LMI-Based Mixed H_2/H_∞ Controller Design with Regional Pole Constraints for Damping Power System Oscillations

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This paper presents the mixed H_2/H_∞ controller design with regional pole constraints for damping power system oscillations. The state feedback gain can be obtained by solving a linear matrix inequality (LMI) feasibility problem that robustly assigns the closed-loop poles in a prescribed LMI region. The proposed technique is illustrated with applications to the design of stabilizer for a typical single-machine infinite-bus (SMIB) and a multimachine power system. The LMI-based control ensures adequate damping for widely varying system operating conditions. The simulation results illustrate the effectiveness and robustness of the proposed stabilizer.

Keywords: mixed H_2/H_∞ control, regional pole constraints, linear matrix inequality, power system stabilizer, robust control

1. Introduction

Power systems are usually large nonlinear systems, which are often subject to low frequency oscillations when working under some adverse loading conditions. The oscillation may sustain and grow to cause system separation if no adequate damping is available. To enhance system damping, the generators are equipped with power system stabilizers (PSSs) that provide supplementary feedback stabilizing signals in the excitation systems. PSSs enhance the power system stability limit by enhancing the system damping of low frequency oscillations associated with the electromechanical modes⁽¹⁾. Many approaches are available for PSS design, most of which are based either on classical control methods⁽¹⁾⁻⁽³⁾ or on intelligent control strategies⁽⁴⁾⁻⁽⁶⁾.

However, as power systems are large nonlinear systems, it is impossible for the system to always run at the preselected operating conditions. When the system is away from the specified operating point, the performance of the PSS will degenerate. Power systems continually undergo changes in the operating condition due to changes in the loads, generation and the transmission network resulting in accompanying changes in the system dynamics. A well-designed PSS has to perform satisfactorily in the presence of such variations in the power system. In other words, the stabilizer should be robust to changes in the power system over its entire operating range.

In the last few years, robust control technique has been applied to power system controller design to guarantee robust performance and robust stability, due to

uncertainty in plant parameter variations. Some of those efforts have been contributed to design robust controllers for PSS and/or FACTS devices using H_∞ concept such as mixed-sensitivity⁽⁷⁾⁻⁽¹⁰⁾; μ -synthesis or structured singular value (SSV)⁽¹¹⁾ and H_2 norm concept such as LQG⁽¹²⁾. Normally, the problem is formulated as a weighted mixed-sensitivity design and solved by a Riccati approach. In addition, robust H_∞ design being essentially a frequency domain approach does not provide much control over the transient behavior and closed-loop pole location. Robust pole placement stabilizer design using linear matrix inequalities (LMIs) has been presented in Ref. (13), where the feedback gain matrix is obtained as the solution of a linear matrix inequality expressing the pole region constraints for polytopic plants.

Design methods based on the H_∞ norm of the closed-loop transfer function have gained popularity, because unlike H_2 methods (best known as LQG), they offer a single framework in which to deal both with performance and robustness. On the other hand, since an H_2 cost function offers a more natural way of representing certain aspects of the system performance, improving the robustness of H_2 -based design methods against perturbations of the nominal plant is a problem of considerable importance for practical applications⁽¹⁴⁾. In the robust H_2 approach, the controller is designed to minimize an upper bound on the worst-case H_2 norm for a range of admissible plant perturbations. Thus, a combination of H_2 control and H_∞ control, called mixed H_2/H_∞ control that minimized the H_2 norm of some closed-loop function subject to the H_∞ norm constraint of another closed-loop function. Khargonekar et al.⁽¹⁸⁾ considered state- and output-feedback problems of mixed H_2/H_∞ control and gave efficient convex optimization approach

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to solve the coupled nonlinear matrix Riccati equations.

With the development of numerical algorithms for solving linear matrix inequality (LMI) problems in the last 8 years, the LMI approach have emerged as a useful tool for solving a wide variety of control problems⁽¹⁶⁾. One of the advantages of linear matrix inequality (LMI) is mixing the time and frequency domain objectives. This paper proposes a mixed H_2/H_∞ controller design with regional pole constraints for damping power system oscillations base on linear matrix inequality. The efficiency of an LMI-based design approach as a practical design tool is illustrated with case studies, including a typical single-machine infinite-bus (SMIB) and a mult-machine power system.

