor-side velocity \(\omega_m\) is calculated from the motor-side encoder, and the reaction force \(\tau_r\) is measured using a torque sensor. Therefore, a state observer is designed using observable outputs of \(\omega_m\) and \(\tau_r\) to enhance the estimation of a state observer. Finally, the state equation of the state observer of a multi-input type is expressed as follows:

\[ \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{KC}) \mathbf{x} + \mathbf{B} \mathbf{I}_{cmd} + \mathbf{K} \begin{bmatrix} \omega_m \\ \tau_r \end{bmatrix} \] \hspace{1cm} (21)

\[ \mathbf{x} = \begin{bmatrix} \dot{\omega}_1 \\ \dot{\theta}_{11} \\ \dot{\omega}_2 \\ \dot{\theta}_{21} \\ \dot{\omega}_3 \\ \dot{\tau}_L \end{bmatrix} \]

\[ \mathbf{A} = \begin{bmatrix} \frac{\partial^2 \omega_1}{\partial \omega_1^2} & \frac{\partial \omega_1}{\partial \omega_1} & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & \frac{\partial \omega_2}{\partial \omega_2} & -\frac{\partial^2 \omega_2}{\partial \omega_2^2} & -\frac{\partial \omega_2}{\partial \omega_2} & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial \omega_3}{\partial \omega_3} & -\frac{\partial^2 \omega_3}{\partial \omega_3^2} & -\frac{\partial \omega_3}{\partial \omega_3} \\ 0 & 0 & 0 & 0 & K_e & 0 \end{bmatrix} \]

\[ \mathbf{B} = \begin{bmatrix} \frac{\partial \omega_m}{\partial \omega_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

\[ \mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ \mathbf{K} = \begin{bmatrix} k_{11} & k_{21} & k_{31} & k_{41} & k_{51} & k_{61} \\ k_{12} & k_{22} & k_{32} & k_{42} & k_{52} & k_{62} \end{bmatrix} \]

where the observer gains are designed such that the observer poles are set to 200 rad/s.
2.3 Numerical Simulations and Experiments

The performance of the force controller based on the conventional three-inertia model is confirmed by numerical simulations and experiments using the industrial robot. This paper conducts experiments using the joint of an actual industrial robot as shown in Fig. 8. A torque sensor (WACOH-TECH Inc.: WEF-6A200-4-RCD-B) is attached to measure the reaction force response. In the experiments, the industrial robot arm pushes on an elastic block using force control.

Figure 9 shows the numerical simulation results of the reaction force response using force controller based on conventional three-inertia model against conventional three-inertia model. In the experimental results shown in Fig. 10, vibration occurs at the transient response. Hence, the reaction force responses spend lots of time 0.52 s to converge the reaction force reference.

The vibration response of the reaction force response occurs despite the stable torque response using a force controller based on the conventional three-inertia model in the numerical simulation. Additionally, the estimated value of the reaction force undershoot and vibration transient response is shown in Fig. 11. Hence, as shown in Fig. 10, the vibration response is attributed to the interaction with the environment of the plant system that is different to that of the conventional three-inertia model.

The characteristics of the resonance frequency is the same that of the motor-side velocity response to the input torque of the motor-side, and that of the motor-side velocity response to the input reaction force. Thereby, to analyze the reason for vibrations occurring at transient response, the characteristics of the anti-resonance frequency based on the motor-side velocity response to the input torque imitated the reaction force is investigated to ascertain the difference between the plant system and the conventional three-inertia model.
3. Proposed Three-Inertia Model for Quick Reaction Force Control

In the previous section, the difference in characteristics of a three-inertia resonant system and conventional three-inertia model is assumed that the anti-resonance frequency of the characteristics based on the motor-side velocity response to the input reaction force. Because of the interaction between the three-inertia resonant system and environment, the presence of some signal pathways against the reaction force in the robot arm is assumed. The new anti-resonance frequency is defined by the presence of signal pathways against the reaction force in the robot arm. The new anti-resonance frequency is confirmed experimentally using a hammer, and the proposed three-inertia model is modeled based on the experimental results.

3.1 Measurement Frequency Characteristics

To demonstrate the existence of a new anti-resonance frequency, the load-side dynamics are measured using a hammer. In the experiments for measuring the load-side dynamic characteristic, the motor-side velocity response is measured when the hammer (PCB Piezotronics: 086C20) struck the end-effector of the industrial robot, as shown in Fig. 12.

