FEM Computation of Magnetic Fields in Anisotropic Magnetic Materials

Akihisa Kameari Member  (Science Solutions Int. Lab., kamearia@ssil.co.jp)
Koji Fujiwara Member  (Okayama University, fujiwara@eplab.elec.okayama-u.ac.jp)

Keywords: anisotropic material, electrical steel sheet, nonlinear magnetostatics, finite element method

The anisotropy of magnetic materials affects the magnetic field distribution and the property of electromagnetic devices. Even the so-called non-oriented electrical steel sheet has weak anisotropy. In order to reduce the anisotropic effect in electric devices such as motors, the steel sheets are usually stacked by rotating the rolling direction. However, it is difficult to stack the steel sheets with rotation in some parts of the device, and then the anisotropic effect could make unexpected results. It is important to develop feasible methods for analysis of the anisotropic effects.

This paper describes a successful analysis of nonlinear anisotropic material using the measured data without special techniques for smoothing. The asymmetric linear equations are solved by using an iterative method aiming at large-scale computation in future. In order to validate the method, the magnetic flux distributions in a proposed ring model was computed for three types of anisotropic materials (the non-oriented (35A300), grain-oriented (35G165), and doubly-oriented (Cube) electrical steel sheets). The resultant calculated distributions in each electrical steel sheet shows characteristic properties, respectively.

Figure 1 shows the distributions of calculated magnetic flux density at the different exciting current in case of the grain-oriented electric steel sheet (35G165). The magnetic flux concentrates in the direction along the easy axis and the concentration is characterized by the magnetic fluxes through inner half of the ring as shown in Fig. 2. Nearly all flux flows through inner side at 90 deg at low excitation.

In the computation, the ICCG or ILUBiCGStab method are adopted as the linear solver and Newton-Raphson method as the nonlinear solver. The measured 2-D property was used directly, but some data must be smoothed to remove the irregularity in the measurement. In the strongly anisotropic materials, the convergence was not attained by the symmetric matrix solver of the ICCG method with symmetrization of the coefficient matrix, but the asymmetric solver of ILUBiCGStab could solve the problem safely. The convergence of the iteration by these methods was not so different from the one in usual isotropic nonlinear analysis.

In conclusion, although the effect of the anisotropy is too exaggerated in the low reluctance model of the ring without a gap, it was shown that the anisotropy affects on the magnetic flux distribution and it could be unexpected by the usual anisotropic field calculation. The proposed method is practical and effective to process measured data in the magnetic field analysis.

Fig. 1. Distributions of magnetic flux density in case of grain-oriented steel sheet (35G165)

Fig. 2. Magnetic fluxes passing through inner half of the ring core at 0 deg and 90 deg, and total flux in case of grain-oriented steel sheet (35G165)
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Akihisa Kameari* Member
Koji Fujiwara** Member

The magnetic fields in nonlinear anisotropic magnetic materials were analyzed by using the Finite Element Method (FEM). The measured data was directly used in the computation without a complicateded smoothing. The resultant asymmetric linear equations were solved by using the ILUBiCGStab method without symmetrization or the ICCG method with symmetrization. The magnetic flux distributions in a ring core model showed the characteristic patterns according to the non-oriented, grain-oriented and doubly-oriented magnetic properties. The good convergence of the Newton-Raphson nonlinear iteration was attained by the iterative solvers without special techniques for the smoothing.

Keywords: anisotropic material, electrical steel sheet, nonlinear magnetostatics, finite element method

1. Introduction

The anisotropy of magnetic materials affects the magnetic field distribution and the property of electromagnetic devices [7]. Even the so-called non-oriented electrical steel sheet has weak anisotropy. In order to reduce the anisotropic effect in electric devices such as motors, the steel sheets are usually stacked by rotating the rolling direction. However, it is difficult to stack the sheets with rotation in some parts of the device, and then the anisotropic effect could make unexpected results.

The so-called two-dimensional magnetic properties, which indicate the inplane anisotropy of electrical sheet, have been measured by using a double-excitation type of single sheet tester. The B-H dependence (B: magnetic flux density, H: magnetic field intensity) is evaluated at multiple angles of B [7]. However, the resultant data has often irregular oscillation due to the measurement error. The magnetic field computation using such measured 2-D properties resulted in a bad convergence in the Newton-Raphson method [7]. In a reference (4), a smoothing technique was introduced to improve the convergence, and the direct solver was adopted to solve asymmetric linear equations obtained from the Newton-Raphson nonlinear iterative procedure.