The paper is organized as follows. A detailed description of the proposed design procedure is given in Section 2. In Section 3, simulation results are given for a typical single-machine infinite-bus (SMIB) and a multi-machine power system to demonstrate the effectiveness of the proposed method. Conclusions are drawn in Section 4.

2. LMI-based Mixed H_2/H_∞ Controller Design

Stability is a minimum requirement for control system. However, in most practical situations, a good controller should also deliver sufficiently fast and well-damped time responses. A customary way to guarantee satisfactory transients (or dynamics) is to place the closed-loop poles in a suitable region of the complex s-plane.

This section discusses state feedback synthesis with a combination of mixed H_2/H_∞ performance and pole assignment specifications. Here, the closed-loop poles are required to lie in some LMI region D contained in the left-half plane. Unconstrained mixed H_2/H_∞ synthesis is considered in Ref. (18), where an LMI-based synthesis procedure is proposed. Excellent background material on LMI may be found in Ref. (15).

2.1 Introduction of Linear Matrix Inequality

A wide variety of problems in control theory and system can be reduced to a handful of standard convex and quasi-convex optimization problems that involve linear matrix inequalities (LMIs), that is constraints of the form⁽¹⁵⁾:

$$F(x) \triangleq F_0 + \sum_{i=1}^m x_i F_i > 0 \dots\dots\dots (1)$$

where $x = [x_1, x_2, \dots, x_m] \in R^m$ is the variable, and the symmetric matrices $F_i = F_i^T \in R^{n \times n}$, $i = 0, \dots, m$, are given. The set $\{x | F(x) > 0\}$ is convex, and need not have smooth boundary.

When the matrices F_i are diagonal, the LMI $F(x) > 0$ is just a set of linear inequalities. Nonlinear (convex) inequalities are converted to LMI form using Schur complements. The basic idea is as follows:

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0 \dots\dots\dots (2)$$

where $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ depend affinely on x , is equivalent to

$$R(x) > 0, Q(x) - S(x)R(x)^{-1}S(x)^T > 0 \dots\dots (3)$$

In other words, the set of nonlinear inequalities Eq. (3) can be represented as the LMI Eq. (2).

Two standard LMI optimization problems are of interest:

- (1) LMI feasibility problem. Given an LMI $F(x) > 0$, the corresponding LMI feasibility problem is to find x^{feas} such that $F(x^{feas}) > 0$ or determine that the LMI is infeasible.
- (2) Semi-Definite Programming problem (SDP). An SDP requires the minimization of a linear objective subject to LMI constraints:

$$\left. \begin{array}{l} \text{Minimize } c^T x \\ \text{Subject to } F(x) > 0 \end{array} \right\} \dots\dots\dots (4)$$

where c is a real vector, and F is a symmetric matrix that depends affinely on the optimization variable x . This is a *convex optimization problem*.

Both these problems can be numerically solved vary efficiently, using currently available software⁽¹⁶⁾⁽¹⁷⁾.

2.2 LMI Formulation for Mixed H_2/H_∞ Performance

Consider the linear plant P with input u , disturbance w , performance output z_∞ and z_2 , the measurement signal x . The input is generated by state feedback, using the controller K . The signal z_∞ is the performance associated with the H_∞ constraint, the signal z_2 is the performance associated with the H_2 criterion. The state space representation of the controlled system can be written as follows:

$$\left. \begin{array}{l} \dot{x} = Ax + B_1 w + B_2 u \\ z_\infty = C_1 x + D_{12} w \\ z_2 = C_2 x + D_{22} u \end{array} \right\} \dots\dots\dots (5)$$

where all the matrices are constant real matrices of appropriate dimension. The illustration of the controlled system is shown in Fig. 1.

