Algebraic Reflections on the Vector Concept in Electrocardiography

Part II. Structure of the Electric Field

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In the previous report, the definition of vector was reconsidered and the method to construct the electric field on the body surface was discussed. The present paper is concerned with the analysis of the structure of the field. Here, a method to decompose the field to the sum of fields of lower degree (Chapter 3), and an "isomorphic image" of the field is demonstrated so as to make its intuitive understanding possible (Chapter 4). The meaning of the expression ELECTROCARDIOGRAM IS SCALAR, which is seemingly inconsistent with the previous statement ELECTROCARDIOGRAM IS VECTOR, is explained. As an application of the results in these 2 chapters, the theoretical limitation of the vectorcardiography is discussed quantitatively.

As was mentioned before,19) the purpose of this study was to find a new principle in the conventional vector theories and to exclude semi-abstract concepts and unnecessary assumptions. The definition of the vector was reconsidered and the method to construct the electric field on the body surface was demonstrated in Part I19) and here, the structure of the field will be analysed algebraically and geometrically.

Chapter 3.

DECOMPOSITION OF THE ELECTRIC FIELD

At the beginning, the fundamentals for the finite-dimensional vector space will be discussed.

All the ideals of $\mathfrak{A}$ form a complementary modular lattice,11,3) when a law of composition $A_1 \cap A_2$ of ideals $A_1$ and $A_2$ and the other law of composition $A_1 \cup A_2$ are defined respectively as their intersection and direct sum $A_1 + A_2$.

The set $\{A\}$ of $k$-ideals $A_1, A_2, \ldots, A_k$ of $\mathfrak{A}$ is named as a "series of ideals", and a subset of $\{A\}$ is a "subseries" of $\{A\}$. Dimension-number of an ideal $A$ is shown by symbol $dA (dA \geq 0)$. The least upper bound of $\{A\}$ is $A = A_1 \cup A_2 \cup \ldots \cup A_k$. When $dA_k = \sum_{i=1}^{k} dA_i$ holds, $\{A\}$ is said "independent". Then the followings are valid.

1) A subseries of $\{A\}$ is independent.

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2) \( A_i \cap A_j = 0 \) \( A_i, A_j \{A\} \), \( i \neq j \)

3) \( A_i \cap A'_i = 0 \) \( \{A' = A_1 \cup A_2 \cup \ldots \cup A_i \cap A_{i+1} \cup \ldots \cup A_k\} \) And conversely, if this relation holds for any \( i \), \{A\} is independent.

4) When an element of \( A_i \) is \( \alpha_i \), \( \sum_{i=1}^{k} \alpha_i = 0 \) holds only in the case when \( \alpha_i = 0 \) \( (i = 1, 2, \ldots, k) \).

Here, the outline of proofs for 3) and for 4) will be demonstrated.

\[
dA_k = d(A_{k-1} \cup A_k) = dA_{k-1} + dA_k - d(A_{k-1} \cap A_k)
\]

In the same manner, the following holds for any \( h \) \( (h = 0, 1, 2, \ldots, k-1) \).

\[
dA_{k-h} = dA_{k-h-1} + dA_{k-h} - d(A_{k-h-1} \cap A_{k-h})
\]

By integrating this formula for all \( h \),

\[
dA_k = \sum_{i=1}^{k} dA_i - \sum_{j=1}^{k-1} d(A_j \cap A_{j+1})
\]

As \{A\} is independent,

\[
\sum_{j=1}^{k-1} d(A_j \cap A_{j+1}) = 0 \quad \text{Then,} \quad d(A_j \cap A_{j+1}) = 0
\]

Hence, we get the relations such as \( A_i \cap A_j = 0 \), \( (A_1 \cup A_2) \cap A_3 = 0 \), \( (A_1 \cup A_2 \cup A_3) \cap A_4 = 0 \), \( \ldots \). Furthermore, the order among the elements in \{A\} is not determined, and any ideal in \{A\} may be appointed as \( A_k \). So 3) holds evidently. The converse may also be proved easily.