This paper describes a successful analysis of nonlinear anisotropic material using the measured data without special techniques for smoothing. The asymmetric linear equations are solved by using an iterative method aiming at large-scale computation in future. Magnetic flux distributions in a ring core model with the non-oriented, grain-oriented and doubly-oriented electrical steel sheets are calculated. The characteristic distributions are obtained according to respective materials.

2. Method of Analysis

The measured data of the nonlinear anisotropic material is represented by the magnitude of magnetic field intensity H and its angle θB to the rolling direction as functions of the magnitude of the magnetic flux density B and its angle θB, i.e., H(B, θB), θH(B, θB), as shown in Fig. 1. The data is assumed to be symmetric to the angles of θB = 0 deg and 90 deg, and is transformed into components Hr and Hθ of H parallel and perpendicular to B, respectively, as

\[
H_r(B, \theta_B) = H \cos(\theta_H - \theta_B),
\]

\[
H_\theta(B, \theta_B) = H \sin(\theta_H - \theta_B).
\]

The x- and y-components of H are given by

\[
H_x(B, \theta_B) = H_r \cos \theta_B - H_\theta \sin \theta_B,
\]

\[
H_y(B, \theta_B) = H_r \sin \theta_B + H_\theta \cos \theta_B.
\]

The components of the derivative matrix \( \partial H/\partial B \) required in the Newton-Raphson procedure are given by

\[
\frac{\partial H_r}{\partial B_x} = \frac{\partial H_r}{\partial B} \cos \theta_B - \frac{\partial H_\theta}{\partial \theta_B} \sin \theta_B,
\]

\[
\frac{\partial H_\theta}{\partial B_x} = \frac{\partial H_r}{\partial B} \sin \theta_B + \frac{\partial H_\theta}{\partial \theta_B} \cos \theta_B,
\]

\[
\frac{\partial H_r}{\partial B_y} = \frac{\partial H_r}{\partial B} \sin \theta_B - \frac{\partial H_\theta}{\partial \theta_B} \cos \theta_B,
\]

\[
\frac{\partial H_\theta}{\partial B_y} = \frac{\partial H_r}{\partial B} \cos \theta_B + \frac{\partial H_\theta}{\partial \theta_B} \sin \theta_B.
\]

Fig. 1. Relationship between B and H in the anisotropic material
where the magnetic flux density is given by

\[ \begin{align*}
\frac{\partial H_y}{\partial B_y} &= \sin \theta_B \left( \frac{\partial H_x}{\partial B} \cos \theta_B - \frac{\partial H_x}{\partial B} \sin \theta_B \right) + \frac{\cos \theta_B}{B} \left( \frac{\partial H_x}{\partial B} \sin \theta_B + \frac{\partial H_x}{\partial B} \cos \theta_B \right)
- H_x \sin \theta_B - H_y \cos \theta_B, \\
\frac{\partial H_x}{\partial B_x} &= \cos \theta_B \left( \frac{\partial H_y}{\partial B} \sin \theta_B + \frac{\partial H_x}{\partial B} \cos \theta_B \right) - \frac{\sin \theta_B}{B} \left( \frac{\partial H_x}{\partial B} \sin \theta_B + \frac{\partial H_x}{\partial B} \cos \theta_B \right)
+ H_y \cos \theta_B - H_x \sin \theta_B, \\
\frac{\partial H_y}{\partial B_y} &= \sin \theta_B \left( \frac{\partial H_x}{\partial B} \sin \theta_B + \frac{\partial H_x}{\partial B} \cos \theta_B \right) + \frac{\cos \theta_B}{B} \left( \frac{\partial H_x}{\partial B} \sin \theta_B + \frac{\partial H_x}{\partial B} \cos \theta_B \right)
+ H_x \cos \theta_B - H_y \sin \theta_B. 
\end{align*} \]

In the third direction, the magnitude of magnetic field intensity \(H\) and its angle \(\theta_H\) are measured and summarized in a table for the magnitude of magnetic flux density \(B\) and its angle \(\theta_B\) of every 5 deg from 0 deg to 90 deg. The components \(H_x, H_y, H_z\) are interpolated bilinearly with \(B\) and \(\theta_B\), and are linearly differentiated to calculate \(\partial H/\partial B\).