After substitution of the state feedback controller $u = Kx$ into Eq. (5), the closed-loop system becomes

$$\left. \begin{array}{l} \dot{x} = (A + B_2 K)x + B_1 w \\ z_\infty = (C_1 + D_{12} K)x \\ z_2 = (C_2 + D_{22} K)x \end{array} \right\} \dots\dots\dots (6)$$

Let $T_{z_\infty w}$ and $T_{z_2 w}$ be the closed-loop transfer matrices from the generalized disturbance w to the performance

Fig. 1. Mixed H_2/H_∞ control

output z_∞ and z_2 , respectively:

$$T_{z_\infty w}(s) = \left[\begin{array}{c|c} A + B_2 K & B_1 \\ \hline C_1 + D_{12} K & 0 \end{array} \right] = \left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl\infty} & 0 \end{array} \right] \quad (7)$$

$$T_{z_2 w}(s) = \left[\begin{array}{c|c} A + B_2 K & B_1 \\ \hline C_2 + D_{22} K & 0 \end{array} \right] = \left[\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl2} & 0 \end{array} \right] \quad (8)$$

The goal of mixed H_2/H_∞ control is to find an internally stabilizing controller K that minimizes the H_2 performance, $\|T_{z_2 w}\|_2$, subject to the H_∞ performance, $\|T_{z_\infty w}\|_\infty < \gamma$ and places the closed-loop poles in some LMI stability region D that will be explained in the next subsection. In this subsection, pure H_2 and H_∞ synthesis are not given. For proofs and more details, see Refs. (19), (20).

We are now ready to give tractable necessary and sufficient conditions for solving the following mixed H_2/H_∞ problem:

$$\min \text{trace}(C_{cl2} P C_{cl2}^T) \quad (9)$$

$$\text{s.t.} \left[\begin{array}{cc|c} A_{cl} P + P A_{cl}^T + B_{cl} B_{cl}^T & P C_{cl\infty}^T & \\ \hline C_{cl\infty} P & -\gamma^2 I & \end{array} \right] < 0 \quad (10)$$

$$P = P^T > 0$$

The optimization problem above is not yet convex because of the products KP arising in terms like $A_{cl} P$. So, defining the new variables $Y = Y^T = P$, $L = KY$ and $W = W^T$ and using Schur's complement it is possible to rewrite the problem above as the LMI problem

$$\min \text{trace}(W) \quad (11)$$

$$\text{s.t.} \left[\begin{array}{cc} H & Y C_1^T + L^T D_{12}^T \\ C_1 Y + D_{12} L & -\gamma^2 I \end{array} \right] < 0 \quad (12)$$

$$\left[\begin{array}{cc} Y & Y C_2^T + L^T D_{22}^T \\ C_2 Y + D_{22} L & W \end{array} \right] > 0 \quad (13)$$

where $H = AY + YA^T + B_2 L + L^T B_2^T + B_1 B_1^T$.

2.3 LMI Formulation for Regional Pole Constraints In the synthesis of control systems, meeting some desired transient performance objectives (to ensure fast and well-damped transient response, reasonable feedback gain, etc.) should be considered. Generally, H_2 -norm and H_∞ synthesis design do not directly deal with the transient response of the closed-loop system. In contrast, a satisfactory transient response can be guaranteed by confining its poles in a prescribe region. For many practical problems, exact pole assignment may not be necessary; it suffices to locate the closed-loop poles in a prescribe subregion in the complex left half plane.

Definition 1. LMI stability region⁽¹⁹⁾. A subset D of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha = [\alpha_{kl}] \in R^{m \times m}$ and a matrix $\beta = [\beta_{kl}] \in R^{m \times m}$ such that

$$D = \{z \in C : f_D(z) < 0\} \quad (14)$$

where the characteristic function $f_D(z)$ is given by

Fig. 2. Region $S(\alpha, r, \theta)$

$f_D(z) = [\alpha_{kl} + \beta_{kl}z + \beta_{kl}\bar{z}]_{1 \leq k, l \leq m}$ (f_D is valued in the space of $m \times m$ Hermitian matrices).