Next, 4) will be proved.

\[
\alpha' + \alpha_k = 0 \quad (\alpha' = \alpha_1 + \alpha_2 + \ldots + \alpha_{k-1} \quad \alpha' \in A'_{k})
\]

If \( \alpha_k \neq 0 \), then \( \alpha' = -\alpha_k \neq 0 \) and \( -\alpha_k \notin A_h \), therefore, \( \alpha' \in A_{k} \cap A'_{h} \) This result is inconsistent with the character 3). Hence, \( \alpha_k = 0 \). Just in the similar way, it is proved that \( \alpha_i = 0 \) holds for any \( i \).

When \{A\} is independent and the relation \( A_k = \mathbb{A} \) holds, we say that \( A_i \) \( (i = 1, 2, \ldots, k) \) in \{A\} is a "component of \( \mathbb{A} \)" and that "\( \mathbb{A} \) is decomposed to \( k \)-components \( A_1, A_2, \ldots, A_k \)".

\[
d\mathbb{A} = n = \sum_{i=1}^{k} dA_i \quad (3-3)
\]

Now the method will be demonstrated here, how to decompose \( \mathbb{A} \) to its components \( A_1, A_2, \ldots, A_k \), so as to comply with given conditions \( dA_i = n_i \), \( \sum n_i = n \) \( (i = 1, 2, \ldots, k) \).

First of all, a \( n_1 \)-dimensional ideal \( A_1 \) is arbitrarily selected \( (dA_1 = n_1) \). As \( \mathbb{A} \) is the complementary modular lattice, \( A'_1 \) is determined though not uniquely, and \( \mathbb{A} = A_1 \cup A'_1 \), \( dA'_1 = n - n_1 \) hold. Next, a \( n_2 \)-dimensional ideal is arbitrarily selected \( (dA_2 = n_2) \). Then, \( A'_2 \) is determined and \( A'_1 = A_2 \cup A'_2 \), \( dA'_2 = n - n_1 - n_2 \). \( \mathbb{A} = A_1 \cup A_2 \cup A'_2 \).

Further, a \( n_3 \)-dimensional \( A_3 \) is selected \( (dA_3 = n_3) \). \( A'_3 = A_3 \cup A'_3 \), \( dA'_3 = n - n_1 - n_2 - n_3 \). \( \mathbb{A} = A_1 \cup A_2 \cup A_3 \cup A'_3 \) By repeated application of these operations, \( \mathbb{A} \) is evidently decomposed as

\[
\mathbb{A} = A_1 \cup A_2 \cup \ldots \cup A_k \quad n = \sum_{i=1}^{k} n_i \quad (3-4)
\]
where obviously \( 1 \leq k \leq n \).

"A finite-dimensional vector space is decomposed to an arbitrary number of components, so far as the number remains within the dimension-number of the space. The sum of dimension-numbers of the components is equal to the dimension-number of the space."

When \( \mathbb{A} \) is decomposed according to formula (3-4), the relation

\[
\alpha = \alpha_1 + \alpha_2 + \ldots + \alpha_k \quad \alpha \in \mathbb{A}, \quad \alpha_i \in A_i \quad (i = 1, 2, \ldots, k)
\]  

(3-5)

holds uniquely for given \( \alpha \). If not so, \( \alpha = \sum \alpha_i = \sum \alpha'_i \) \( \therefore \sum (\alpha_i - \alpha'_i) = 0 \)

Because \( \alpha_i - \alpha'_i \in A_i \) and because \( \{A\} \) is independent, \( \alpha_i - \alpha'_i = 0 \) \( \therefore \alpha_i = \alpha'_i \) from the above mentioned character 4). Hence, \( \alpha_i \) corresponds uniquely to \( \alpha \) given in \( \mathbb{A} \), according to formula (3-5). By making \( \alpha \) correspond to \( \alpha_i \), a homomorphic mapping \(^{1,3} \) of \( \mathbb{A} \) to \( A_i \) is obtained. "\( A_i \) is homomorphic to \( \mathbb{A} \)." \( \alpha_i \) is called as the "component" of \( \alpha \) in direction to \( A_i \) or the "projection" of \( \alpha \) in \( A_i \).

