Electromechanical Analysis of a Micromachined Comb-Drive Actuator by Admittance Measurement

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Keywords : comb-drive actuator, Electromechanical Analysis, admittance measurement

Introduction

Electromechanical analysis of a micromachined comb-drive actuator based on equivalent circuit method is described. From the equivalent circuit, we formulated electrical admittance for a comb-drive actuator as a function of AC driving frequency and derived some relation between specific frequencies which can be obtained by admittance measurement. We have also found that mechanical parameters of a comb-drive actuator such as spring constant, rigid mass and mechanical resistance could be estimated by the relation of the specific frequencies. Using this theory we evaluated mechanical properties of a fabricated comb-drive actuator and confirmed its practical usefulness for characterization of a comb-drive actuator.

Modeling of Comb Drive Actuator, and Formulation by Equal Circuit Method

Fig. 1 shows a model of a comb-drive actuator to be analyzed, having a gap between opposing comb fingers of d , structure height of b , and initial overlap of comb fingers of x_0 . C_0 and C_{stray} are a total capacitance between opposing comb-drive fingers and a stray capacitance of the electrical system, respectively. The actuator is supposed to be driven by DC bias E_0 superposing small AC power

Fig. 1. Analysis model of a comb-drive actuator

Fig. 2. The equivalent circuit of the comb-drive actuator

e. The analysis is started from introduction of Lagrangian L for the comb-drive actuator as functions of velocity v , displacement x , and charge q . The equivalent circuit can be drawn as Fig. 2 for the comb-drive actuator. As shown in Fig. 2, the parameter A is important factor to couple the electrical part with the mechanical part. If DC bias E_0 is zero, the coupling doesn't occur, while for large E_0 strong coupling is expected.

Frequency Dependence on Admittance of a Comb-drive Actuator

As described the previous section, electrical and mechanical circuits are coupled when DC bias E_0 is not zero. Therefore, it is possible to observe mechanical motion by measuring electrical characteristics. The calculation result obtained from the theoretical formula is shown in Fig. 3. It turned out that the resonant frequency ω_0 of a machine system exists between the frequency ω_1 of zero point, equivalent to peak ω_p of admittance, and found out that it was possible to perform machine characteristics evaluation of a comb-drive actuator.

Conclusion

When admittance was measured the actually created comb-drive actuator in LCR meter (H.P.4192A), as it was shown in Figure 4, the same graph as Figure 3 was obtained. It became clear theoretically from the relational expression of frequency and admittance that parameters, such as the mass m of a comb-drive actuator, mechanical resistance r_m , compliance C_m , and sharpness-of-resonance Q , are computable based on this graph. A very useful thing is shown in the theory of this paper evaluating the mechanical parameter of a comb-drive actuator.

Fig. 3. Frequency dependence on absolute value of admittance

Fig. 4. Measured admittance as a function of ω

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Electromechanical analysis of a micromachined comb-drive actuator based on equivalent circuit method is described. From the equivalent circuit, we formulated electrical admittance for a comb-drive actuator as a function of AC driving frequency and derived some relation between specific frequencies which can be obtained by admittance measurement. We have also found that mechanical parameters of a comb-drive actuator such as spring constant, rigid mass and mechanical resistance could be estimated by the relation of the specific frequencies. Using this theory we evaluated mechanical properties of a fabricated comb-drive actuator and confirmed its practical usefulness for characterization of a comb-drive actuator.

Keywords: comb-drive actuator, admittance measurement

1. Introduction

Comb-drive actuator is widely used in micro electro mechanical systems(MEMS) as a key element of actuation or sensing. From the view point of electromechanical transducers, comb-drive actuators is thought as a machine converting electrical energy into mechanical energy, and vice versa. In the case of electrostatic actuators, an electric field acts as medium coupling electrical power with mechanical power; when a mechanical part is moving with a short distance l in an electric field E , the electromotive force $V(=El)$ will be appeared in the system, while when there are some electrical charges q in an electric field E , the force $F=(qE)$ will act on the system.

The equivalent circuit method⁽¹⁾⁽²⁾, in which both the electrical and mechanical portions of the actuator are represented by electrical equivalents, is a useful method to analyze the behavior of electromechanical transducers. We formulated electrical admittance using the equivalent circuit method of a comb-drive actuator with taking account of stray capacitance of the electrical part and drew the admittance curve as a function of AC driving frequency. The graph shows a clear peak near mechanical resonant frequency if the stray capacitance is relatively small compared with the capacitance between comb-drive figures. By recent MEMS technology, especially progress of deep reactive ion etching (DRIE) technique, we can easily obtain deep structures with small gap, thereby the device structure to observe such a peak have been realized.