The location of the closed-loop poles of $(A + B_2 K)$ in Eq. (6) concern with the performance of the closed-loop system, i.e., the stability, the decay rate, the maximum overshoot, the rise time and settling time. Therefore, it is interesting work for control engineers to design the control gain K such that the closed-loop poles of $(A + B_2 K)$ lie in a suitable subregion of the left half plane. The interesting region for control purposes is the set $S(\alpha, r, \theta)$ of complex number $x + jy$ such that

$$x < -\alpha < 0, |x + jy| < r, \text{ and } \tan(\theta)x < -|y| \quad (15)$$

as shown in Fig. 2. Confining the closed-loop poles to this region ensures a minimum decay rates α , a minimum damping ratio $\zeta = \cos \theta$, and a maximum undamped natural frequency $\omega_d = r \sin \theta$ (θ in radian).

The LMI formulations for the poles of $(A + B_2 K)$ lie in the region $S(\alpha, r, \theta)$ are characterized as the following LMIs^{(19) (20)}: if there exists symmetric $P > 0$ such that

$$(A + B_2 K)P + P(A + B_2 K)^T + 2\alpha P < 0 \quad (16)$$

$$\left[\begin{array}{cc} -rP & (A + B_2 K)P \\ P(A + B_2 K)^T & -rP \end{array} \right] < 0 \quad (17)$$

and

$$\left[\begin{array}{c} \sin(\theta)((A + B_2 K)P + P(A + B_2 K)^T) \\ \cos(\theta)(P(A + B_2 K)^T - (A + B_2 K)P) \\ \cos(\theta)((A + B_2 K)P - P(A + B_2 K)^T) \\ \sin(\theta)((A + B_2 K)P + P(A + B_2 K)^T) \end{array} \right] < 0 \quad (18)$$

with $L = KP$; $Y = P$, the above LMIs are equivalent to

$$AY + YA^T + B_2 L + (B_2 L)^T + 2\alpha Y < 0 \quad (19)$$

$$\left[\begin{array}{cc} -rY & AY + B_2 L \\ Y A^T + (B_2 L)^T & -rY \end{array} \right] < 0 \quad (20)$$

$$\begin{bmatrix} \sin(\theta)(AY + B_2L + YA^T + (B_2L)^T) \\ \cos(\theta)(YA^T + (B_2L)^T - AY - B_2L) \\ \cos(\theta)(AY + B_2L - YA^T - (B_2L)^T) \\ \sin(\theta)(AY + B_2L + YA^T + (B_2L)^T) \\ \dots\dots\dots \end{bmatrix} < 0 \quad (21)$$

From the analysis above, if there exists Y and L for Eqs. (19)~(21), then the poles of $(A + B_2K)$ lie in the region $S(\alpha, r, \theta)$.

2.4 Mixed H_2/H_∞ Controller with Regional Pole Constraints The combination objectives of robust mixed H_2/H_∞ control with regional pole constraints can be characterized as follows:

$$\begin{aligned} & \min_{\{Y, L\}} \text{trace}(W) \\ & \text{s.t. Eqs. (12), (13), and (19)~(21)} \dots\dots\dots \end{aligned} \quad (22)$$

From the analysis above, the most important task in this paper is to find the variables Y , L , γ and W can be solved using standard optimization techniques. Once a feasible solution (Y, L) satisfying Eq. (22) is found, the required state feedback gain matrix can be computed as

$$K = LY^{-1} \dots\dots\dots (23)$$

which leads to

$$\|T_{z_\infty w}\|_\infty \leq \gamma, \|T_{z_2 w}\|_2 \leq \sqrt{\text{trace}(W)} \dots\dots (24)$$

The Lyapunov shaping paradigm for multi-objective design provides a greater flexibility than single-objective optimal design techniques such as H_∞ synthesis or H_2 -norm technique.

3. Simulation Results

To test the performance of the proposed stabilizer, simulation studies were performed on the two systems: a single-machine infinite-bus (SMIB) and a 3-machine 9-bus power system.