These general characters of the vector space will be applied to the electric field \( \mathbb{B} \).

The extension \( \mathbb{B} \) of \( \mathbb{A} \) is a \( n \)-dimensional vector space, and is decomposed to \( k \)-components \( (1 \leq k \leq n) \) as

\[
\mathbb{B} = \mathbb{B}_1 + \mathbb{B}_2 + \ldots + \mathbb{B}_k \quad , \quad n = \sum_{i=1}^{k} \mathbb{B}_i
\]  

(3-6, 7)

The component \( p_i \) of \( p \in \mathbb{B} \) in direction to \( \mathbb{B}_i \) is determined uniquely and the formula

\[
p = p_1 + p_2 + \ldots + p_k
\]  

(3-8)

holds. The set \( \mathbb{B}_i \) consisting of \( p_i \)'s corresponding to all \( p \)'s in \( \mathbb{B} \) is the "component (field)" of the field \( \mathbb{B} \) in direction to \( \mathbb{B}_i \) or the "projection" of \( \mathbb{B} \) in \( \mathbb{B}_i \). Then

\[
\mathbb{B} = \mathbb{B}_1 + \mathbb{B}_2 + \ldots + \mathbb{B}_k
\]  

(3-9)

\[
d \mathbb{B} = d \mathbb{B}_1 + d \mathbb{B}_2 + \ldots + d \mathbb{B}_k
\]  

(3-10)

hold, where the addition in formula (3-9) is defined as that between the elements of \( \mathbb{B}_i \)'s.

"The electric field has its component in any given direction and of any lower dimension. It is reduced to the sum of its components."

As the special cases, the following two are the most important characters of the field.

1) The \( n \)-dimensional field is composed of \( n \)-one-dimensional fields.

2) The field is reduced to the sum of three-dimensional fields or to the sum of three-dimensional fields and a one- or two-dimensional one. The three-dimensional field is regarded as equivalent to that with a single electric dipole (Chapter 2), and the one- or two-dimensional field is considered to be a special field with a dipole. Hence, the electric field may be regarded as equivalent to that with electric dipoles.

In connection with the concept "component of the field", the following results are interesting.

An electrocarodiogram, a vector in the vector space (Chapter 2), has hitherto been named as a "scalar electrocardiogram" and regarded as a component or a projection of some vectorial quantity. In the author's opinion, too, an electro-
cardiogram $p$ may agree with such view. To say in other words, 

"An electrocardiogram $p$ represents a one-dimensional component field of the field $\mathbb{V}$."

Or this is said in catchy expression that ELECTROCARDIOGRAM IS SCALAR.

This is the result corresponding to the conventional view "scalar electrocardiogram", and the meaning is as follows. An element $p^*$ of the principal ideal $\mathbb{V}^*$ generated by $p$ ($\mathbb{V}^* = [p]$) is expressed as $ap$ ($a \in \mathbb{F}$). Then an element $p_1$ of the one-dimensional component field $\mathbb{V}_1$ of $\mathbb{V}$, in direction to $\mathbb{V}^*$, is expressed as $a'p$ ($a' \in \mathbb{F}$). Or projection $\mathbb{V}_1$ of the field is composed of elements proportionate to $p$.

