Electric Field Optimization Using NURB Curve and Surface

Non-member Eung-Sik Kim (Hoseo University)
Non-member Byeong-Yoon Lee (Seoul National University)
Member Jong-Keun Park (Seoul National University)

The objective of this paper is to propose a new method for the optimal design of HV equipments. In this method, a shape function called NURB-spline (Non-Uniform Rational B-spline) is introduced to represent the contour of the electrode or insulator. NURB-spline has much better controllability, locality and continuity than other shape functions. The first two properties can significantly reduce the number of geometric variables and the last is a necessary condition for the smoothness of a field.

There are two distinctive processes in this algorithm. One is the determination of an optimal desired field which should be the smallest and uniformly implemented value. The other is the adaptive addition of a geometric variable to a region where the uniformity fails. Due to the above mentioned features, the entire process guarantees stable convergence. IEM (Integral Equation Method) is used for field calculations. Three examples are chosen from the conventional researches and simulated to verify the advantages of the proposed method.

Key words: NURB-spline, Optimization, IEM

1. Introduction

So far, various kinds of algorithms for electric field optimization have been proposed(1)-(3). Each of them has its own characteristics, but most of them usually have two weak points in common. One is that a large number of variables are required. It is not suitable for the iterative field optimization that numerous variables have to be used. The other is the discontinuous curvature of a contour, which directly affects the uniformity of the field distribution. So an extra algorithm may be required to smooth the optimized contour(4). The reason why these problems occur can be understood from the fact that variables are nodes on the curve or surface and a suitable interpolation is required.

This paper is motivated from the recognition of these problems. To overcome them, a function called NURB-spline is introduced(5)(6). It has very useful properties such as controllability, locality and continuity. These properties greatly contribute to solve these problems and make NURB-spline suitable for the design of HV equipments.

In the conventional methods, there has not been any comment about how to decide the desired field value. Furthermore, the desired field determined has been used without verifying the optimality, in other words, whether the desired field value is the smallest and can be uniformly implemented. In this paper we suggest how the optimal desired field should be specified.

2. NURB-spline as a contour function(7)-(9)

Fig.1 shows the surface charge models of IEM. These pictures show how many variables should be used for optimization. To make matters worse, the discontinuity or fluctuation of the curvature occurs in most cases and causes a nonuniform field. Such
problems can be solved with NURB-spline. NURB-spline and its properties are explained in the following and for simplicity the illustration is mainly concentrated on the NURB-spline curve.

RB-spline curve is defined as Eq. (1). \( P(u) \) is a parametric curve and consists of \( x \) and \( y \) components.

\[
P(u) = \frac{\sum_{i=0}^{n} h_i C[i] N_{i,n}(u)}{\sum_{i=0}^{n} h_i N_{i,n}(u)} \quad (0 \leq u < n-k+2)
\]

where \( h_i \) are weighting, \( C[i] \) called control point are 2-dimensional vectors and \( N_{i,n}(u) \) are blending functions. They are defined recursively by the following expressions,

\[
N_{i,1}(u) = \begin{cases} 
1, & t_i \leq u < t_{i+1} \\
0, & \text{otherwise}
\end{cases}
\]

\[
N_{i,n}(u) = \frac{(u-t_i) N_{i,n-1}(u)}{t_{i+k-1}-t_i} + \frac{(t_{i+k}-u) N_{i+1,n-1}(u)}{t_{i+k}-t_{i+1}}
\]

\[
t_i = \begin{cases} 
0, & \text{if } i < k \\
i-k+1, & \text{if } k \leq i \leq n \\
n-k+2, & \text{if } i > n
\end{cases}
\]

where \( k \) controls the degree \((k-1)\) of the resulting polynomial in \( u \) and thus also controls the continuity of the curve. The \( t_i \) are called knot values. If they are distributed nonuniformly, Eq. (1) is called a NURB-spline curve. Blending functions for \( n=5, k=3 \) are shown in Fig. 2.

As can be seen, each blending function has dominant values in a particular interval of \( u \) and depends on a specified control point. So each control point has a local influence. Interpolation and approximation are two groups of curve-defining techniques. The difference between them is whether the curve passes through the nodes or not. Most curves which belong to the former do not provide for the local controllability. Consequently, local changes of shape tend to be strongly propagated throughout the entire curve. This means that most of curve-defining variables keep staying as unknown variables to the end of optimization.

Fig. 3 (a) shows an example. Being an approximation curve, the NURB-spline curve only approaches the characteristic polygon which consists of the given control points. Fig. 3 (b) illustrates that the curve can be controlled not by the nodes on it but by the control points outside. Furthermore, weighting is also used as variable to change the curve with a characteristic polygon fixed. Fig. 3 (c) shows the change of the curve as the weighting varies. The larger the weighting is, the closer the approximation is. Fig. 3 (b) and (c) show the locality and controllability of NURB curve. Due to the above-mentioned properties the entire contour can be controlled effectively with a small number of control points.