3.1 A Single-machine Infinite-bus System A typical single-machine infinite-bus (SMIB) power system⁽²¹⁾ is chosen for analysis of the proposed controller as shown in Fig. 3. The static fast exciter is shown in Fig. 4. The overall system is of fourth order, with the synchronous machine represented by third order model and exciter-voltage regulator system represented by first order dynamics. The state variables of this model are $\Delta\omega$, $\Delta\delta$, $\Delta E'_q$, ΔE_{fd} , respectively, angular speed, rotor angle, voltage behind transient, and excitation voltage. The power input to the generator shaft is assumed constant, the network is represented by a set of algebraic equations and the loads are modeled by constant impedance. The machine data, exciter data, block diagram of CPSS, and CPSS constants are given in the Appendix.

The operating condition: $P_e = 1.0$ pu, $Q_e = 0.015$ pu and $v_t = 1.05$ pu are chosen as the nominal operating condition and other operating points are regarded as perturbations of the nominal system. Four different loading conditions representing nominal, heavy, light,

Fig. 3. A SMIB power system

Fig. 4. Static fast exciter model

Table 1. Operating conditions

Case	P (pu)	Q (pu)
1. Nominal	1.0	0.015
2. Heavy	1.2	0.20
3. Light	0.7	0.10
4. Leading PF	0.7	-0.20

and leading power factor (PF) are considered as given in Table 1. The eigenvalues of the nominal system are $0.295 \pm j4.96$ and $-10.4 \pm j3.28$. It is observed that the electromechanical mode (characterized by the pair of eigenvalues $0.295 \pm j4.96$) is negatively damped and the eigenvalues for this mode should be shifted leftward to more desirable locations into the left half s-plane.

It is proposed to apply a full state feedback controller using the LMI based approach described in Section 2. There is some limitation for application of state feedback design in comparison with output feedback design because state feedback design needs all states to be measured. However, there are many instruments that could be used to measure the state variables accurately. For the 4th order model used here, the states are the deviations in the rotor speed ω , load angle δ , field voltage E_{fd} and the internal voltage E'_q . Of these, ω , δ and E_{fd} can be directly measured using appropriate transducers. The internal voltage E'_q can be computed from the instantaneous values of the stator currents and the equivalent circuit parameters. On the other hand, there are some essential problems in output feedback design. For instance, a higher-order controller resulted from output feedback design would lead to difficulties in practical implementation. Therefore, state feedback design would

be a reasonable choice than output feedback design, if all states can be measured.

The feasibility problem was solved for (Y, L) and the required state feedback matrix was obtained as $K = LY^{-1}$, where Y is a symmetric, positive definite matrix and L is the matrix introduced to obtain linearity.

To determine the gain of the state feedback controller, the minimization problem Eq. (22) is solved using LMI Control Toolbox⁽¹⁶⁾. We can solve the eigenvalue problem in Eq. (22) with regional pole constraints in the region of $S(2, 16, 1.1593)$. In this case, the associated matrices and scalars are

$$Y = \begin{bmatrix} 0.0151 & -0.2730 & 1.4668 & -100.90 \\ -0.2730 & 10.8880 & -16.902 & 618.11 \\ 1.4668 & -16.902 & 197.26 & -19365 \\ -100.90 & 618.11 & -19365 & 2524100 \end{bmatrix}$$

$$L = [0.0151 \quad -4.7418 \quad -25.4840 \quad 2486.9]$$

$$\gamma = 0.8864, \sqrt{\text{trace}(W)} = 1.1388$$

and thus the feedback gain matrix is

$$K = [288.65 \quad 1.9197 \quad -3.7586 \quad -0.01678]$$

which makes the poles of $(A + B_2K)$ locate at $-5.77 \pm j10.2$ and $-12.70 \pm j5.12$. As a final check, we can calculate

$$\|T_{z_\infty w}\|_\infty = 0.8748, \quad \|T_{z_2 w}\|_2 = 0.5616$$

that satisfies the inequality Eq. (24).

