Paper

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Calculation of Grounding Resistance of a Combination Electrode

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Summary

The grounding resistance of a combination electrode composed of a horizontal linear wire or a mesh and vertical rods, can be calculated by the current simulation method using average potential on contour surface. In the procedure, the combination electrode is divided into n sections and a linear current source is located in each divided section. Then simultaneous equations where the current sources are unknowns, are obtained. Solution of the equations gives the currents injected from the electrode's surface, so the grounding resistance can be calculated. In the method, as n increases, the error of the calculated grounding resistance decreases, but computing time and memory capacity of computer increase. Thus, for effective calculation, the choice of n is very important, so the relation of n to the error is investigated concerning various types of the electrode and various configurations of the earth. For an actual example of a combination electrode composed of a mesh and two vertical rods, the calculated grounding resistance well agreed with the measured one.

1. Introduction

When the grounding resistance of a horizontal linear wire electrode alone or a mesh electrode alone does not satisfy a required value, a combination electrode composed of a mesh or a horizontal linear wire and vertical rods may be adopted. It is well known that the addition of vertical rods to a horizontal linear wire or a mesh is very effective for decreasing the grounding resistance especially when the resistivity of the upper layer of the earth is high and that of the lower layer is low. However, calculation of the grounding resistance of the combination electrode is not well known. Only a simple method of calculation in which a combination factor is employed has been proposed⁽¹⁾⁽⁵⁾. For practical use of this method, the combination factor needs to be previously known for various electrode's types and dimensions, and various

configurations of the earth. For a combination electrode of two or three vertical rods in homogeneous earth, tables of the factor were published⁽⁶⁾, but for a combination electrode of a horizontal wire and vertical rods, or that of a mesh and vertical rods, a table of the combination factor has not been published yet.

By the way, it was reported that the grounding resistance of a mesh electrode alone could be calculated by the current simulation method using average potential on contour surface⁽²⁾. This method can be easily applied to the combination electrode, as described below. In the method, the electrode is divided into n sections. And, as n increases, the error of the calculated resistance decreases, but computing time and memory capacity of computer increase. Thus the choice of n is very important in the application of the method and discussed here.

By the proposed method, if an electrode is divided into sections in the same manner as described below, the grounding resistance of the electrode could be computed very precisely, with about 1% error. In the design of the electrode, such a good accuracy of the calculation may not be expected, because the other influential factors such as the estimated resistivity and

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configuration of the earth are with bad accuracies at present. However, the proposed method improves the accuracy of the design to some degree and will be employed effectively in the analysis of the other influential factors.

In this paper, first the method of calculation is described briefly. Next, number of sections, n , is choosed reasonably from numerical calculations. Furthermore it is described that the method was applied to an actual combination electrode and the calculated resistance well agreed with the measured one. Last the combination factor employed in the simple method^{(1),(5)} is discussed.

2. Method of Calculation

The method of calculation by the current simulation method using average potential on contour surface⁽²⁾ will be described briefly.

A ground electrode injects a total current I from the electrode's surface into the earth. For calculation, the current injected from the electrode's surface is assumed to be injected from current sources in the electrode. By the way, the injected current densities are not equal along the electrode's surface. For representing this unequality, the current sources need to be of different amplitudes. Thus, in the procedure, (i) the electrode is divided into n sections, that is, n cylinders of length $l_j (j=1, 2, \dots, n)$. And, (ii) in each cylinder, a linear current source J_j of length l_j is located coincidently with the axis of the cylinder of length l_j . In the following descriptions, it is noted that the number of divided sections of the electrode, n , denotes the number of current sources in the electrode too.

The space surrounding the electrode consists of two or more media: in the case of homogeneous earth, two media (air and earth); in the case of two-layer earth, three media (air and two layers of earth); and so on. It is assumed that the boundaries between different layers of the earth are parallel to the earth's surface. Thicknesses of the layers of the earth are $h_1, h_2, \dots$, and resistivities of them are $\rho_1, \rho_2, \dots$, respectively from the top layer to the bottom layer. In this situation, the method of images can be applied. Then, in the procedure, (iii) images of each current source J_j are generated. The details of the generation of the images in the case of homogeneous earth are described in reference⁽²⁾ and those in the case of multi-layer earth are described in references^{(3),(4)}. It is clear from the superposition theorem that the images of a horizontal current source of length l_j are horizontal and of length l_j , and the images of a vertical current source of length l_j are vertical and length l_j .