Figure 13 shows the experimental results of the motor-side velocity responses when the hammer struck the end-effector. The frequency component from the results of the motor-side velocity as shown in Fig. 13 is analyzed. Using the results of frequency analysis, the load-side dynamic characteristics is calculated. Figure 14 shows the frequency characteristics of the motor-side velocity response to the input torque imitated the reaction force.

As shown in Fig. 14, the load-side dynamic characteristics involve two resonance frequencies and one anti-resonance frequency. The resonance frequencies and anti-resonance frequencies of the dynamic characteristics of the motor-side torque and the dynamic characteristics of the reaction force are expressed in Table 1. As shown in Fig. 14, the existence of a new anti-resonance frequency is confirmed experimentally.

3.2 Modeling Based on New Transmission Factors

To model the three-inertia system considering the new anti-resonance frequency, the transfer function of the load-side dynamics is defined as follows. Based on the load-side dynamics expressed in Eq. (22), the block diagram of the three-inertia system is transmuted.

\[
\omega_1 = \frac{G_L(s^2 + 2\zeta_1\omega_1s + \omega_1^2)}{(s+\alpha)(s^2 + 2\zeta_2\omega_2s + \omega_2^2)}
\]

New transmission factors are added to the block diagram of a conventional three-inertia model shown in Fig. 3 considering a new anti-resonance frequency because new transmission factors are required to an anti-resonance frequency in the transfer function. The proposed three-inertia model considering a new anti-resonance frequency is represented in Fig. 15. Based on the proposed three-inertia model represented in Fig. 15, the state equation considering the load-side dynamics is defined as Eqs. (23) and (24).

\[
x = Ax + B_1\tau_L
\]

\[
y = Cx
\]
The transfer function shown in Eq. (25) is rewritten as follows:

\[
\begin{align*}
\frac{\omega_{m}}{\tau_{L}} &= \frac{b_{2}s^{2} + b_{1}s + b_{0}}{s^{3} + \alpha_{3}s^{3} + \alpha_{2}s^{2} + \alpha_{1}s + \alpha_{0}}. \\
\end{align*}
\]

Eq. (26). Hence, by comparing Eqs. (22) and (26), transmission factors \( tA \) and \( tB \) are obtained:

\[
\begin{align*}
\frac{\omega_{m}}{\tau_{L}} &= \frac{K_{1}b_{2}s^{2} + (D_{1}s + D_{2})s + \frac{K_{1}b_{1}s}{J_{1}} + 1)}{s^{3} + \alpha_{3}s^{3} + \alpha_{2}s^{2} + \alpha_{1}s + \alpha_{0}}, \\
tA &= 1 - \frac{K_{2}}{J_{3}\omega_{La1}}, \\
tB &= \frac{K_{2}}{J_{3}\omega_{La1}}.
\end{align*}
\]

In this paper, the sum of transmission factors \( tA \) and \( tB \) is defined as one because the total reaction force is constant.

Figure 16 and Table 2 show transmission factors \( tA \) and \( tB \) based on the proposed three-inertia model and conventional three-inertia model. As shown from Table 2, the reaction force in the proposed three-inertia model is divided between torsional torque \( \tau_{T1} \) (30%) and \( \tau_{T2} \) (70%). Thus, the difference in behavior of the reaction force and torsional torque \( \tau_{T1} \) becomes too important to ignore. The proposed three-inertia model is modeled based on new transmission factors, as presented in Table 2 and Fig. 16.

4. Numerical Simulations and Experiments

The proposed three-inertia model is used to achieve a quick reaction force control. The accuracy of the proposed three-inertia model is confirmed based on numerical simulations and experiments using the state observer. In addition, the quick reaction force response is validated through the numerical simulations and experimental results of a force controller based on the proposed model.

4.1 Design of State Observer Based on Proposed Model

The state equation of the multi-input type state observer based on the proposed three-inertia model is expressed as follows:

\[
\dot{x} = (A - KC)\hat{x} + B_{cmd} + K_{m} \begin{bmatrix} \tau_{m} \\ \tau_{L} \end{bmatrix}
\]

Table 2. Transmission factors of reaction force

<table>
<thead>
<tr>
<th></th>
<th>( tA )</th>
<th>( tB )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed model</td>
<td>0.70</td>
<td>0.30</td>
</tr>
<tr>
<td>Conventional model</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

where the observer gains are designed such that the observer poles are set to 200 rad/s.