For evaluation purposes, the performance of the system with the proposed controller was compared to the CPSS⁽²¹⁾ and H_∞ PSS. A small disturbance of 10% step increase in the reference voltage (V_{ref}) was applied to the SMIB power system at four different operating conditions. The system eigenvalues and damping ratios of electromechanical modes at various operating conditions are given in Table 2. Note that the damping ratio as shown in Table 2 is written in the brackets. It is clear that the system stability is greatly enhanced with the proposed stabilizer. It can also be seen that all eigenvalues and damping ratios with the proposed stabilizer lie in an LMI region of S . Simulation results shown in Figs. 5~8 illustrate the performance and robustness of the proposed PSS under different operating conditions. It can be seen that the proposed PSS yields the best damping performance compared to the CPSS as well as H_∞ PSS. It is less sensitive to changes in the system parameters and more robust against model uncertainties.

Table 2. System eigenvalues and damping ratios at various operating conditions

Case	CPSS	Proposed PSS
1. Nominal	$-1.52 \pm j 3.17; (0.43)$	$-5.77 \pm j 10.2; (0.49)$
2. Heavy	$-1.07 \pm j 3.02; (0.33)$	$-6.32 \pm j 11.3; (0.49)$
3. Light	$-1.53 \pm j 3.40; (0.41)$	$-3.64 \pm j 7.17; (0.45)$
4. Leading PF	$-1.98 \pm j 3.39; (0.51)$	$-4.05 \pm j 8.45; (0.43)$

For completeness and verifications, all controllers were tasted at the following disturbances and loading conditions.

- Nominal loading $(P, Q) = (1.0, 0.015)$ pu with one-line fault.
- Heavy loading $(P, Q) = (1.2, 0.3)$ pu with one-line fault.

A line fault is assumed; one of the transmission lines (as

Fig. 5. Response with 10% step in V_{ref} for case 1

Fig. 6. Response with 10% step in V_{ref} for case 2

Fig. 7. Response with 10% step in V_{ref} for case 3

Fig. 8. Response with 10% step in V_{ref} for case 4

Fig. 9. Response with fault disturbance for case (a)

Fig. 10. Response with fault disturbance for case (b)

shown in Fig. 3) met a line-fault and the circuit breaker operated. The simulation results for cases (a) and (b) as shown in Fig. 9 and Fig. 10, respectively. Fig. 10 shows that both CPSS and H_∞ PSS fail to stabilize the system with disturbance (b), the proposed PSS provide good damping characteristics and system is stable under this disturbance. It is clear that the proposed PSS exhibits better damping properties and guarantees robust

Fig. 11. Three-machine nine-bus power system

stability of the power systems.

3.2 A 3-machine 9-bus Power System In this part of the study, the 3-machine 9-bus power system shown in Fig. 11 is considered. Details of the system data are given in Ref. (22).

Each machine has been represented by a 3rd-order generators equipped with a static exciter. Without power system stabilizers, the system damping is poor and the system exhibits highly oscillatory response. It is therefore necessary to install one or more PSSs to improve the dynamic performance. To identify the optimum locations of PSSs, the participation factor method⁽²³⁾ was used. The results of the method indicate that all generators (G1, G2 and G3) are the optimum locations for installing PSSs to damp out the electromechanical modes of oscillations.

To design the proposed controller, three operation conditions, i.e. a heavy loading condition, a nominal loading condition, and a light loading condition, are considered as shown in Table 3. The open loop eigenvalues (dominant eigenvalues) of the study system for three operating conditions are given in Table 4. As each pair of conjugate eigenvalues corresponds to an oscillation mode, there are three modes in this study system. Mode 1, 2 and 3 are the rotor oscillation modes (the electromechanical modes). It can be seen that the damping of the rotor oscillation modes for all the operating conditions are poor. In the power systems, a damping ratio (ζ) of at least 10% and the real part of eigenvalue (σ) not greater than -0.5 for the troublesome low frequency electromechanical mode, guarantees that the low frequency oscillations, when excited, will die down in a reasonably short time.