In this paper, we present an analysis method of the comb-drive actuator based on the equivalent circuit method, and also show an estimation method of mechanical property including spring constant, moving mass, mechanical resistance by the admittance measurement.

2. Modeling of Comb Drive Actuator, and Formulization by Equal Circuit Method

Fig. 1 shows a model of a comb-drive actuator to be analyzed, having a gap between opposing comb fingers of d , structure height of b , and initial overlap of comb fingers of x_0 . C_0 and C_{stray} are a total capacitance between opposing comb-drive fingers and a stray capacitance of the electrical system, respectively. The actuator is supposed to be driven by DC bias E_0 superposing small AC power e . The analysis is started from introduction of Lagrangian L for the comb-drive actuator as functions of velocity v , displacement x , and charge q .

$$L = \frac{1}{2}mv^2 - \frac{x^2}{2C_m} - \frac{q^2}{2\{C(x) + C_{stray}\}} \dots\dots\dots (1)$$

Where m and C_m are a rigid mass and a compliance of the comb-drive actuators, respectively, and $C(x)$ is a changing capacitance between opposing comb-drive figures.

In this case, $C(x)$ is given by

$$C(x) = \frac{2n\epsilon_0 b(x_0 + x)}{d}, \dots\dots\dots (2)$$

where x is the small displacement of the comb-drive actuator, n is the number of comb-drive fingers, and ϵ_0 is the permittivity in vacuum.

Dissipation function F is also given as functions of v and current i .

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Fig. 1. Analysis model of a comb-drive actuator

$$F = \frac{1}{2}r_mv^2 + \frac{1}{2}Ri^2 \dots \dots \dots (3)$$

Where r_m is mechanical resistance, and R is electrical resistance.

From the Lagrangian and the dissipation function, two Lagrange's equations can be then derived for the comb-drive actuator applying DC bias E_0 and AC driving voltage e :

$$\begin{aligned} f_e &= \frac{d}{dt} \left(\frac{\partial L}{\partial v} \right) - \frac{\partial L}{\partial x} + \frac{\partial F}{\partial v} \\ &= m \frac{dv}{dt} + r_m v + \frac{1}{C_m} \int v dt \\ &\quad - \frac{1}{2} \left(\frac{q}{C(x) + C_{stray}} \right)^2 \frac{dC(x)}{dx} \dots \dots \dots (4) \end{aligned}$$

$$E_0 + e = Ri + \frac{1}{C(x) + C_{stray}} \int i dt \dots \dots \dots (5)$$

where f_e is an external force acting on the comb-drive actuators.

If we suppose the AC driving voltage e is much smaller than E_0 ($E_0 \gg e$) having a single angular frequency ω and f_e is synchronized with e , the equations are rewritten in complex forms in case of $R \approx 0$,

$$f_a = j\omega m v_a + r_m v_a + \frac{v_a}{j\omega C_m} - A e_a \dots \dots \dots (6)$$

$$i_a = j\omega (C_0 + C_{stray}) e_a + A v_a \dots \dots \dots (7)$$

where $C_0 = \frac{2n\epsilon_0 b x_0}{d}$, $A = \frac{C_0 E_0}{x_0}$, and the suffix a denotes maximum amplitude of the each parameters.

Hence the equivalent circuit can be drawn as Fig. 2 for the comb-drive actuator. As shown in Fig. 2, the parameter A is important factor to couple the electrical part with the mechanical part. If DC bias E_0 is zero, the coupling doesn't occur, while for large E_0 strong coupling is expected.

3. Frequency Dependence on Admittance of a Comb-Drive Actuator

As described the previous section, electrical and mechanical circuits are coupled when DC bias E_0 is not zero. Therefore, it is possible to observe mechanical motion by measuring electrical characteristics. By solving

Fig. 2. The equivalent circuit of the comb-drive actuator

Eqs. 6 and 7 supposing $f_a = 0$, we can obtain admittance function as follows:

$$\begin{aligned} Y(\omega) &= \frac{i_a}{e_a} \\ &= j\omega (C_0 + C_{stray}) + \frac{A^2}{r_m + j \left(\omega m - \frac{1}{\omega C_m} \right)} \dots \dots \dots (8) \end{aligned}$$

The absolute value of the equation 8, which can be measured by an impedance analyzer directly, is then given by

$$\begin{aligned} |Y(\omega)| &= \sqrt{\frac{A^2 \left\{ A^2 - 2\omega(C_0 + C_{stray}) \left(\omega m - \frac{1}{\omega C_m} \right) \right\}}{r_m^2 + \left(\omega m - \frac{1}{\omega C_m} \right)^2} + \omega^2 (C_0 + C_{stray})^2} \dots \dots \dots (9) \end{aligned}$$

Fig. 3 shows a schematic representation of the function $|Y(\omega)|$. The admittance curve is on the line of a linear function $Y(\omega) = \omega(C_0 + C_{stray})$ at lower and higher angular frequencies, while shows one peak around the mechanical resonant frequency:

$$\omega_0 \left(= 1/\sqrt{mC_m} \right) \dots \dots \dots (10)$$

Note that the peak angular frequency ω_p is not corresponding to the resonant angular frequency ω_0 which is slightly higher than ω_p since ω_0 will be placed at the prolongation of the peak curve as shown in Fig. 3.