Thus, (iv) the following simultaneous equations are obtained.

$$\begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ \vdots & \vdots & & \vdots \\ p_{i1} & p_{i2} & \dots & p_{in} \\ \vdots & \vdots & & \vdots \\ p_{n1} & p_{n2} & \dots & p_{nn} \end{bmatrix} \begin{bmatrix} J_1 \\ \vdots \\ J_i \\ \vdots \\ J_n \end{bmatrix} = \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} \quad (1)$$

In Eq. (1), V_i is the potential of the section i , and $V_1 = V_2 = \dots = V$. The coefficient p_{ij} is the sum of average potentials due to the current source J_j and its images, on the contour surface of the section i . When a current source or an image, J , is assumed to be coincident with the x -axis with its center at the origin, the average potential⁽⁶⁾ due to J is calculated as follows. If the section i is parallel to the x -axis and laid between a point (x_1, y, z) and a point (x_2, y, z) , it is calculated by⁽²⁾

$$\begin{aligned} V = & \frac{\rho J}{4\pi l} \frac{1}{x_2 - x_1} \left[\left(x + \frac{l}{2} \right) \log \left\{ x + \frac{l}{2} \right. \right. \\ & + \sqrt{\left(x + \frac{l}{2} \right)^2 + y^2 + z^2} \left. \right\} - \left(x - \frac{l}{2} \right) \\ & \times \log \left\{ x - \frac{l}{2} + \sqrt{\left(x - \frac{l}{2} \right)^2 + y^2 + z^2} \right\} \\ & - \sqrt{\left(x + \frac{l}{2} \right)^2 + y^2 + z^2} \\ & \left. + \sqrt{\left(x - \frac{l}{2} \right)^2 + y^2 + z^2} \right]_{x=x_1}^{x=x_2} \quad (2) \end{aligned}$$

where l is the length of the current source or the image and ρ is the resistivity of the layer where the section i is laid.

If the section i is parallel to the y -axis and laid between a point (x, y_1, z) and a point (x, y_2, z) , it is calculated by⁽²⁾

$$\begin{aligned} V = & \frac{\rho J}{4\pi l} \frac{1}{y_2 - y_1} \left[y \log \frac{x + l/2 + \sqrt{a^2 + y^2}}{x - l/2 + \sqrt{b^2 + y^2}} \right. \\ & + \left(x + \frac{l}{2} \right) \log \left\{ \frac{y}{a} + \sqrt{1 + \left(\frac{y}{a} \right)^2} \right\} \\ & - \left(x - \frac{l}{2} \right) \log \left\{ \frac{y}{b} + \sqrt{1 + \left(\frac{y}{b} \right)^2} \right\} \\ & + 2z \arctan \left\{ \frac{a - (x + l/2)}{z} \right\} \\ & \times \left(\sqrt{1 + \left(\frac{a}{y} \right)^2} - \frac{a}{y} \right) \\ & - 2z \arctan \left\{ \frac{b - (x - l/2)}{z} \right\} \\ & \left. \times \left(\sqrt{1 + \left(\frac{b}{y} \right)^2} - \frac{b}{y} \right) \right]_{y=y_1}^{y=y_2} \quad (3) \end{aligned}$$

where

$$\begin{aligned} a = & \sqrt{(x + l/2)^2 + z^2} \\ b = & \sqrt{(x - l/2)^2 + z^2} \end{aligned} \quad (4)$$

If the section i is parallel to the z -axis and laid

between a point (x, y, z_1) and a point (x, y, z_2) , it is calculated by Eqs. (3) and (4) where y and z are replaced one another, and, y_1 and y_2 are replaced with z_1 and z_2 respectively.