In the nonlinear magnetostatic analysis by the FEM, the following weak form is solved.

\[ r(A) = \iiint (\nabla \times N \cdot H(B))dV - \iiint N \cdot JdV = 0, \]

where the \(A, J\) and \(N\) are the magnetic vector potential, current density and the weighting function, respectively. The magnetic flux density is given by \(B = \nabla \times A\). In the Newton-Raphson method, the equation is linearized as

\[ r(\delta A) = r(A) + \iiint \nabla \times N_n \cdot \frac{\partial H}{\partial B} \nabla \times \delta A = 0. \]

The global matrix (each entry: \(A_{nm}\)) is derived from the second term in Eq. (6) by the discretization of the FEM. The matrix is asymmetric because of the asymmetry of \(\partial H/\partial B\). \(A_{nm}\) is given by

\[ A_{nm} = \iiint \nabla \times N_n \cdot \frac{\partial H}{\partial B} \nabla \times N_m dV, \]

where \(N_i (i = n, m)\) is the edge shape function.

In the weakly anisotropic material, the ICCG method could solve the linearized equations by taking the symmetric part of the matrix. In the strongly anisotropic material, however, the asymmetric solver such as the ILU(BiCG)Stab method was required.

3. Computational Model

A ring core model shown in Fig. 2 is analyzed to validate the above-mentioned method. The inner and outer radii of the ring core are 50 mm and 100 mm, respectively. The rolling direction is along 0 deg (x-direction). The coil is wound at 180 deg with 18 deg angular width and 5 mm thickness. The position and width of the coil affect the magnetic field distribution very little because of the high permeability of the core. For the isotropic material, the field distribution becomes nearly uniform in the angular direction. For the anisotropic material, characteristic patterns are observed as shown below.

4. Results and Discussion

4.1 Non-Oriented Steel Sheet

Figure 3 shows the measured 2-D magnetic property of a non-oriented electrical steel sheet of 35A300. Even the so-called non-oriented electrical steel sheet has weak anisotropy. Especially in the low \(B\) region, the permeability in the rolling direction \((\theta_B = 0\deg)\) is several times higher than the transverse direction \((\theta_B = 90\deg)\) as shown in Fig. 3(c).

Figure 4 shows the magnetic flux distributions. At low excitation (30 AT), the flux concentrates near the region at 90 deg because of the relatively higher permeability in the rolling direction. At higher excitation (60 AT), the flux distribution becomes uniform. Fig. 5 shows the magnetic fluxes passing through the inner half of the ring core at 0 deg and 90 deg, and total flux. The flux concentration can be clearly understood.

In the computation, the coefficient matrix of the linearized equations for the Newton-Raphson procedure was symmetrized and the ICCG method was enough to solve it. The symmetrization was carried out by \(\frac{\partial H_i}{\partial B_j} = (\frac{\partial H_j}{\partial B_i} + \frac{\partial H_i}{\partial B_j})/2\) for \(i, j = x, y\). The average numbers of the Newton-Raphson iterations with allowable relative residual of \(10^{-3}\) for the computation of 20 coil currents were 5.2 by the ICCG method and 4.2 by the ILU(BiCG)Stab.
method without the symmetrization.

In order to validate the computation, the rolling direction was changed as shown in Fig. 6. When the figures are rotated so that their rolling directions coincide with each other, their distributions show fairly good agreement.

When the conventional method is applied, in which the \( B - H \) curves in the rolling and transverse directions measured independently are taken into account (3), the flux distribution is completely different as shown in Fig. 7, which corresponds to Fig. 6(a) by the proposed method. The flux concentrates near the region at 45 deg and 135 deg because of the reason shown in the reference (3).

4.2 Grain-Oriented Steel Sheet

Figure 8 shows the 2-D property of grain-oriented electrical sheet of 35G165. The material has high permeability in the rolling direction. Figure 9 shows the magnetic flux distributions. Abrupt change of the direction of the magnetic flux density is observed and the flux concentrates near region at 90 deg. Nearly all flux flows through the inner side at 90 deg at low excitation as shown in Fig. 10.