Figure 17 shows the numerical simulation results of the state observer when the step torque response is input to the three-inertia system using the force controller. In the results of the estimated reaction force shown in Fig. 17, the state observer estimates the accurate state values. Figure 18 shows the experimental results of the state observer. As shown in Fig. 18, the vibration response of the estimated reaction force is small when using the state observer based on the proposed model as compared to the state observer based on the conventional model as shown in Fig. 11. This is attributed to the
force controller based on the proposed model is designed to achieve a quick reaction force control. Moreover, the bandwidth of the force controller is 6.63 Hz, as indicated in Fig. 20.

Additionally, the pole placement of the force controller based on the conventional three-inertia model and the proposed three-inertia model is represented in Fig. 21. The red line shows the pole of the force control system based on the proposed three-inertia model. The blue line shows the pole of the force control system based on the conventional three-inertia model. The poles of the force controller based on the proposed three-inertia model are set to the poles listed Table 3 to achieve a stable torque response. Additionally, the poles of the force controller based on the conventional three-inertia model are set to the poles listed in Table 3 similar to the force controller based on the proposed three-inertia model.

In design condition of Fig. 7, the pole placement of force control system based on conventional three-inertia model is changed in case of including $t_B$ as shown in Fig. 21. Hence, the vibration response of the reaction force response occurs in using the force controller based on the conventional three-inertia model.

The effectiveness of the force controller based on the proposed model is confirmed from the numerical simulations and experiments. Figure 22 shows the numerical simulation results. In Fig. 22, the black line indicates the reaction force reference, the blue line represents the results of the reaction force response using a force controller based on the conventional three-inertia model, and the red line shows the results of the reaction force response using a force controller based on the proposed three-inertia model against proposed three-inertia model. As shown in Fig. 22, the quick reaction force response is confirmed when using the force controller based on the proposed model.

Figure 23 shows the experimental results. In Fig. 23, the
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Fig. 22. Numerical simulation results of reaction force response of input step-command against proposed three-inertia model

Fig. 23. Experimental results of reaction force response of input step-command

black line indicates the reaction force reference, the blue line shows the results of the reaction force response when using a force controller based on the conventional three-inertia model, and the red line shows the results of the reaction force response when using a force controller based on the proposed three-inertia model. As shown in Fig. 23, the reaction force converges the reaction force reference at 0.38 s. By comparing the results shown in Fig. 10, the quick reaction force response is confirmed when using the force controller based on the proposed model. Therefore, the experimental results indicate that the quick reaction force control is improved when using the force controller based on the proposed model.

5. Conclusion

To improve the performance of the force control for a three-inertia resonant system, a new three-inertia model with new transmission factors is proposed. To analyze the load-side dynamic characteristic, the frequency characteristic is measured using the results of a hammer strike on the end-effector of an industrial robot. The measurement results of the load-side dynamic characteristics confirm that three-inertia resonant system is modeled as new transmission factors. The effectiveness of the force controller based on the proposed model is confirmed by the numerical simulations and experiments. The results indicated that the proposed model achieves the quick force controller for the three-inertia resonant system.

References


Appendix

1. Force Control Based on Two-Inertia Model

This section describes the design of a force controller based on a conventional two-inertia model. In Fig. 2, the black line shows the frequency characteristic of the joint in app. Fig. 1. In addition, the resonance and anti-resonance frequencies shown in app. Fig. 2 are listed in app. Table 1.

According to the list in app. Table 1, a conventional two-inertia model is modeled. The green line in app. Fig. 2 shows the frequency characteristic of the conventional two-inertia model. When a force control system is implemented based on the two-inertia model using the feedback, integrator, and feedforward systems, the block diagram is as shown in app. Fig. 1. Equations (A1) and (A2) show the state equations of the two-inertia model.

\[ x = Ax + B_M I_{cmd} \]  
\[ y = Cx \]  
\[ x = \begin{bmatrix} \omega_M & \theta_s & \omega_L \end{bmatrix} \]

\[ A = \begin{bmatrix} D_{ss} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -D_{ss} & 0 \end{bmatrix} \]

\[ B_M = \begin{bmatrix} \frac{K_F}{J_L} b_0 L_1 \end{bmatrix} \]

The force controller is designed based on the two-inertia model shown in Eqs. (A1) and (A2). The transfer functions from a current-reference to each state variable are derived as follows.