Finally in the procedure, (v) Eq. (1) is solved concerning J_j , and then grounding resistance R is given by

$$R = \frac{V}{I} = V / \sum_{j=1}^n J_j \quad (5)$$

As described in introduction, the choice of n is very important in the application of the method, and will be investigated in the following section.

3. Choice of n

As preliminaries of consideration of a combination electrode, a vertical rod electrode and a horizontal linear wire electrode will be considered.

3.1 Vertical Rod Electrode

(1) In homogeneous earth A rod electrode of radius r_R and length L_R is buried vertically at depth D in the homogeneous earth as shown in Fig. 1(a). The calculated grounding resistance and its error is shown in Table 1, where the rod electrode is divided into n sections of the same length L_R/n and the error is estimated for the calculated resistance when $n=200$. In all the following descriptions, the errors are estimated for the calculated resistances when $n=200$. It is found in Table 1 that as n increases, the error decreases. And the error is less than about 1% even if $n=1$, that is, even if the electrode is not divided at all.

(2) In two-layer earth The above rod electrode is buried vertically in two-layer earth. It is assumed that the electrode extends to the bottom layer, that is, $0 < h_1 - D < L_R$. In this situation, the density of the current injected from the electrode's surface has a discontinuity at the boundary between the two layers of the earth, as shown in Fig. 2. For

representing this discontinuity, the electrode needs to be divided at the boundary. That is, it needs that $n \geq 2$. Dividing the electrode at the boundary needs also for the method of images.

Table 2 shows the error of the calculated resistance. When n is small, the error in case of (a) is greater than that in case of (b). This difference between the error in the two cases is caused by the difference between the injected current densities on the electrode's surface in the two cases. Figure 2 shows the injected current density distribution on the electrode's surface in the two cases. The density increases near the top edge ($z \approx 0$) and near the bottom edge ($z \approx L_R$) in each case. However, the density in case of (a) extremely increases near the bottom edge. Then, when n is small, it is caused mainly from this extremely increasing density that the error in case of (a) is greater than that in case of (b). Thus it is found, from other numerical examples as well as the

Table 1. Error of calculated grounding resistance of a vertical rod electrode in homogeneous earth.

n	$L_R=20$ m		$L_R=50$ m		$L_R=200$ m	
	Calculated resistance (Ω)	Error (%)	Calculated resistance (Ω)	Error (%)	Calculated resistance (Ω)	Error (%)
1	9.407	1.1	4.385	0.7	1.324	0.4
2	9.397	1.0	4.380	0.6	1.323	0.3
5	9.367	0.7	4.371	0.4	1.321	0.2
10	9.350	0.5	4.366	0.3	1.320	0.1
20	9.336	0.4	4.363	0.2	1.320	0.1
50	9.321	0.2	4.359	0.1	1.319	0.0
100	9.311	0.1	4.357	0.0	1.319	0.0
200	9.302	—	4.355	—	1.319	—

$\rho=200\Omega\cdot\text{m}$, $r_R=0.07$ m, $D=1$ m

Fig. 1. Electrodes.

(a) $\rho_1/\rho_2=1600/200$, (b) $\rho_1/\rho_2=200/1600$.
 $r_R=0.07$ m, $L_R=40$ m, $D=1$ m, $h_1=21$ m

Fig. 2. Distribution of injected current density on the electrode's surface of a vertical rod electrode in two-layer earth.

Table 2. Error of calculated grounding resistance of a vertical rod electrode in two-layer earth.

n	(a)		(b)	
	$\rho_1 = 1600\Omega \cdot \text{m}, \rho_2 = 200\Omega \cdot \text{m}$		$\rho_1 = 200\Omega \cdot \text{m}, \rho_2 = 1600\Omega \cdot \text{m}$	
	Calculated resistance (Ω)	Error (%)	Calculated resistance (Ω)	Error (%)
2	8.6961	0.9	11.214	0.4
4	8.6745	0.7	11.206	0.3
10	8.6529	0.4	11.195	0.2
20	8.6412	0.3	11.188	0.2
50	8.6303	0.1	11.180	0.1
100	8.6239	0.1	11.175	0.0
200	8.6182	—	11.170	—

$r_R = 0.07 \text{ m}, L_R = 40 \text{ m}, D = 1 \text{ m}, h_1 = 21 \text{ m}$

above example, that the error when $\rho_1 > \rho_2$ is greater than that when $\rho_1 < \rho_2$. So, next the error when $\rho_1 > \rho_2$ will be investigated in detail.