In order to improve the damping of electromechanical modes, a decentralized controller was designed for all generators at the nominal loading conditions based on the proposed design technique in Section 2. In this scheme, each of the generators is fitted with a partial state feedback controller so that only locally available states are feedback at each generator. This implies that

Table 3. Loading condition (in pu)

Generator	Heavy		Nominal		Light	
	P	Q	P	Q	P	Q
G1	1.330	0.630	0.716	0.270	0.362	0.162
G2	1.900	0.361	1.630	0.066	0.800	-0.109
G3	1.200	0.120	0.850	-0.109	0.450	-0.204
Load						
A	1.750	0.700	1.250	0.500	0.650	0.550
B	1.200	0.400	0.900	0.300	0.450	0.350
C	1.400	0.500	1.000	0.350	0.500	0.250

Table 4. Open loop eigenvalues of the study system

Modes	Heavy	Nominal	Light
1	$-0.379 \pm j 3.34$	$-0.401 \pm j 3.46$	$-0.468 \pm j 3.53$
2	$-0.158 \pm j 7.60$	$-0.159 \pm j 7.63$	$-0.377 \pm j 7.44$
3	$-0.924 \pm j 13.2$	$-0.759 \pm j 13.30$	$-0.983 \pm j 13.40$

Fig. 12. Schematic of decentralized state feedback controller

the state feedback matrix K of the overall system is block diagonal. This is schematically shown in Fig. 12, where the submatrices K_1 , K_2 , and K_3 are the feedback gain of each of the three generators. The locally measured states: $\Delta\omega$, $\Delta\delta$, $\Delta E'_q$, and ΔE_{fd} are feedback at the AVR reference input of each machine after multiplication by suitable feedback gains. The LMI problem was constructed by writing the LMI formulation Eq.(22). The feasibility problem was solved for (Y, L) with regional pole constraints in the region of S (2,121, 1.369) and the required state feedback matrix was obtained as $K = LY^{-1}$. If the matrices L and Y used in LMI formulation are restricted to be block diagonal then the product LY^{-1} will also have a block diagonal structure. Such a requirement on the L and Y matrices merely means restricting certain off diagonal elements to be zero which is an easily implemented additional constraint in the optimization problem.

In order to facilitate comparison with CPSS, the design and tuning of CPSS for this multimachine system

Table 5. CPSS parameters

Gen#	K_c	T_w	T_1	T_2
1	83.24	8.0	0.329	0.140
2	33.01	8.0	0.169	0.072
3	78.18	8.0	0.124	0.053

Table 6. Closed-loop eigenvalues and damping ratios at various operating conditions

Modes	Heavy	Nominal	Light
1	$-3.870 \pm j 6.70$ (0.496)	$-6.540 \pm j 5.01$ (0.793)	$-3.800 \pm j 1.45$ (0.934)
2	$-5.820 \pm j 9.05$ (0.541)	$-12.40 \pm j 7.35$ (0.860)	$-5.950 \pm j 7.06$ (0.362)
3	$-10.50 \pm j 16.30$ (0.541)	$-7.740 \pm j 12.20$ (0.535)	$-5.590 \pm j 14.40$ (0.645)

were used method in Ref. (24). In this paper, a CPSS with transfer function

$$G(s) = K_c \frac{sT_w}{1 + sT_w} \frac{(1 + sT_1)^2}{(1 + sT_2)^2} \dots \dots \dots (25)$$

was used and the parameters of stabilizer have been tuned to provide an adequate amount of damping for mode of oscillation. The CPSS data for a three-machine system is given in Table 5. The output of all CPSS is limited to ± 0.1 pu.

With the proposed PSS, the closed-loop eigenvalues and damping ratios at various operating conditions are given in Table 6. Note that the damping ratio as shown in Table 6 is written in the brackets. It is quite clear that the system stability is greatly enhanced with the proposed stabilizer. It can also be seen that all eigenvalues and damping ratios associated with the electromechanical modes have been successfully shifted to lie in an LMI region of S with the proposed PSS.