Another useful property is that the same NURB curve can be represented using a different number of control points. This process is called knot insertion. With this process, a new control point can be added...
to the desired region where an accurate control of the contour is needed. For instance, Fig. 3(a) and (d) represent same curves. But Fig. 3(d) has one more control point by knot insertion.

3. Field optimization

3.1 Optimization technique

IEM is used for field calculations. The numerical formulation is similar to Misaki's method. For the electrode model, the system equation is given as Eq. (4). The problem is to find the optimal position of geometric variable vector $x$ under specified field condition $(E=\sigma/e)$. $X$ is $n$-dimensional vector whose elements are selected from the components of each control point. The objective function $\|F\|$ in Eq. (5) is taken as the sum of square errors.

$$[A(X)]\sigma=[\Phi] \quad \text{......................................(4)}$$

$A$: potential coefficient matrix
$X$: geometric variable determining the shape
$\{x_1, x_2, \ldots, x_{n-1}, x_n\}^T$
$\sigma$: charge density vector
$\Phi$: applied voltage vector
$\|F\|=[\sigma-A^{-1}\Phi]^T[\sigma-A^{-1}\Phi]$ \quad \text{......................................(5)}$

$\|F\|$ : objective function
$\sigma$: desired charge density vector

The Gauss method is used to find $X$ which gives minimum value of $\|F\|$. The following is the algorithm of the Gauss method.

(1) $\Delta X = -(J^TJ)^{-1}J^T(F_k)$

$$J = \left[ \frac{\delta F}{\delta X} \right]$$

(2) $J_{ii} = \frac{\delta F_i}{\delta x_i} = A_{i1}\sigma_1 + A_{i2}\sigma_2 + \cdots + A_{i\sigma}\sigma_{\sigma}$

$$= \left. \frac{1}{\delta x_i} \sum_{\sigma=1}^{\sigma} (A_{i\sigma}(x_1+\Delta x_1)-A_{i\sigma}x_1)\sigma \right|_{x=x_i}$$

(3) $X_{k+1} = X_k + \lambda \Delta X$

$F_k$ is computed from the $k$-th step and $X_{k+1}$ is determined by a line search in the direction of $\Delta X$. The Jacobian matrix $J$ is computed with numerical differentiation.

3.2 Optimization algorithm

Fig. 4 is the flowchart. A constant $\delta$ is a given tolerance limit. The details of important blocks are going to be explained. At first the given contour is represented with a NURB-spline curve for a preliminary optimization. The position and number of control points are determined according to the variation of curvature. Since most models draw arcs, 3 control points are enough to represent the contours. But in case a slope is specified at an end of curve, the addition of an auxiliary control point is needed to keep the boundary condition.

With given NURB-spline curve, the preliminary optimization is carried out. Its purpose is only to decide an optimal desired field. The optimal value should be the smallest one which can be uniformly implemented. This procedure is as follows.

(i) Calculate the average field from the field distribution of a given contour.

(ii) Try to optimize the contour under the field level which is less than the average field.

(iii) After convergence of step (ii), calculate a new average field from the converged contour. This value is taken as the optimal desired field.
To verify the optimality, a full-scale optimization process has been tried with a desired field which is 2~3% below the optimal level. In this case, the contour diverged or the uniformity failed even though the contour converged. Forcing the contour not to diverge resulted in a fluctuation of the field. This proves that the desired field value obtained by the preliminary optimization is very close to the real optimum.

Before the real optimization the distribution of field must be checked. If the uniformity fails as a result of the preliminary optimization, another control point is added by knot insertion for an accurate control of contour. Even though this causes the increase in the number of geometric variables, the number of variables for optimization does not increase in consideration of locality. The rest of the procedure is as shown in the flowchart.

### 3.3 Application to optimal design

Fig. 5 (a) shows a rod plane gap electrode model. The objective of design is to obtain a uniform field distribution along the contour. Five control points are used as geometric variables. The auxiliary control point C(3) is used to maintain a vertical slope at C(4) and another auxiliary control point C(1) can move with a horizontal slope fixed at C(0). C(1) and C(2) are used for the preliminary optimization variables. The optimal desired field was 13.2 V/m through this procedure as shown in Fig. 5 (b).

Fig. 5 (b) shows a partial fluctuation of the field. The addition of a control point to the fluctuating region is carried out by knot insertion. C(2) and C(3) are chosen as new optimization variables shown in Fig. 5 (c).