Figure 3 shows the error when $\rho_1 > \rho_2$ and $n=2$. l_2 is the length laid in the second layer of earth. It is clear that, as l_2/L_R decreases, the error increases, and the error may be greater than 2% if l_2/L_R is very small and $\rho_1 \gg \rho_2$. However, in actual cases, if $\rho_1 \gg \rho_2$, the electrode is designed to be $l_2/L_R > 0.1$. The reason is that, if $\rho_1 \gg \rho_2$, the grounding resistance decreases approximately in proportion to l_2 and such electrode as $l_2/L_R < 0.1$ has a high resistance with high cost. Thus, in actual cases where $l_2/L_R > 0.1$, and when $n=2$, the error is less than 1.7% (see Fig. 3).

In addition to Fig. 3, if the divided section of length l_2 is further divided into two sections of the same length, namely $n=3$, the error is reduced to 1.2% when $\rho_1/\rho_2=50$.

Thus, if the rod is divided at the boundary between the two layers and the divided lower section is further divided into two sections of the same length, namely $n=3$, the error is less than about 1%.

Fig. 3. Error of calculated grounding resistance of a vertical rod electrode in two-layer earth when $n=2$.

(3) In three-layer earth The above electrode is buried vertically in three-layer earth and extends to the bottom layer. In this situation, the injected current density on the electrode's surface has two discontinuities respectively at two boundaries between three layers of the earth. For representing the discontinuities and for the method of images, the electrode needs to be divided at the two boundaries; it needs that $n \geq 3$.

It is found from numerical calculations (not shown here), that the injected current density increases near the top and bottom electrode's edges, and extremely increases near the bottom edge when $\rho_2 > \rho_3$. And the error when $\rho_2 > \rho_3$ is greater than that when $\rho_2 < \rho_3$. The error when $\rho_2 > \rho_3$ and $n=3$ in the three-layer case is nearly equal to the error when $\rho_1 > \rho_2$ and $n=2$ in the two-layer case.

Thus, if the rod is divided at the two boundaries of the earth, into three sections, and the divided bottom section is further divided into two sections of the same length, namely $n=4$, the error is less than about 1%.

3.2 Horizontal Linear Wire Electrode

(1) In homogeneous earth A linear wire of radius r_w and length L_w is buried horizontally at depth D in the homogeneous earth, as shown in Fig. 1 (b). In this situation, it is found from numerical calculations that, even if $n=1$, the error is less than 1%. For examples, when $D=1 \text{ m}$ and $r_w=0.0056 \text{ m}$, the error is 0.5% when $L_w=200 \text{ m}$, 0.6% when $L_w=50 \text{ m}$, and 0.7% when $L_w=20 \text{ m}$.

(2) In two-layer earth The above electrode is buried in the first layer of two-layer earth. Now the path of the current injected from the electrode's surface to infinity is considered. If $\rho_1 < \rho_2$, the current flows more easily in the first layer than in the second layer, and if $\rho_1 > \rho_2$, the current flows more easily in the second layer than in the first layer. Naturally the current flows most easily in the electrode in each case. Thus the path of the injected current from the electrode's surface is supposed to be as shown in Fig. 4. Accordingly, if $\rho_1 < \rho_2$, the injected current density on the electrode's surface is much greater near the electrode's both edges than at the electrode's center, and if $\rho_1 > \rho_2$, that is nearly equal along the electrode. An example is shown in Fig. 5.

It is clear from the above considerations that, when

Fig. 4. Path of the injected current from the electrode's surface of a horizontal linear wire electrode in two-layer earth.