\[ \omega_M = \frac{b_{in3} s^3 + b_{in2} s^2 + b_{in1} s + b_{in0}}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]  
\[ I_{cmd} = \frac{b_{d1} s^2 + b_{d1} s + b_{d0}}{s^2 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]  
\[ \theta_s = \frac{b_{f2} s^2 + b_{f1} s + b_{f0}}{s^2 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]  
\[ I_{cmd} = \frac{b_{d1} s^2 + b_{d1} s + b_{d0}}{s^2 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]  
\[ I_{cmd} = \frac{b_{d1} s^2 + b_{d1} s + b_{d0}}{s^2 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]  
\[ I_{cmd} = \frac{b_{d1} s^2 + b_{d1} s + b_{d0}}{s^2 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \]

The state feedback system is derived using Eqs. (A3)–(A6). The transfer function of the state feedback system and integral controller is shown as follows.

\[ \frac{\tau_{ref}}{\tau_L} = \frac{b_{f0}}{s^3 + a_4 s^2 + a_3 s + a_2 s + a_1 s + a_0} \]  
\[ a_{f4} = a_3 + f_{in} b_{in3} \]  
\[ a_{f3} = a_3 + f_{in} b_{in2} + f_{in} b_{in2} \]  
\[ a_{f2} = a_3 + f_{in} b_{in1} + f_{in} b_{in1} + f_{in} b_{in1} \]  
\[ a_{f1} = a_3 + f_{in} b_{in0} + f_{in} b_{in0} + f_{in} b_{in0} + f_{in} b_{in0} \]  
\[ a_{f0} = K_f b_{d0} L_c \]  
\[ b_{f0} = K_f b_{d0} L_c \]

To design the controller gains of the feedback and integral, the denominator polynomial \( D_2(s) \) of the force controller based on the feedback and integral controller is defined as follows.

\[ D_2(s) = (s + p_1)^2 (s^2 + 2 \zeta_i s + \omega_i^2) \]  
\[ s^5 + a_{p4} s^4 + a_{p3} s^3 + a_{p2} s^2 + a_{p1} s + a_{p0} \]  

By comparing Eq. (A7) with Eq. (A9) expanded from Eq. (A8), the controller gains are obtained as follows.

\[ f_{in} = \frac{1}{b_{in1}} (a_{p4} - a_3) \]  
\[ f_s = \frac{1}{b_{s14}} (a_{p3} - a_2 - f_{in} b_{d14}) \]  

The transfer functions from a current-reference to each state variable are derived as follows.

The state feedback system is derived using Eqs. (A3)–(A6). The transfer function of the state feedback system and integral controller is shown as follows.

\[ \frac{\tau_{ref}}{\tau_L} = \frac{b_{f0}}{s^3 + a_4 s^2 + a_3 s + a_2 s + a_1 s + a_0} \]  
\[ a_{f4} = a_3 + f_{in} b_{in3} \]  
\[ a_{f3} = a_3 + f_{in} b_{in2} + f_{in} b_{in2} \]  
\[ a_{f2} = a_3 + f_{in} b_{in1} + f_{in} b_{in1} + f_{in} b_{in1} \]  
\[ a_{f1} = a_3 + f_{in} b_{in0} + f_{in} b_{in0} + f_{in} b_{in0} + f_{in} b_{in0} \]  
\[ a_{f0} = K_f b_{d0} L_c \]  
\[ b_{f0} = K_f b_{d0} L_c \]

To design the controller gains of the feedback and integral, the denominator polynomial \( D_2(s) \) of the force controller based on the feedback and integral controller is defined as follows.

\[ D_2(s) = (s + p_1)^2 (s^2 + 2 \zeta_i s + \omega_i^2) \]  
\[ s^5 + a_{p4} s^4 + a_{p3} s^3 + a_{p2} s^2 + a_{p1} s + a_{p0} \]  

By comparing Eq. (A7) with Eq. (A9) expanded from Eq. (A8), the controller gains are obtained as follows.

\[ f_{in} = \frac{1}{b_{in1}} (a_{p4} - a_3) \]  
\[ f_s = \frac{1}{b_{s14}} (a_{p3} - a_2 - f_{in} b_{d14}) \]
The transfer function of the feedforward system is as follows.

\[
FF(s) = \frac{K_f \prod_{i=1}^{2} (s^2 + 2\zeta_i s + \omega_i^2)}{\prod_{i=1}^{2} (s^2 + 2\zeta_{f,i} s + \omega_{f,i}^2)} \quad \text{(A15)}
\]