Fig. 6 (a) shows an insulator model supporting the HV electrode. The objective is to obtain a uniform tangential field distribution along the dielectric contour. Five control points are used as geometric variables. C(1) and C(3) can move vertically to maintain the vertical slopes at both ends. C(3) and C(4) are used for the preliminary optimization variable. The optimal desired field becomes 12.6 V/m as shown in Fig. 6 (b).

Fig. 6 (b) show a partial fluctuation of the field. The addition of a control point is carried out. C(4)
and C(5) are chosen as new optimization variables shown in Fig. 6(c).

The preparation for the real optimization has been finished. The results of the real optimization will be discussed in the following section.

4. Results of optimization

Three examples are chosen from the conventional researches. They are the rod plane gap electrode, HV insulator and the shielded electrode model. The first two of them are 2-dimensional models and the last is a 3-dimensional model. The Gaussian curvature is used among others to show its relation to the field.

4.1 Definition of field error and uniformity

In order to estimate the optimized field intensity of the electrode or insulator, field error and uniformity are defined as follows.

\[
\text{field error} = \frac{E_{\text{max or min}} - E_{\text{ref}}}{E_{\text{ref}}} \times 100 \, \% \tag{6}
\]

\[
E_{\text{max or min}}: \text{maximum or minimum field}
\]

\[
E_{\text{ref}}: \text{desired field}
\]

\[
\text{uniformity} = \frac{E_{\text{max}}}{E_{\text{min}}} \tag{7}
\]

4.2 Rod plane gap electrode

Fig. 7(a) shows an optimized contour. The distributions of the field and curvature at 50 check points along the rod are illustrated in Fig. 7(b). Field error and uniformity were 0.05% and 1.0006, respectively.

4.3 HV insulator

Fig. 8(a) shows the optimized shape. The distributions of field and curvature at 100 check points along the dielectric are shown in Fig. 8(b). They are located according to Chebyshev zeros. Field error and uniformity were 1.11% and 1.019, respectively.

4.4 Shielded electrode

Fig. 9(a) shows a shielded electrode model. In this case, the inner sphere electrode is to be optimized with an outer shield electrode fixed. It is represented with a NUBR-spline surface using only 7 control points. Each of the control points on the axes is restricted in its movement along each axis. The optimization is performed under the desired field condition 38.0 V/m. Fig. 9(b) shows the initial and optimized shapes along the direction from...
Fig. 9. Results of optimization.

A to B. Fig. 9 (c) illustrates the distributions of field and curvature along the above-mentioned direction. The maximum field error was 1.43% and uniformity was 1.020.

5. Conclusion

The merits of this paper are summarized as follows.

(1) The continuity of derivatives being satisfied, NURB-spline guarantees the continuity of curvature which is a necessary condition for uniform field distribution. Therefore, higher accuracy of the optimized contour compared with the former researches is achieved.

(2) The outstanding controllability of NURB-spline makes it possible to control the entire contour with a small number of control points. So the number of variables are greatly reduced.

(3) A control point can be easily added to a desired region by knot insertion for more accurate control of a contour. The increase of control points, however, does not result in that of optimization variables due to locality.

(4) The way an optimal desired field should be determined is suggested. This value should be the smallest as well as uniformly implemented. This optimal desired field determined through the preliminary optimization ensures a stable convergence.

The authors appreciate T. Takuma (CREPI) and H. Okubo (Nagoya University) for their careful advices and comments.

(Manuscript received March 26, '93, revised June 1, '93)

References

(9) L. Piegl: “Modifying the shape of rational B-spline. Part II: surfaces”, ibid., 21, No. 9, 538 (1989)

Eung-Sik Kim (Non-member)

He was born in Seoul, Korea. He received the B.S., M.S. and Ph. D. degree in Seoul National University, Korea in 1984, 1986 and 1991, respectively. Since 1991, he has been a Senior Lecture at the Dept. of Industrial Safety Engineering in Hoseo University. His present research interests are numerical field analysis.
Byeong-Yoon Lee (Non-member)

He was born in the Jeonbuk Province, Korea. He received the B.S. and M.S. degree from Seoul National University, Korea in 1990 and 1992, respectively. He is working toward a Ph.D. degree in Seoul National University from 1992. His present research interests are field analysis and optimization of high voltage equipments.

Jong-Keun Park (Member)

He was born in the Chungnam Province, Korea. He received the B.S. degree from Seoul National University, Korea in 1973. He received the M.S. and Ph. D. degree from the University of Tokyo, Japan in 1979 and 1982 respectively. Since 1983, he has been a Professor at Seoul National University, Korea. His present research interests are power system analysis, control and protection, expert system and neural networks.

He is a member of IEEE and IEE of Japan.