In the computation, the symmetric solver using the ICCG method has failed to converge because of highly asymmetry of the derivative matrix. Then the asymmetric solver such as the ILU(BiCGStab) method was adopted to get convergence. The average number of iterations for 20 coil currents was 6.6. The convergence was difficult in some other geometry due to the flatness of \( \theta_H \) near 90 deg. When \( \theta_H \) is so flat, a small angle variation of \( H \) causes large angle variation of \( B \).
Fig. 6. Distributions of magnitude of magnetic flux density at 30 AT with different rolling directions

Fig. 7. Distribution of magnitude of magnetic flux density at 30 AT in case of non-oriented steel sheet (35A300) calculated by the conventional method

Fig. 8. 2-D magnetic property of grain-oriented material (35G165)

Fig. 9. Distributions of magnetic flux density in case of grain-oriented steel sheet (35G165)
Fig. 10. Magnetic fluxes passing through inner half of the ring core at 0 deg and 90 deg, and total flux in case of grain-oriented steel sheet (35G165)

Fig. 11. 2-D magnetic property of doubly-oriented steel sheet (Cube)

4.3 Doubly-Oriented Steel Sheet A doubly-oriented electrical steel sheet was investigated. Figure 11 shows its measured 2-D property. The material was made by Sumitomo Metal Industries, Ltd., which is called the “Cube” in this paper. The measured data of Fig. 11(b) was considerably irregular and was required to be smoothed simply by shifting the data values so that the irregularity could be removed. The material has two easy magnetization axes at 0 deg and 90 deg. The resultant magnetic flux density concentrates near regions at 0 deg and 90 deg as shown in Fig. 12. In the computation, the asymmetric solver with the ILUBiCGStab method was required to get convergence. The average number of the iterations was 5.9.

5. Conclusions

The method of analysis for the nonlinear anisotropic material by using the FEM with iterative linear solvers was proposed. The magnetic flux distributions for non-oriented, grain-oriented and doubly-oriented electrical steel sheets were computed in a ring core model. Each material showed a characteristic magnetic flux distribution. In the computation, the measured 2-D property was used directly, although some data must be smoothed to remove the irregularity in the measurement. In the case, the data values are shifted a little. In case of weakly anisotropic material (35A300), the ICCG method could get convergence by symmetrizing the coefficient matrix for the Newton-Raphson procedure. For strongly anisotropic materials (35G165 and Cube), the asymmetric solver such as the ILUBiCGStab method was required.

In conclusion, although the effect of the anisotropy is too exaggerated in the low reluctance model of the ring core without gap, it was shown that the anisotropy affects on the magnetic flux distribution and they could be unexpected by the conventional anisotropic field calculation.

The nonlinear anisotropic and asymmetric magnetic property has solved successfully by the presented method. However, the asymmetric derivative matrix $\partial H / \partial B$ is questionable and not physical as the static magnetic property without hysteresis, because this results in energy generation for some B-H history. Neglecting the hysteresis in measurements possibly caused the asymmetric property independent on the field history. The asymmetric property and the hysteresis cannot be separated at present and will be a future issue in measurement and field analysis. The proposed method is a practical and effective one to process measured data in the magnetic field analysis.

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References


Akihis Kameari (Member) was born in Osaka Prefecture, Japan, on August 1 in 1950. He received the B.S. degree in physics from Kyoto University, Japan, in 1973. From 1973 to 1996, he was with Mitsubishi Atomic Power Industries, Inc. and Mitsubishi Heavy Industries, Ltd. Since 1996, he has been with Science Solutions International Laboratory, Inc. His major fields of interest are the research and development of numerical methods for electromagnetic analysis. He is a member of the IEEE.

Koji Fujiwara (Member) was born in Hiroshima Prefecture, Japan, on 26 January in 1960. He received the B.S. and M.S. degrees in electrical engineering from Okayama University, and the D.E. degree from Waseda University, Japan, in 1982, 1984 and 1993, respectively. From 1985 to 1986, he was with Mitsui Engineering and Shipbuilding Co., Ltd. Since 1994, he has been an Associate Professor at the Department of Electrical and Electronic Engineering, Okayama University. His major fields of interest are the development of the 3-D finite element method for nonlinear magnetic field analysis including eddy currents and its application to electrical machines, and the development of standard methods of measurement of magnetic properties of magnetic materials.