Zero Moment Point Estimation Based on Resonant Frequencies of Wheel Joint for Wheel-Legged Mobile Robot

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The zero moment point (ZMP) is a measure for realizing dynamic motion in legged robots. An important concern in ZMP-based motion control is the measurement of the ZMP itself. General humanoid robots use sensors attached to their feet to measure the reaction forces. However, this method is not applicable for wheel-legged mobile robots because it is difficult to attach force sensors inside the wheels. Therefore, this study proposes a method to determine the ZMP based on the relationship between the load and resonant frequency of the tires. The resonant frequency of the tires depends on the tire pressure. First, the relationship between the load and resonant frequency is confirmed by a frequency analysis and experiments on the wheel joints. Second, a method used to estimate the ZMP based on the relationship between the load and resonant frequency is described. Third, an online estimation technique for the ZMP is described. Finally, factors other than the vertical force that affect the frequency characteristics of the wheel are discussed. The average of the root-mean-square errors from the sensor value in the online-ZMP estimation results is 0.0361 m, which is sufficiently smaller than the typical stability margin of the support polygon of the robot.

Keywords: wheel-legged mobile robot, zero moment point, resonant frequency, load estimation.

1. Introduction

Robots that can perform unanticipated tasks and move by themselves in different environments are anticipated. A wheel-legged mobile robot has both leg and wheel mechanisms, and its modes of movement meet this expectation: it can use the wheel mechanism to move over flat surfaces and the leg mechanism to move over uneven terrain. Using the leg or wheel mechanism selectively, the robot can move efficiently in a variety of environments. Boston Dynamics has developed a wheel-legged mobile robot called Handle that realizes high-speed and effective motion in various environments using its two wheel-legged mechanisms⁽¹⁾. Moreover, wheel-legged mobile robots have been discussed in several related studies. For example, this type of robot has been studied for rescue applications⁽²⁾, for movement in extreme environments⁽³⁾⁽⁴⁾, for personal mobility⁽⁵⁾, and for human-to-human support environments⁽⁶⁾⁽⁷⁾.

Stability must be measured when the robot moves. Several static stability measures have been proposed for the robot^{(8)~(11)}, as well as dynamic stability measures⁽¹²⁾⁽¹³⁾. However, the downside to these stability measurements is that the robot dynamics and the external forces must be known, and they need to be transformed into the measurement frame. To realize effective motion in various environments, this study focuses on motion control based on the zero moment point

(ZMP), which is the center of pressure of the ground reaction force⁽¹⁴⁾. The ZMP is simple to calculate when the external forces are known compared with other measures. ZMP-based methods are used for dynamic movement stabilization, thus allowing robots to perform unanticipated tasks and movements in different environments. For wheel-legged mobile robots, there have been several studies on ZMP-based wheeled locomotion^{(15)~(17)}. However, none of these studies have attempted dynamic motion stabilization using feedback of ZMP measured from reaction forces in a real environment.

Accurate measurement of the ZMP is necessary for ZMP-based motion control. However, measuring the ZMP with force sensors attached inside the feet, as in a conventional humanoid robot^{(18)~(22)}, is difficult in a wheel-legged mobile robot. Nevertheless, ZMP-based motion control has already been realized in a wheeled mobile robot by calculating the ZMP with a robot model⁽²³⁾. In addition, methods have been proposed for measuring the ZMP using force sensors attached between the body and the wheel joint⁽²⁴⁾⁽²⁵⁾. These methods were realized the effectiveness of motion control for the wheeled mobile robot based on the ZMP. However, it can be difficult to realize walking motion in a wheel-legged mobile robot using these methods because they do not use the ground reaction force at the contact point of the wheel. Therefore, it is necessary to consider a new technique in measuring the ZMP. Measuring the force at the contact point of the tire is necessary when calculating the ZMP of a wheel-legged mobile robot. Several methods have been proposed for obtaining this force in mobile robots and automobiles, including methods using mathematical models⁽²⁶⁾⁽²⁷⁾, methods using observers in addition to mathematical models^{(28)~(31)}, and a method using special sensors in addition to mathematical models⁽³²⁾.

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Fig. 1. Overview of the wheel-legged mobile robot. The robot has four legs with the same structure, each with five degrees of freedom. The yaw crotch and steer joints have a reduction gear and a DC motor. The roll crotch, knee, and wheel joints have a reduction gear and a DC motor connected by a timing belt and pulley.