(a) $\rho_1/\rho_2=4000/200$, (b) $\rho_1/\rho_2=200/4000$, (c) $\rho_1=\rho_2$, $r_w=0.0056$ m, $L_w=200$ m, $D=1$ m, $h_1=3$ m

Fig. 5. Distribution of injected current density on the electrode's surface of a horizontal linear wire electrode in two-layer earth.

Fig. 6. Error of calculated grounding resistance of a horizontal linear wire electrode in two-layer earth.

n is small, the error when $\rho_1 < \rho_2$ is greater than that when $\rho_1 > \rho_2$. The error when $\rho_1 < \rho_2$ and the electrode is divided into n sections of the same length L_w/n , is shown in Fig. 6. It is found that, as ρ_2/ρ_1 increases and as h_1 decreases, the error increases. And it is also found that, when $n=6$, the error is less than 1.1%.

(3) In three-layer earth The above electrode is buried in the first layer of three-layer earth. The error in this situation is nearly equal to the error in the case of two-layer earth.

3.3 A Combination Electrode of a Horizontal Linear Wire and Vertical Rods

(1) In homogeneous earth A linear wire of radius r_w and length L_w is buried horizontally at depth D and a rod of radius r_R and length L_R is buried

(a) Distance from wire's center
(b) Distance from rod's top

(a) On the surface of the horizontal linear wire.
(b) On the surface of the vertical rod.

$r_w=0.0056$ m, $L_w=50$ m, $r_R=0.07$ m, $L_R=50$ m, $D=1$ m

Fig. 7. Distribution of injected current density.

perpendicularly so that the top of the rod touches the wire. This system is called here a combination electrode of a horizontal linear wire and a vertical rod. For calculating the grounding resistance of the combination electrode in the homogeneous earth, the wire is divided into n_w sections of the same length L_w/n_w and the rod is divided into n_R sections of the same length L_R/n_R .

For reference to choose n_w and n_R , an example of the density of the current injected from the wire's surface and the rod's surface, when the rod is buried to touch the wire at the wire's center, is shown in Fig. 7. It is noted that the abscissa indicates, in (a), the distance from the center toward one end of the wire, and in (b), the distance from the top toward the bottom of the rod respectively. It is found that the injected current densities vary widely along both the wire and the rod. So, if $n_w=1$ and $n_R=1$, the error is supposed to be much greater than the error when the wire is alone and $n=1$, or the error when the rod is alone and $n=1$. The error when $n_w=1$ and $n_R=1$ is shown in Table 3. Errors in some cases are greater than 2%.

For more precise calculation, the electrode needs to be divided into more sections. $n_w=6$ and $n_R=3$ are recommended. Concerning the wire, each half of the wire is divided into three sections of the same length. The divided three sections roughly correspond to the section of little injected current density, the section of middle injected current density and the section of much injected current density respectively. Concerning the rod, it is divided into three sections of the same length. The divided three sections roughly correspond

Table 3. Error of calculated grounding resistance of a combination electrode of a horizontal linear wire and a vertical rod in homogeneous earth (when the rod is buried to touch the wire at the wire's center).

Wire L_w (m)	Rod L_R (m)	$n_w=100, n_R=100$	$n_w=1, n_R=1$		$n_w=6, n_R=3$	
		Calculated resistance (Ω)	Calculated resistance (Ω)	Error (%)	Calculated resistance (Ω)	Error (%)
50	50	3.3409	3.4172	2.3	3.3585	0.5
	100	2.1101	2.1453	1.7	2.1195	0.4
	200	1.2370	1.2506	1.1	1.2414	0.3
100	50	2.6098	2.6728	2.4	2.6236	0.5
	100	1.8297	1.8677	2.1	1.8384	0.5
	200	1.1484	1.1658	1.5	1.1531	0.4
200	50	1.7973	1.8347	2.1	1.8062	0.5
	100	1.4231	1.4539	2.2	1.4300	0.5
	200	0.9926	1.0112	1.9	0.9970	0.4

$$\rho = 200 \Omega \cdot \text{m}, D = 1 \text{ m}, r_w = 0.0056 \text{ m}, r_R = 0.07 \text{ m}$$

Table 4. Error of calculated grounding resistance of a combination electrode of a horizontal linear wire and a vertical rod in two-layer earth (when the rod is buried to touch the wire at the wire's center).