There is also a specific angular frequency ω_1 determined by the following equation:

$$\begin{aligned} A^2 - 2\omega_1(C_0 + C_{stray}) \left(\omega_1 m - \frac{1}{\omega_1 C_m} \right) &= 0 \dots \dots \dots (11) \end{aligned}$$

This is the condition that the first term in the square root of Eq.9 becomes zero, hence ω_1 can be plotted on the linear function $Y(\omega) = \omega(C_0 + C_{stray})$ in Fig. 3.

From Eq. 11, the relation between ω_1 and ω_0 is given by:

$$\omega_1^2 - \omega_0^2 = \frac{A^2}{2m(C_0 + C_{stray})} \dots \dots \dots (12)$$

Since the right side of Eq. 12 always takes a positive value, we can know the relation $\omega_0 < \omega_1$, consequently $\omega_p < \omega_0 < \omega_1$. The strict relation between these angular frequencies will be discussed in the next section.

Fig. 3. Frequency dependence on absolute value of admittance

4. Derivation of Device Parameters from Admittance Curve

In order to obtain the relation between the device parameters and the specific angular frequency, we considered a function of ω^2 which is made by subtracting $\omega^2(C_0 + C_{stray})^2$ from $|Y(\omega)|^2$:

$$f(\omega^2) = |Y(\omega)|^2 - \omega^2(C_0 + C_{stray})^2$$

$$= \frac{A^2 \{ A^2 + \frac{2(C_0 + C_{stray})}{C_m} - 2m(C_0 + C_{stray})\omega^2 \}}{r_m^2 - \frac{2m}{C_m} + m^2\omega^2 + \frac{1}{\omega^2 C_m^2}} \quad \dots \dots \dots (13)$$

Fig. 4 shows a typical variation of the $f(\omega^2)$ as a function of ω^2 . We can easily obtain ω_1^2 as a point at which the function crosses the horizontal axis. As shown in Fig. 4, the graph has one peak and one bottom at the squared angular frequency of ω_p^2 and ω_b^2 , respectively. By solving the equation $\frac{df(\omega^2)}{d\omega^2} = 0$, we can obtain ω_p^2 and ω_b^2 as follows:

$$\omega_p^2 = \frac{\omega_0^4 - \omega_0^2 \sqrt{\omega_0^4 - \omega_1^2 \left(2\omega_0^2 - \omega_1^2 - \frac{r_m^2}{m^2} \right)}}{2\omega_0^2 - \omega_1^2 - \frac{r_m^2}{m^2}} \quad \dots \dots \dots (14)$$

$$\omega_b^2 = \frac{\omega_0^4 + \omega_0^2 \sqrt{\omega_0^4 - \omega_1^2 \left(2\omega_0^2 - \omega_1^2 - \frac{r_m^2}{m^2} \right)}}{2\omega_0^2 - \omega_1^2 - \frac{r_m^2}{m^2}} \quad \dots \dots \dots (15)$$

On the other hand, by making a Taylor expansion of the $f(\omega^2)$ around ω_0^2 and taking a first order approximation, the slope (denoted by a) at ω_0^2 is given by:

$$a = \frac{-2mC_0A^2}{r_m^2} = \frac{-4m^2(C_0 + C_{stray})^2(\omega_1^2 - \omega_0^2)}{r_m^2} \quad \dots \dots \dots (16)$$

This can be rewritten in the following form:

Fig. 4. Schematic diagram of the function $f(\omega^2)$

$$\frac{r_f^2}{m^2} = -\frac{4(C_0 + C_{stray})^2(\omega_1^2 - \omega_0^2)}{a} \quad \dots \dots \dots (17)$$

Adding Eq. 14 to Eq. 15 and successively substituting Eq. 17, we can obtain the expression of ω_0^2 as follows:

$$\omega_0^2 = \frac{1}{4} \left\{ \left(2 - \frac{4(C_0 + C_{stray})^2}{a} \right) (\omega_p^2 + \omega_b^2) - \sqrt{(\omega_p^2 + \omega_b^2) \left\{ \left(2 - \frac{4(C_0 + C_{stray})^2}{a} \right)^2 (\omega_p^2 + \omega_b^2) + 8 \left(\frac{4(C_0 + C_{stray})^2}{a} - 1 \right) \omega_1^2 \right\}} \right\} \quad \dots \dots \dots (18)$$

where the relation $\omega_p < \omega_0 < \omega_1$ is used when driving this equation.