The present paper considers a method for estimating the tire force by concentrating on changes in tire pressure. In most automobiles, tire pressure is measured using a tire-pressure monitoring system (TPMS)⁽³³⁾. In an automobile, the resonant frequency of an inflated tire changes according to the tire pressure⁽³⁴⁾⁽³⁵⁾. Therefore, the method proposed in this paper is used to estimate the ZMP using changes in the resonant frequency of the tire as detected at the wheel joint.

This paper is organized as follows. Section 2 shows how a frequency analysis and experiments were used to assess the dependence of the frequency characteristics of a wheel-legged mobile robot on the contact-point load. Section 3 describes how we used the changes in the frequency characteristics to estimate the load, which is then used to estimate the ZMP. The performance of the proposed method is assessed experimentally. Section 4 discusses the technique used for the online estimation of the ZMP. Section 5 discusses factors that affect the frequency characteristics of the wheel joint other than the vertical force. Finally, Sect. 6 presents our conclusions.

2. Resonant Frequency of a Wheel Joint

We begin this section by describing the wheel joint of our robot. A photograph of our wheel-legged mobile robot is shown in Fig. 1. We then describe our frequency analysis and experiments to measure the frequency characteristics and to establish a relationship between the resonant frequency and the load. In this section, we confirm this relationship for tubular and solid tires.

The wheel joint of our robot is shown in Fig. 2. The output of a DC motor is connected to the wheel using a synchronous belt pulley and a reduction gear (harmonic drive). In addition, it is possible to change the tire in the wheel joint from a solid one to a tubular one.

2.1 Frequency Analysis of the Wheel The frequency analysis used to establish the relationship between the resonant frequency and the load was performed using the SolidWorks modeling software. The wheel model comprises

Fig. 2. Structure of the wheel joint.

Fig. 3. Resonant frequencies from the frequency analysis.

a tire and a steel wheel. Circumferentially, the tire has a cylindrical surface with a planar contact portion at the bottom. In addition, the tubular tire model has a hollow structure that contains an air model. The wheel model has constraints that permit circumferential motion of the tire and the steel wheel. A distributed load of 5.0 or 10.0 kg was applied to the steel wheel.

From the frequency analysis, the resonant frequencies of the solid and tubular tires for each mode order are shown in Fig. 3. In addition, the effective mass participation factor (EMPF) for each mode order for loads of 5.0 and 10.0 kg is shown in Fig. 4 for the solid and tubular tires. The EMPF is the percentage of the mass that contributes to a specific mode. Therefore, the larger the EMPF of a mode, the greater the influence of that mode on the dynamic response of the system. In this paper, the EMPF is calculated for the longitudinal, lateral, and normal axes of the tire.

For a solid tire, the results in Fig. 3 show that the resonant frequency increases with the load for modes higher than second order. The change in EMPF with load is the largest for the second- and third-order modes, as shown in Fig. 4(a). For a tubular tire, the results in Fig. 3 show the resonant frequency increasing with the load for modes higher than third order. The change in EMPF with load is the largest for the

Fig. 4. Effective mass participation factor (EMPF) for loads of 5.0 and 10.0 kg from the frequency analysis: (a) solid tire and (b) tubular tire.

Fig. 5. Experimental setup for measuring the frequency characteristics of the wheel joint.

third- and fourth-order modes, as shown in Fig. 4(b). Hence, we conclude from the above that the resonant frequency depends on the load on the tire.

2.2 Frequency Characteristics of the Wheel Joint

In this section, the frequency characteristics of the wheel joint are measured to verify the results from the frequency analysis. The frequency characteristics of the wheel in the first leg were measured five times when the load on the contact point was either 5.0 or 10.0 kg. This load was measured using a load meter. The experimental setup is shown in Fig. 5. The measurement conditions are listed in Table 1.