Finally, the force controller shown in app. Fig. 1 is designed to set to the poles listed as app. Table 2. The pole placement of the force controller based on a two-inertia model is shown in app. Fig. 3. However, the pole placement shown in app. Fig. 3 becomes different when force control is implemented against a three-inertia system because the force controller is designed based on a two-inertia model. The pole placements of the force control against a three-inertia system using a force controller based on a two-inertia model are shown in app. Figs. 4 and 5. As shown in app. Figs. 4 and 5, the force controller is a stable pole placement when the bandwidth is set to condition A listed app. Table 2. Additionally, the force controller is an unstable pole placement when the bandwidth is set to condition B listed app. Table 2. Based on the analyses of the pole placement, it is difficult to achieve a quick reaction force control against a three-inertia resonant system using the force controller based on a two-inertia model.

App. Fig. 6 show the numerical simulation results of the reaction force control using a force controller based on a two-inertia model against a three-inertia resonant system. The numerical simulation results in app. Fig. 6(a) indicate a stable force response when the bandwidth of the force controller is set to condition A listed app. Table 2. However, the numerical simulation results in app. Fig. 6(b) indicate instability when the bandwidth of the force controller is set to condition B listed app. Table 2.

In addition, app. Fig. 7 shows the experimental results of the reaction force control using a force controller based on a two-inertia model. The experimental results in app. Fig. 7(a) are the stable force response when the bandwidth is set to condition A listed app. Table 2. However, the experimental results in app. Fig. 7(b) are unstable when the bandwidth is...
set to condition B listed app. Table 2.

As shown in the simulation and experimental results in app. Figs. 6–7, it is difficult to achieve a quick force control system based on a two-inertia model against a three-inertia resonant system.

2. Force Control System Considering Nonlinear Friction

This paper focuses the new transmission factor $t_B$ of the three-inertia resonant system, which is not included to conventional three-inertia model. Therefore, under the design condition in app. Fig. 9, the pole placement of force control system is changed to include $t_B$, as shown in app. Fig. 8.

app. Fig. 10 shows the numerical simulation results of reaction force control using the controllers based on conventional three-inertia model against the conventional three-inertia model. The force controllers based on conventional three-inertia model achieved quick reaction force response and no overshoot response.

However, as shown in app. Fig. 11, the performance of the force controller based on conventional three-inertia model is reduced against the plant system because the new transmission factor $t_B$ is not included in the conventional three-inertia model. Therefore, the overshoot in experiments is reduced because the proposed three-inertia model is highly consistent to plant system.

In addition, the effect of nonlinear friction is ignored in the numerical simulation in this paper. However, nonlinear friction exists in the axis of the robot arm. To confirm the effect of nonlinear friction, the authors conducted numerical simulation. The block diagram of the force control system considering nonlinear friction is shown in app. Fig. 12. The nonlinear friction is defined by static friction clone, friction, and viscous friction.

The nonlinear friction $\tau_f$ is expressed as follows:

$$
\tau_f = \begin{cases} 
\text{sgn}(\omega_m)\tau_c + D\omega_m & (|\omega_m| \geq \omega_s) \\
\text{sgn}(\omega_m)\tau_c + D\omega_m + \text{sgn}(\omega_m)(\tau_s - \tau_c)e^{-\frac{(|\omega_m| - \omega_s)^2}{2\omega_s^2}} & (\omega_s > |\omega_m|) \\
\tau_M - \tau_{s1} & (|\tau_M - \tau_{s1}| < \tau_s \cap |\omega_m| < 10^{-1})
\end{cases}
$$

(A16)

where $\tau_c$, $\tau_s$, $\tau_{s1}$, $\omega_s$, $\omega_{fm}$, and $D$ denote the clone friction torque, static friction torque, torsional torque, Stribeck velocity, velocity that produces the maximum static friction force,
shows the numerical simulation results considering the nonlinear friction. The results show that vibration occurs in the transient response because of the effect of nonlinear friction.

app. Fig. 12. Block diagram of force control system considering nonlinear friction

app. Fig. 13. Model of nonlinear friction

app. Fig. 14. Simulation results of force control considering nonlinear friction and viscous friction, respectively. In this paper, the nonlinear friction is modeled as shown in app. Fig. 13. app. Fig. 14

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