Wire L_w (m)	Rod L_R (m)	$\rho_1 (\Omega \cdot \text{m})$	$\rho_2 (\Omega \cdot \text{m})$	$n_w=100, n_R=100$	$n_w=6, n_R=3$	
				Calculated resistance (Ω)	Calculated resistance (Ω)	Error (%)
50	50	1600	200	4.2935	4.3164	0.5
	"	200	1600	13.861	13.952	0.7
	200	1600	200	1.3176	1.3208	0.2
	"	200	1600	7.7437	7.7989	0.7
200	50	1600	200	3.4463	3.4647	0.5
	"	200	1600	6.6690	6.7205	0.8
	200	1600	200	1.2465	1.2500	0.3
	"	200	1600	5.1785	5.2262	0.9

$$D = 1 \text{ m}, h_1 = 3 \text{ m}, r_w = 0.0056 \text{ m}, r_R = 0.07 \text{ m}$$

to the sections of little, middle and much injected current densities respectively. The error when $n_w=6$ and $n_R=3$ is shown in Table 3. The error is less than 0.5%. It is noted that, for improvement of the accuracy of the calculation, only a little more labor, a little more computing time and a little more memory capacity of computer are required.

When the rod is buried to touch the wire at any place (except the center) of the wire, the wire and the rod are divided in the same manner as described above ($n_w=6$ and $n_R=3$). Then the error is a little less than the error in Table 3. When two or more rods are buried to touch the wire, the wire and each rod are divided respectively in the same manners as described above ($n_w=6$ and $n_R=3$ for each rod). Then the error is scarcely related to number of rods, and a little less than the error in Table 3.

Thus, if the electrode is divided so that $n_w=6$ for the wire and $n_R=3$ for each rod, a good result is obtained in short computing time and little memory capacity of computer.

When the combination electrode is buried in two or three-layer earth, the electrode is also divided so that $n_w=6$ for the wire and $n_R=3$ for each rod. The wire is divided into six sections of the same length, but each rod is divided into three sections of different lengths as

described below.

(2) In two-layer earth The combination electrode is buried in two-layer earth so that the wire is in the first layer and the rod extends to the bottom layer. In this situation, first the rod is divided into two sections at the boundary between the two layers of the earth, and next the divided lower section is divided into two sections of the same length. An example of the error when $n_w=6$ and $n_R=3$ is shown in Table 4. Errors are less than 1%.

(3) In three-layer earth The combination electrode is buried in three-layer earth so that the wire is in the first layer and the rod extends to the bottom layer. In this situation, it is recommended that the rod is divided into three sections at two boundaries between the three layers; $n_w=6$ for the wire and $n_R=3$ for each rod. It is disappointed that errors when $n_w=6$ and $n_R=3$ could not be estimated. The reason is that the grounding resistance when $n=200$ ($n_w=100$ and $n_R=100$) could not be calculated because of limitations of computer.

3.4 A Combination Electrode of a Mesh and Vertical Rods

(1) In homogeneous earth A mesh, which is composed of wires of radius r_M and length $L_X(L_Y)$ as shown in Fig. 1 (c), is buried horizontally at depth D ,

(a) On the surface of a wire of the mesh's borders.
(b) On the surface of the rod.
 $r_M=0.0056$ m, $L_x=L_y=40$ m, $l_x=l_y=10$ m,
 $D=1$ m, $r_R=0.07$ m, $L_R=50$ m

Fig. 8. Distribution of injected current density.

and a vertical rod of radius r_R and length L_R is buried perpendicularly so that the top of the rod touches the mesh. This system is called here a combination electrode of a mesh and a rod. In this situation, the injected current density on the electrode's surface varies along each wire of the mesh as well as the rod, for example, as shown in Fig. 8. For representing this current density variation, the rod and each wire of the mesh will need to be divided into some sections respectively. However, in general, as the mesh is composed of many wires of length $L_x(L_y)$, dividing each wire of the mesh into sections causes that number of the divided sections increases extremely, so computing time and memory capacity increase extremely. Then, for effective calculation, it is recommended that each wire of the mesh is not divided into sections ($n_w=1$, where n_w is number of the divided sections of each wire of the mesh).