The measurement results for the frequency characteristics of the solid tire and the tubular tire are shown in Fig. 6. For the solid tire, from the measurement results in Fig. 6(a), the

Fig. 6. Frequency characteristics of the wheel: (a) solid tire and (b) tubular tire. The solid lines and shaded areas indicate the mean and the standard deviation for five measurements.

resonant frequency of the gain characteristic is seen to change from 50.0 to 60.0 Hz when the load is increased. In addition, the resonant frequency of the phase characteristic also changes from 45.0 to 50.0 Hz. As with the solid tire, the resonant frequency of the tubular tire changed with the load, as shown in Fig. 6(b). From the above, the dependence of the resonant frequency on the load has been confirmed, which agrees with the frequency analysis. Therefore, we now proceed to consider how to estimate the ZMP using the relationship between the load and the resonant frequency. The following discussion will consider only the solid tire because both tires have similar characteristics.

3. Method for Estimating the Zero Moment Point

From the results of the frequency analysis and the frequency characteristics of the wheel joints in section 2, the relationship between the load and the resonant frequency was confirmed. In this section, the method for estimating the ZMP using the relationship between the load on the contact point and the resonant frequency of the wheel is described. Firstly, the calibration data are measured and are used to create an approximation model of the gain and phase characteristics based on the resonant frequencies. Next, the load on each leg is estimated from the test data using the approximation model. Then, the ZMP for the robot is calculated from the estimated load on each leg.

Table 1: Measurement Conditions

Subject	Section 2.2	Section 3	Section 4 (Calibration / Test)
Calculation type	Offline	Offline	Offline / Online
Input signal for the frequency characteristics	Motor torque	Motor torque	Motor torque
Kind of input signal	Swept sine wave from 0 to 250 Hz	White Gaussian noise	White Gaussian noise
Output signal for the frequency characteristics	Motor angular velocity [†]	Motor angular velocity [†]	Motor angular velocity [†]
Sampling interval of input and output signal	0.5 ms	1.0 ms	0.05 ms
Control interval of each joint ^{††}	—	1.0 ms	1.0 ms
Calculation interval of the frequency characteristics	—	—	— / 0.5 s
Sampling length of input and output signal	5.0 s	8.192 s	3.2768 s / 0.4096 s
Smoothing of the frequency characteristics	Moving average filter	Moving average filter Gaussian filter	Moving average filter Gaussian filter

[†] The motor angular velocity was calculated from the angle provided by the encoder attached to the motor.

^{††} Each joint other than the wheel joint was controlled by a joint space control using proportional-derivative control and a disturbance observer.

Fig. 7. Experimental conditions for the ZMP estimation.

3.1 Measurement of Calibration Data

This section describes the measurement of the calibration data for creating the approximation model. Figure 7 shows the experimental conditions for estimating the ZMP. The frequency characteristics of the wheel were measured five times using a load meter for contact-point loads of 5.0, 7.5, and 10.0 kg. In addition, the contact-point load was changed by moving the posture of the robot using a chain block. The chain block can move in the lateral and longitudinal directions relative to the robot, and it can lift the robot. Therefore, the experiment was conducted by static motion. This is because this paper focuses on the verification of the ZMP estimation from the relationship between the resonant frequency of the wheel and the load. The load was changed in the same way for the subsequent measurements. The measurement conditions are shown in Table 1. Here, we cannot obtain the resonant frequency only with the joint servo signal, and the swept sine wave cannot be measured while the joint is being driven. Therefore, white Gaussian noise is used because it has a uniform frequency component.

The measurement results for the frequency characteristics of the calibration data are shown in Fig. 8. Here, the frequency characteristics of the first leg are shown as an example. In addition, the resonant frequencies of the calibration data are shown in Fig. 9. Data are excluded if we cannot obtain the resonant frequency because of noise. From the results, the frequency characteristics become the noisy shapes because the white Gaussian noise was applied as the

Fig. 8. Frequency characteristics of the calibration data in the first leg. The solid lines and shaded areas indicate the mean and the standard deviation for five measurements.

Fig. 9. Resonant frequencies of the calibration data. The dots and error bars indicate the mean and the standard deviation for five measurements. The solid lines show the fitting line of the mean values by the least-squares calculation.

input signal. Therefore, the resonant frequency obtained after smoothing was varied. Nevertheless, the resonant frequency of each leg clearly depends on the load on the contact point.