To demonstrate the capability of the proposed PSS to enhance system damping over a wide range of operating conditions, three different loading conditions were considered. A 10% step change in the reference voltage was applied at generator 2 (G2) as follows.

(1) Nominal loading condition The system responses at the nominal loading condition are shown in Figs. 13(a)~(c), respectively, speed deviation of generator 1, speed deviation of generator 2, and speed deviation of generator 3. It is obvious that the system performance with the proposed PSS is better than CPSS.

(2) Heavy loading condition The system responses at the heavy loading condition are shown in Figs. 14(a)~(c), respectively, speed deviation of generator 1, speed deviation of generator 2, and speed deviation of generator 3. Regarding to the control efforts, the voltage response of generator following the disturbance under heavy loading condition as shown in Figs. 15(a)~(c), respectively. The results here show the superiority of the proposed PSS to the CPSS. It can be concluded that the proposed PSS provides very good

Fig. 13. Generator response under nominal loading condition

Fig. 14. Generator response under heavy loading condition

damping over a wide range of operating conditions.

(3) Light loading condition The system responses at the light loading condition are shown in Figs. 16(a)~(c), respectively, speed deviation of generator 1, speed deviation of generator 2, and speed deviation of generator 3. It is clear that the proposed PSS provide good damping characteristics to low-frequency

oscillations and greatly enhance the dynamic stability of power system.

Studies have shown that a well-tuned PSS can improve power system dynamic stability effectively. Conventional power system stabilizer (CPSS) of the lead-lag compensation type has been adopted by most utility companies because of their simple structure, flexibility

(a)

(a)

(b)

(b)

(c)

(c)

Fig. 15. Voltage response under heavy loading condition

Fig. 16. Generator response under light loading condition

and easy of implementation. Unfortunately, the major disadvantage of the CPSS design method is that it *does not guarantee* system stability under varying operating conditions. The main objective of mixed H_2/H_∞ controller design is that the controller must ensure adequate damping at different operating conditions. The adequacy of the damping is governed by the to settle the oscillations following a disturbance, including a

short circuit with clearance. It can, therefore, be said that the LMI-based robust control will always guarantee adequate robust damping performance whereas conventional PSS may fail to provide this over a range of operating conditions. The LMI method is very systematic and methodical and should therefore be the *preferred option*.

4. Conclusions

This paper has presented the design of mixed H_2/H_∞ control with regional pole constraints for damping power system oscillations. The required state feedback gain has been obtained by solving a linear matrix inequality (LMI) feasibility problem that robustly assigns the closed-loop poles in a prescribed LMI region. The performance of the proposed stabilizer on a SMIB and a multimachine power system are seen to be robust over a wide range of operating conditions. Finally, simulation results show the effectiveness and robustness of the proposed stabilizer to enhance the damping of low frequency oscillations.

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Appendix

System equations and machine parameters

Generator:

$$\dot{\omega} = \frac{1}{M}(T_m - T_e + D(\omega - 1)) \dots\dots\dots (A1)$$

$$\dot{\delta} = \omega_b(\omega - 1) \dots\dots\dots (A2)$$

$$\dot{E}'_q = \frac{1}{T'_{d0}} \{E_{fd} - (x_d - x'_d)i_d - E'_q\} \dots\dots\dots (A3)$$

$$T_e = E'_q i_q + (x_q - x'_d)i_d i_q \dots\dots\dots (A4)$$

Exciter:

$$\dot{E}_{fd} = \frac{1}{T_A} \{K_A(v_{ref} - v_t + u_e) - E_{fd}\} \dots\dots (A5)$$

app. Fig. 1. Block diagram of conventional PSS used for comparison

app. Table 1. Machine data

M	T'_{d0}	D	x_d	x'_d	x_q
9.26	7.76	0	0.973	0.19	0.55

app. Table 2. Exciter data

K_A	T_A
50	0.05

app. Table 3. CPSS constants

T_w	T_1	T_2	K_c
3.0	0.685	0.1	7.09

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