All of variables in the right side of Eq. 18 are measurable from the graph as shown in Fig. 4, consequently ω_0^2 can be calculated. Once we find ω_0^2 , the moving mass m can be estimated using Eq. 12, successively the compliance C_m and the mechanical resistance r_m are calculated by Eqs. 10 and 17, respectively. In this calculation, it is convenient to use a relation $\frac{2}{\omega_1^2} = \frac{1}{\omega_p^2} + \frac{1}{\omega_b^2}$, which reduces one variable from Eq. 18.

5. Analysis of a Comb-Drive Actuator by Admittance Measurement

As described in the previous sections, a comb-drive actuator can be analyzed by admittance measurement showing a peak related to the mechanical resonance. However, if the stray capacitance C_{stray} , which is mainly made between electrodes and a substrate, is much larger than the capacitance of opposing comb-drive fingers C_0 , the peak height become very small. In the worst case, almost electrical current flow through the stray capacitance; as a result, no peak will be observed. Fig. 5 shows numerical admittance curves with the ratio $\frac{C_{stray}}{C_0}$ as parameter. As shown in the Fig. 5, the clear peak will be observed when C_0 and C_{stray} are compatible. Therefore, we designed a comb-drive actuator with very small pad electrode as shown in Fig. 6.

When calculating the parameter A , a value of the gap d is required. Since the gap usually become wider than the designed value due to undesirable under-etching, scan-

Fig. 5. Numerical admittance curves with the ratio $\frac{C_{stray}}{C_0}$ as parameter

Fig. 6. A SEM photograph of the fabricated comb-drive actuator

Fig. 7. Measured admittance as a function of ω . DC bias and AC driving voltage are 17.5V and 0.5 V_{pp}, respectively. Total capacitance is approximately 2.00 pF

ning electron microscope (SEM) observation should be conducted in order to obtain correct value; for our fabricated device, d was measured to be $3.50\mu\text{m}$, then A was calculated to be $5.53 \times 10^{-7}\text{FV/m}$.

Fig. 7 shows the admittance as a function of angular frequency measured by LCR meter (HP4192A) when DC bias and AC driving voltage are 17.5V and 0.5 V_{pp}, respectively. The frequency step was 0.1Hz. By fitting the linear function, it was found that summation of C_0 and C_{stray} was 2.00pF.

Using this capacitance value, a graph of the function $f(\omega^2)$ was drawn as Fig. 8. From Fig. 8, the angular frequencies of $\omega_1, \omega_p, \omega_b$, and slope a were determined to $1.4997 \times 10^4\text{rad/s}$, $1.4951 \times 10^4\text{rad/s}$, $1.5036 \times 10^4\text{rad/s}$,

Fig. 8. The graph of the $f(\omega^2)$ converted from Fig. 7

Table 1. Measured device parameters

f_0 (Hz)	m (kg)	C_m (m/N)	r_f (kg/s)	Q
2383.6	1.23×10^{-7}	3.62×10^{-2}	9.82×10^{-6}	188

and $-1.56 \times 10^{-21} \frac{\text{S}^2}{\text{rad}^2/\text{s}^2}$, respectively. The resonant frequency of the fabricated comb-drive actuator was hence found to be 2.3836 KHz by substituting those value to Eq.18. Calculated device parameters are summarized in Table 1. The obtained moving mass showed good agreement with the mass which is estimated by the designed structure. However, the measured resonant frequency was smaller than the designed value (3KHz), eventually the compliance C_m didn't agree with the designed one. We suppose the compliance is more sensitive to the fabrication process (especially undercut) than the mass due to cubed parameter dependency on the compliance.

6. Conclusion

We have described an analysis method of a comb-drive actuator by measuring admittance curve as a function of angular frequency. The admittance curve has a peak at frequency closing to the mechanical resonance. We showed an equation which can calculate resonant angular frequency from the admittance curve, and also introduced equations by which we can estimate device parameters such as mass, compliance, and mechanical resistance. The theory was examined by comparing experimental values with designed ones, and found that the presented method is useful to estimate mechanical parameters of a comb-drive actuator.

Acknowledgment

The authors would like to thank VDEC of the University of Tokyo for providing masks of comb-drive actuators.

(Manuscript received Aug. 22, 2005,
revised Jan. 19, 2006)

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