3.2 Approximation Model for the Resonant Frequency The approximation model for each leg is created using the resonant frequency of the calibration data. The coefficients of the following bivariate function are obtained:

Table 2: Applied Load in the Test Data

Case	Applied load (kg)			
	1st leg	2nd leg	3rd leg	4th leg
Front	9.0	6.0	9.0	6.0
Rear	6.0	9.0	6.0	9.0
Left	9.0	9.0	6.0	6.0
Right	6.0	6.0	9.0	9.0

$$W = a_1\omega_G + a_2\omega_P + a_3 \dots \dots \dots (1)$$

where W is the load on the contact point, ω_G is the resonant frequency of the gain characteristic, ω_P is the resonant frequency of the phase characteristic, and a_1 , a_2 , and a_3 are the coefficients of the bivariate function. Equation (1) can be expressed in matrix form as

$$W = \begin{bmatrix} \omega_G & \omega_P & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \dots \dots \dots (2)$$

Next, on the basis of n sets of measurement data, Eq. (2) can be expressed as

$$\begin{bmatrix} W_1 \\ \vdots \\ W_n \end{bmatrix} = \begin{bmatrix} \omega_{G1} & \omega_{P1} & 1 \\ \vdots & \vdots & \vdots \\ \omega_{Gn} & \omega_{Pn} & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \dots \dots \dots (3)$$

Equation (3) can then be summarized as $W = Aa$. Then, the coefficient vector a is calculated using a least-squares calculation as follows:

$$a = (A^T A)^{-1} A^T W \dots \dots \dots (4)$$

The approximation models are obtained using least-squares calculations from the calibration data for each leg as follows:

$$W_{leg1} = 0.2393\omega_{G_{leg1}} + 0.3762\omega_{P_{leg1}} - 27.1874 \dots (5)$$

$$W_{leg2} = 0.4265\omega_{G_{leg2}} + 0.3522\omega_{P_{leg2}} - 32.8957 \dots (6)$$

$$W_{leg3} = 0.3176\omega_{G_{leg3}} + 0.1450\omega_{P_{leg3}} - 18.8801 \dots (7)$$

$$W_{leg4} = 0.2319\omega_{G_{leg4}} + 0.3914\omega_{P_{leg4}} - 25.6484 \dots (8)$$

where W_{legi} , $\omega_{G_{legi}}$, and $\omega_{P_{legi}}$ are the load on the contact point, the resonant frequency of the gain characteristic, and the resonant frequency of the phase characteristic of the i th leg, respectively.

3.3 ZMP Estimation Using the Approximation Models

In this section, load estimation using the approximation models and the calculation of the ZMP are described. The frequency characteristics of the test data used for estimating the ZMP were measured under the same conditions as those for the calibration data. Here, the load was applied to each leg for the four cases where the ZMP of the robot moves to the front or rear and to the left or right, as shown in Table 2. Five measurements were recorded. The resonant frequencies of the test data are shown in Fig. 10. Then, the load on each leg was estimated using the approximation models and the resonant frequencies of the test data. The estimated loads are shown in Fig. 11. In addition, the root-mean-square errors (RMSEs) of the estimated loads are given in Table 3.

From the results shown in Fig. 11, the estimated load on each leg is seen to depend on the applied load. However, the variation in the estimated load is large in Fig. 11 and Table 3

Fig. 10. Resonant frequencies of the test data. The dots and error bars indicate the mean and the standard deviation for five measurements.

Fig. 11. Estimated loads for each leg in each direction. The dots and error bars indicate the mean and the standard deviation for five measurements.

Table 3: RMSEs of the Estimated Loads

Case	Estimated load (kg)			
	1st leg	2nd leg	3rd leg	4th leg
Front	0.8525	1.0632	0.2795	1.7361
Rear	0.7282	1.5023	1.4435	1.1730
Left	0.6703	1.1811	1.1367	1.0716
Right	0.8114	0.9156	0.7589	0.6871

because the noise in the frequency characteristics affects the resonant frequency. In particular, the amount of change in the resonant frequencies of the gain and phase characteristics is small, as shown in Fig. 9. In addition, a load meter was used to measure the vertical force. However, the proposed method cannot be used to give an estimate by considering the reaction force in each direction. In the experiment, the center of gravity was moved toward the lateral and longitudinal directions

Fig. 12. Schematic of ZMP calculation parameters: (a) reaction forces on each leg from the load estimation and (b) contact positions of each leg in the x and y directions.