On the other hand, in general, as number of the rods in the combination electrode system is one or some, dividing each rod into sections causes that number of the divided sections increases just a little, so computing time and memory capacity increase just a little. Furthermore, the injected current density on the rod's surface varies more than that on the wire's surface of the mesh (see Fig. 8). Thus it is recommended that each rod is divided into three sections of the same length. The divided three sections roughly correspond to the sections of little, middle and much injected current densities respectively. The error when $n_w=1$ and $n_R=3$ is shown in Table 5. The estimation method of the error is detailed in reference⁽²⁾. The errors are equal to or less than 1.2%.

Table 5. Error of calculated grounding resistance of a combination electrode of a mesh and a vertical rod in homogeneous earth (when the rod is buried to touch the mesh at the mesh's center).

Mesh $L_x(m) \times L_y(m)$	Rod $L_R(m)$	$n_w=1, n_R=3$	
		Calculated resistance (Ω)	Error (%)
120×120	50	0.7575	1.1
	100	0.7193	1.1
	200	0.6223	1.2
80×80	50	1.1224	1.0
	100	1.0080	1.1
	200	0.7985	1.2
40×40	50	2.0165	0.8
	100	1.5621	1.0
	200	1.0562	1.2

$\rho=200 \Omega \cdot m$, $D=1$ m, $r_M=0.0056$ m, $l_x=l_y=10$ m, $r_R=0.07$ m

Table 6. Error of calculated grounding resistance of a combination electrode of a mesh and a vertical rod in two-layer earth (when the rod is buried to touch the mesh at the mesh's center).

Mesh $L_x(m) \times L_y(m)$	Rod $L_R(m)$	$\rho_1(\Omega \cdot m)$	$\rho_2(\Omega \cdot m)$	$n_w=1, n_R=3$	
				Calculated resistance (Ω)	Error (%)
80×80	50	1600	200	2.5643	0.9
	"	200	1600	3.9629	1.2
	200	1600	200	1.1546	0.9
	"	200	1600	3.5332	1.4
40×40	50	1600	200	3.9717	0.9
	"	200	1600	5.9312	0.9
	200	1600	200	1.3044	0.8
	"	200	1600	4.7409	1.2

$D=1$ m, $h_1=10$ m, $r_M=0.0056$ m, $l_x=l_y=10$ m, $r_R=0.07$ m

When a vertical rod is buried to touch the mesh at any place (except the mesh's center), or two or more vertical rods are buried to touch the mesh, the mesh and each rod are divided in the same manner as above ($n_w=1$ and $n_R=3$). Then the error is a little less than the error in Table 5.

(2) In two or three-layer earth When the above combination electrode is buried in two-layer earth so that the mesh is in the upper layer and the rod extends to the lower layer, $n_w=1$ and $n_R=3$ are also recommended. Each rod is divided into three sections in the same manner as in section 3.3(2). The error in this case is shown in Table 6.

When the above combination electrode is buried in three-layer earth, $n_w=1$ and $n_R=3$ are recommended too. Each rod is divided into three sections in the same manner as in section 3.3(3). The error in this case could not be estimated because of limitations of computer. However, the following actual example shows that the calculated grounding resistance well agreed with the measured one.

Table 7. Combination factor η for a combination electrode of a horizontal linear wire and a vertical rod in homogeneous earth (when the rod is buried to touch the wire at the wire's center).

$L_w(m)$	$L_R(m)$	$R_w(\Omega)$	$R_R(\Omega)$	$R_c(\Omega)$	η
50	50	7.427	4.355	3.341	1.22
50	200	7.427	1.319	1.237	1.10

$\rho=200 \Omega \cdot m$, $D=1 m$, $r_w=0.0056 m$, $r_R=0.07 m$

Table 8. Combination factor η for a combination electrode of a horizontal linear wire and a vertical rod in two-layer earth (when the rod is buried to touch the wire at the wire's center).