Table 4: RMSEs of Estimated ZMP

Case	x_{zmp} (m)	y_{zmp} (m)	xy_{zmp} (m) [†]
Front	0.0366	0.0127	0.0274
Rear	0.0144	0.0216	0.0184
Left	0.0132	0.0120	0.0126
Right	0.0149	0.0205	0.0179

[†] xy_{zmp} is the RMSEs that combines the x and y directions.

of the robot by altering the posture. Therefore, a load larger than the measured value was estimated because the external force is applied in multiple directions.

The position of the ZMP is calculated from the following equations using the estimated load:

$$x_{zmp} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 + x_4 f_4}{f_1 + f_2 + f_3 + f_4} \dots \dots \dots (9)$$

$$y_{zmp} = \frac{y_1 f_1 + y_2 f_2 + y_3 f_3 + y_4 f_4}{f_1 + f_2 + f_3 + f_4} \dots \dots \dots (10)$$

where x_{zmp} and y_{zmp} are the ZMP positions in the x and y directions, respectively, x_i and y_i are the contact positions of the i th leg viewed from the center of the support polygon in the x and y directions, respectively, and f_i is the reaction force of the i th leg. Here, the reaction force is calculated from the estimated load. The relationships among the variables are shown in Fig. 12, and the ZMP positions are shown in Fig. 13. In addition, the RMSEs of the ZMP positions are given in Table 4.

From the results shown in Fig. 13 and Table 4, it is clearly possible to calculate the ZMP position using the estimated load. The errors in the ZMP position given in Table 4 are due to the large variation in the estimated load. However, a multi-legged wheel-legged mobile robot has a large support polygon. For our robot, the support polygon extends 0.620 and 0.665 m in the x and y directions, respectively. Therefore, an error of approximately 0.02 m is considered to be acceptable.

4. Online Estimation of the Zero Moment Point

The online ZMP estimation is required to actually use for robot motion control based on the ZMP because the ZMP estimation results in section 3 were the offline estimation. Therefore, this section describes an approach for the online ZMP estimation. To estimate the ZMP online, the frequency characteristics need to be computed online. This method uses a fast Fourier transform (FFT), the resolution of which is determined by the data length and sampling frequency. Thus,

Fig. 13. Estimated ZMP of the robot in each direction. The dots and error bars indicate the mean and the standard deviation for five measurements.

obtaining a sufficient data length for the FFT in a short time is necessary for online ZMP estimation. To accomplish this, we use zero padding, a technique that increases the data length by adding a zero to the FFT data to increase the apparent FFT resolution⁽³⁶⁾. This results in a sufficient data length for the FFT in a short time. Therefore, the frequency characteristics for estimating the ZMP can be obtained in a short time using zero padding.

Next, the calculation of the online estimation is verified through an experiment. The calibration data are measured using the same conditions as those for the offline case. The measurement conditions are shown in Table 1. Here, the data length was 65,536 points. The approximation models were obtained as follows:

$$W_{leg1} = 0.3786\omega_{G_{leg1}} + 0.4270\omega_{P_{leg1}} - 33.1656 \dots (11)$$

$$W_{leg2} = 0.3175\omega_{G_{leg2}} + 0.3511\omega_{P_{leg2}} - 25.6501 \dots (12)$$

$$W_{leg3} = 0.2057\omega_{G_{leg3}} + 0.2072\omega_{P_{leg3}} - 11.8049 \dots (13)$$

$$W_{leg4} = 0.2911\omega_{G_{leg4}} + 0.2234\omega_{P_{leg4}} - 18.2135 \dots (14)$$

These approximation models are different from those of Eqs. (5) to (8) because the conditions for the FFT and the filter are different from the offline calculation. The online ZMP estimation experiment was conducted using these equations. The measurement conditions are shown in Table 1. Here, zero padding was used to obtain the FFT data in a short time. The sampling length was measured to be 0.4096 s using a sampling interval of 0.05 ms. The data length was 8,192 points, which further increased to 65,536 points after zero padding. Table 2 gives the load on each leg. In addition, a multi-link model-based ZMP calculation⁽¹⁶⁾ was also performed for comparison. The experimental results for the estimated load and ZMP are shown in Figs. 14 and 15, respectively. In addition, the RMSEs of the estimated load and the estimated ZMP are shown in Tables 5 and 6, respectively.

From the results shown in Fig. 14 and Table 5, the load can be estimated, though the variation is large. The cause of

Fig. 14. Online estimated loads of each leg in each direction. The dots and error bars indicate the mean and the standard deviation for five seconds measurements.

Fig. 15. Online estimated ZMP of the robot in each direction. The dots and uncertainty bars indicate the mean and the expanded uncertainty for five seconds measurements.

Table 5: RMSEs of the Online Estimated Load

Case	Estimated load (kg)			
	1st leg	2nd leg	3rd leg	4th leg
Front	1.9725	1.6013	0.5551	1.7212
Rear	1.7311	2.1615	0.6253	1.9572
Left	1.2144	3.0812	0.3932	1.8120
Right	3.0276	1.8057	1.2169	1.3908

this variation is that the frequency component was not sufficiently large because the original FFT signal was too short. From the ZMP results in Fig. 15 and Table 6, there was an estimation error in the multi-link model-based ZMP calculation.

Table 6: RMSEs of the Online Estimated ZMP

Method	Case	Estimated ZMP		
		x_{zmp} (m)	y_{zmp} (m)	xy_{zmp} (m) [†]
Proposed	Front	0.0308	0.0457	0.0389
	Rear	0.0363	0.0344	0.0354
	Left	0.0400	0.0367	0.0383
	Right	0.0355	0.0303	0.0319
Multi-link model-based	Front	0.0575	0.0011	0.0407
	Rear	0.0593	0.0012	0.0420
	Left	0.0014	0.0704	0.0498
	Right	0.0020	0.0631	0.0447

[†] xy_{zmp} is the RMSE that combines x and y directions.

tion. This method is affected by the change in posture and the initial conditions because the ZMP is calculated using the joint angle. In these experiments, the reaction force has been changed because of the external force, which is affected by the chain block attached to the robot. The multi-link model-based ZMP calculation cannot take the influence of the reaction force due to such an external force into account, and there is a large error. The ZMP estimated using the proposed method tends to be close to the measured ZMP because the influence of the reaction force due to the external force at the contact point can be considered as the estimated load; however, the estimated ZMP has variations caused by the variation in the estimated load. Comparing the totals from the x- and y-axes, the average of the RMSEs for the proposed method is 0.0361 m, and for the multi-link model-based ZMP calculation, it is 0.0443 m. The proposed method reduced the ZMP error to about 82% of the multi-link model-based ZMP calculation. The accuracy of the estimated ZMP is evaluated to compare with the case of assuming the force sensor used in general ZMP measurement. Here the stability room⁽³⁷⁾ is used for the evaluation measure. This measure is the minimal distance from ZMP to the boundary of the stable region. In this paper, the stable region was calculated by subtracting the error as the margin from the support polygon. The position of the ZMP is assumed at the most stable point which is the center of the support polygon, and the stability room is 0.274 m when the average of the RMSE of the online ZMP estimation is considered as the margin. Next, the stability room is obtained under the assumption of the force sensor. The weight of our robot is about 55 kg, therefore, the allowable load in the vertical direction is 1000 N to have a sufficient margin. When assuming the allowable load in the vertical direction is 1000 N, the resolution of the force sensor is 0.125 to 0.5 N⁽³⁸⁾⁻⁽⁴⁰⁾, and the median is 0.313 N. Then, the error of ZMP caused by the resolution of the force sensor in the case of the load in Table 2 applied on each leg is 0.00132 m in the x direction, 0.00142 m in the y direction, and 0.00137 m on average. The stability room considering the error from the resolution of the force sensor is 0.309 m. From the above, the proposed method can achieve an accuracy of 88.7 % when assuming the use of the force sensor.

Finally, we considered the uncertainty in the experimental results, which is shown in Table 7. The uncertainties in the measurement by the load meter, the proposed method, and the multi-link model-based ZMP calculation were most affected by the accuracy of the load meter, the measurement repeatability, and the resolution of the encoder, respectively.

Table 7: Uncertainty of the Online Estimated ZMP

Method	Case	$x_{zmp}(mm)$	$y_{zmp}(mm)$
Measured by load meter	Front	59.7±2.7	1.10±2.9
	Rear	-60.9±2.7	1.22±2.9
	Left	0.517±2.7	70.4±2.9
	Right	-0.982±2.7	-63.1±2.9
Proposed	Front	38.4±15	-35.5±18
	Rear	-92.5±12	1.35±23
	Left	-9.56±26	48.8±20
	Right	-7.30±22	-64.6±20
Multi-link model-based	Front	2.17±1.3	0.00133±0.87
	Rear	-1.59±1.3	0.00788±0.87
	Left	0.951±1.3	0.0249±0.87
	Right	0.582±1.2	0.00414±0.87

The first term of each value indicates the average. The second term shows the expanded uncertainty when the coverage factor is 2.0.