$\rho_1(\Omega \cdot m)$	$\rho_2(\Omega \cdot m)$	$R_w(\Omega)$	$R_R(\Omega)$	$R_c(\Omega)$	η
1600	200	39.18	4.531	4.294	1.06
200	1600	16.91	26.76	13.86	1.34

$D=1 m$, $h_1=3 m$, $r_w=0.0056 m$, $L_w=50 m$, $r_R=0.07 m$, $L_R=50 m$

(3) An actual example In Ojima Substation, Chubu Electric Power Company, a combination electrode of a mesh and two vertical rods was buried in depth $D=1 m$ in four-layer earth. Concerning the mesh, $r_M=0.0056 m$, $L_x=70 m$, $L_y=25 m$, $l_x=70 m/6$ and $l_y=25 m/2$. Concerning each of the two rods, $r_R=0.075 m$ and $L_R=92.5 m$. One rod was at a point ($x=10 m$, $y=25 m$) and the other rod at a point ($x=55 m$, $y=5 m$). The configuration of the earth was measured to be composed of four layers: $\rho_1=300 \Omega \cdot m$, $h_1=2 m$, $\rho_2=900 \Omega \cdot m$, $h_2=3.6 m$, $\rho_3=260 \Omega \cdot m$, $h_3=4.4 m$ and $\rho_4=540 \Omega \cdot m$. For efficient calculation of the grounding resistance, the four-layer earth was replaced with three-layer earth: $\rho_1=300 \Omega \cdot m$, $h_1=2 m$, $\rho_2=468 \Omega \cdot m$, $h_2=8 m$ and $\rho_3=540 \Omega \cdot m$ (the method of the replacement will be reported in the near future). The electrode was divided so that $n_w=1$ for each wire of the mesh and $n_R=3$ for each rod. The calculated resistance was 3.039Ω and the measured one was 3.0Ω at $1 A$. The calculated result well agreed with the measured one.

4. Combination Factor

Grounding resistance of a combination electrode, R_c , has been frequently calculated by⁽¹⁾

$$R_c = \eta \frac{1}{\sum_i \frac{1}{R_i}} \quad (6)$$

where R_i is grounding resistance of each electrode alone, and calculated by an approximate equation. η is called combination factor and $\eta > 1$. For practical use of Eq. (6), η needs to be previously known for various electrodes' types and dimensions, but a table of η has not been published yet.

Examples of η , when a combination electrode of a horizontal linear wire and a vertical rod is buried in homogeneous earth, are shown in Table 7. R_w is

grounding resistance of the wire alone, and R_R is that of the rod alone. As shown in Table 7, even if the wires are of the same length, η s are different dependently of the rods' lengths.

Examples of η , when the above combination electrode is buried in two-layer earth, are shown in Table 8. Even if the same electrode is buried, η s are different dependently of the configurations of the earth.

Thus it is noted that η varies dependently of the type and the dimension of the combination electrode, and the configuration of the earth. Accordingly, it will be laborious to make out a table of η . On the other hand, it is not so laborious to calculate the grounding resistance of the combination electrode directly by the method proposed here.

5. Conclusions

The grounding resistance of a combination electrode composed of a mesh or a horizontal linear wire and vertical rods can be calculated by the current simulation method using average potential on contour surface. In the method, it is recommended from numerical calculations that a combination electrode is divided into sections in the following manner.

(1) For a combination electrode of a horizontal linear wire and vertical rods, the wire is divided into six sections of the same length, and each rod into three sections.

(2) For a combination electrode of a mesh and vertical rods, each wire of the mesh is not divided at all, and each rod is divided into three sections.

Then the grounding resistance can be calculated in error of less than about 1%, short computing time and small memory capacity of computer.

For an actual example of a combination electrode of a mesh and two vertical rods, the calculated grounding resistance well agreed with the measured one.

Concerning a simple method of calculation in which a combination factor is employed, the combination factor varies dependently of the type and dimension of the combination electrode, and also dependently of the configuration of the earth. Accordingly, it will be laborious to make out a table of the combination factor.

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