From Table 7, the estimated ZMP of the proposed method was closer to the value measured by the load meter than the value from the multi-link model-based ZMP calculation. In addition, the expanded uncertainty of the proposed method is larger than that of the multi-link model-based ZMP calculation; however, the uncertainty is acceptable because the expanded uncertainty is sufficiently smaller than the support polygon of the robot. From the above, the ZMP of a wheel-legged mobile robot is possible to estimate by the proposed method. The proposed method estimates the ZMP based on the relationship between the resonant frequency of the wheel and the load. This relationship is held on the wheel joint of other mobile systems that has the active wheel joint. Therefore, the proposed method can be applied to the active-wheeled mobile system other than the wheel-legged mobile robot and can be used for its motion control.

5. Discussion

The only vertical force was considered by the offline and online ZMP estimation in the previous sections. In a real environment, several factors affect the resonant frequency of the tire other than the vertical force. These factors include that the force is in multiple directions (multiple-direction force) as well as the vertical direction, temperature changes, and the stiffness and roughness of the ground. The effect of each factor was examined experimentally. The measurement conditions for the frequency characteristics were the same as those in Sect. 2.2. The load at the contact point was changed by adjusting the posture at the chain block.

5.1 Effect of the Multiple-Direction Force The effects of the lateral and longitudinal forces on the wheel joint were then explored. The frequency characteristics were measured when the external force in the lateral and longitudinal directions was applied as shown in Fig. 16. Each applied load weighed 2.5 kg. The resonant frequencies of the vertical force alone and with the addition of the external force in the lateral and longitudinal directions are shown in Fig. 17. The first resonant frequencies in Fig. 17 correspond to the resonant frequencies used for the ZMP estimation in the previous section. These results indicate that the change in the resonant frequencies was small in the 10.0 kg test; however, they increased slightly in the 5.0 kg test. The correspondence is difficult to explain using the conventional model of a single vertical force. This problem was addressed using the multi-

Fig. 16. Experimental conditions to confirm the effect of the multiple-direction force: (a) lateral direction and (b) longitudinal direction.

Fig. 17. Resonant frequencies of the multiple-direction force. The dots and error bars indicate the mean and the standard deviation for five measurements.

ple resonant frequencies seen in the frequency characteristics. A quartic model was considered for the multiple-direction force. This model included the calibration data for the lateral and longitudinal forces. The number of degrees of freedom of the model was increased by including another resonant frequency between 20.0 and 40.0 Hz. By considering the two resonant frequencies, the following quartic equation was obtained through an expansion of Eq. (1):

$$W = b_1\omega_{G1} + b_2\omega_{P1} + b_3\omega_{G2} + b_4\omega_{P2} + b_5 \dots \dots (15)$$

where ω_{G1} and ω_{G2} are the resonant frequencies of the first and second gain characteristics. ω_{P1} and ω_{P2} are the resonant frequencies of the first and second phase characteristics. The coefficients of the quartic equation are b_1 , b_2 , b_3 , b_4 , and b_5 . Equation (15) was created by including the resonant frequencies from Fig. 17 as follows:

$$W = -0.1819\omega_{G1} + 0.5181\omega_{P1} + 0.2338\omega_{G2} - 0.0473\omega_{P2} - 11.9825 \dots \dots (16)$$

Next, the effect was confirmed by estimating the load using Eq. (16), and the resonant frequencies from another data set were used as a test. The estimated load both without external forces and with an external force with a lateral or longitudinal component is shown in Fig. 18. The load was estimated using multiple resonant frequencies, even when the external

Fig. 18. Estimated load for the multiple-direction force. The dots and error bars indicate the mean and the standard deviation for five measurements.

Fig. 19. Resonant frequencies for a temperature change. The dots and error bars indicate the mean and the standard deviation for five measurements. The solid lines show the fitting line for the mean values by a least-squares calculation.

force was affected. From Fig. 18, it can be seen that a correlation for the external force is possible by preparing a model in advance.

5.2 Effect of Temperature The effect of a temperature change was confirmed by measuring the frequency characteristics while changing the temperature of the room. Measurements were conducted when the surface temperature of the wheel was 18.0 °C, 21.0°C, 24.0 °C, 27.0 °C, 29.0 °C, and 31.0 °C. The resonant frequencies at each temperature are shown in Fig. 19. From the results, a change in the resonant frequency corresponding to a change in temperature can be confirmed. This change is caused by the change in the hardness of the rubber due to temperature. Moreover, the change in the resonant frequency corresponding to the load is constant at each temperature. This means that the reference value for the resonant frequency can be modeled. The relationship between temperature and the resonant frequency can be modeled almost linearly, although there were slight varia-

Fig. 20. Experimental conditions to confirm the effect of the contact plane: (a) sand and (b) gravel.

Fig. 21. Resonant frequencies for each contact plane. The dots and error bars indicate the mean and the standard deviation for five measurements.

tions, as shown in Fig. 19. Accounting for this relation using a linear model has been examined in a TPMS of automotive applications⁽³⁴⁾. Therefore, the effect of a temperature change can be accounted for by measuring it using a sensor and applying an offset to the resonant frequency corresponding to the measured temperature.

5.3 Effect of Contact Plane The effect of the contact plane was also considered. Frequency characteristic measurements were conducted on sand and gravel, as shown in Fig. 20. The resonant frequencies for a flat plane, sand, and gravel are shown in Fig. 21. The first resonant frequencies in Fig. 21 also correspond to the resonant frequencies used to estimate the ZMP in the previous section. From the results, it can be seen that the resonant frequency increased for sand and gravel. This is caused by the apparent weight of the contact plane decreasing through small slips on the sand and gravel.

Here, we considered increasing the number of degrees of freedom using multiple resonant frequencies, like the method used in Sect. 5.1, to account for the effect of the contact plane. The resonant frequencies for the entire contact plane from 20.0 to 40.0 Hz, as shown in Fig. 21, were used to create the following equation:

$$W = -0.1690\omega_{G1} + 0.1954\omega_{P1} + 0.0752\omega_{G2} + 0.4593\omega_{P2} - 5.2420 \dots \dots (17)$$

Next, the load was estimated using Eq. (17), and the resonant frequency for the test data was measured to confirm the effect. The estimated loads for the flat plane, sand, and gravel are shown in Fig. 22. The load was estimated using multiple resonant frequencies even when the environment of the contact plane varied. This shows that the effect of the environ-

Fig. 22. Estimated load for each contact plane. The dots and error bars indicate the mean and the standard deviation for five measurements.

ment can be calculated by preparing a model of the contact plane in advance.

5.4 Effect of Multiple Factors The previous sections discussed the effects of the force acting in multiple directions as well as the vertical direction, and the effects of a temperature change, and the stiffness and roughness of the ground. These effects occur simultaneously during actual motion. Therefore, we considered the composite effects. Firstly, the effect of temperature can be solved independently based on the temperature measured by an additional sensor because this effect can be approximated as a linear model. Next, the resonant frequencies of the tire are below 100 Hz from the results of Sect. 2. Therefore, the model used to account for the effects of the multiple-direction force and the contact plane has about four degrees of freedom. In addition, the effects of the multiple-direction force and the contact plane can be solved using the respective effects simultaneously for the calibration data because the change in the resonant frequencies was different, as well as Figs. 17 and 21. From the above, the estimation is possible even when there are multiple simultaneous factors.

6. Conclusions

This paper focused on the relationship between the wheel load and resonant frequency to obtain the reaction force at the contact point. Firstly, frequency analyses and experiments were conducted to confirm the relationship between the load and resonant frequency in the wheel joint of a wheel-legged mobile robot. Next, a method used to estimate the ZMP using a model based on the relation between the load and the resonant frequency was proposed. Then, the effectiveness of the proposed method was confirmed by an experiment with online estimation. From the experimental results of the on-line estimation, the average of the RMSEs in the x and y directions was 0.0361 m. This value is acceptable because it is sufficiently smaller than the support polygon of the robot. Furthermore, the factors that affected the resonant frequency of the wheel joint were taken into account. From the above, we can estimate the ZMP of a wheel-legged mobile robot.

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