On the other hand, a scalar electrocardiogram is usually thought to be deduced from a "vectorcardiogram", provided that the electric field is generated by a single electric dipole, because a vectorcardiogram is in reality synthesized from three independent scalar electrocardiograms $p^1$, $p^2$, and $p^3$ by certain method. Generally to say, the vectorcardiography is based on the view, the "vectorial electrocardiography", in our terminology, that the electric field is to be reconstructed by three scalar electrocardiograms. Vectorcardiogram is a sort of "vectorial electrocardiogram", the combination of three basic scalar electrocardiograms. This view is surely true, so far as the equivalency of electric field on the body surface to the single dipole field holds. But no one knows whether the equivalency does hold or not. Then, if not so, what represents or means this vectorial electrocardiogram? This question is generally a very complicated one, but under the condition that the electric field is linear (Chapter 2), the following may be allowed to say.

"A vectorial electrocardiogram represents a three-dimensional component of the electric field", corresponding to the former result that a scalar electrocardiogram represents a one-dimensional component of the field. Thus, "A vectorcardiogram represents a three-dimensional component of the electric field." Because an element of the three-dimensional component $\mathbb{V}_3$ of $\mathbb{V}$, in direction to $\mathbb{V}^*$, is expressed as $a_1p^1 + a_2p^2 + a_3p^3$ ($a_1, a_2, a_3 \in \mathbb{F}$) as well as an element of the ideal $\mathbb{V}^* = [p^1, p^2, p^3]$ generated by three scalar electrocardiograms $p^1$, $p^2$, and $p^3$, a vectorcardiogram (a sort of vectorial electrocardiogram) represents a three-dimensional component of the field, and does no more than this. This is the law which sets limits quantitatively to the vectorcardiography. Various methods of deriving the vectorcardiogram only determine the direction of component field which the vectorcardiogram represents.

As to the concept component or projection, more concrete explanation is demonstrated in the next chapter (Chapter 4).

Chapter 4.

ISOMORPHIC IMAGE OF THE ELECTRIC FIELD

This chapter was prepared in order to make intuitive understanding of the structure of the field possible.

As to $\mathbb{S}$ and mapping $f$ in formula (2-2), the following postulations may empirically be accepted easily.

1) $\mathbb{S}$ is a convex closed surface and does not cross any straight line at more than two points.

2) Mapping $f$ is one-to-one and bicontinuous. In other words, $f$ is a topological mapping.

As the extension $\mathbb{V}$ of the field $\mathbb{V}$ is a $n$-dimensional vector space, an isomorphic
correspondence\(^{1,3}\) between \(\mathfrak{F}\) and \(V_n\) is obtained. And
\[
V_n = g(\mathfrak{F})
\]  
(4-1)

shows this correspondence, an example of which is shown when in formula (2-13)\(^{19}\) \(A(a_1, a_2, \ldots, a_n)\) is made to correspond to \(p\). Particularly, the most important isomorphic correspondence is that which makes \(p\) correspond to its field-position-vector \(P(p_1, p_2, \ldots, p_n)\). Such an isomorphic correspondence is obviously possible, because formula (2-13)\(^{"\prime\prime}\) means an automorphism in \(V_n\). This relation is specially expressed as
\[
V_n = \pi(\mathfrak{F})
\]  
(4-2)

From formula (4-1),
\[
\vec{V}_n = g(\mathfrak{F})
\]  
(4-3)

holds, where \(\vec{V}_n\) is called as an "isomorphic image or \(g\)-image" of the electric field \(\mathfrak{F}\). And from formula (4-2),
\[
V_n^* = \pi(\mathfrak{F})
\]  
(4-4)

holds, where \(V_n^*\) is called as the "\(\pi\)-image" of \(\mathfrak{F}\). The \(\pi\)-image is a sort of the isomorphic image.

From formulas (2-2) and (4-3),
\[
\vec{V}_n = g f(\mathfrak{F})
\]  
(4-5)

where \(\vec{V}_n\) is called as a "\(gf\)-image" of the body surface \(\mathfrak{F}\). The mapping-\(gf\) is also a topological mapping, because \(f\) is topological and \(g\) is isomorphic.

For the purpose of intuitive understanding, the vector space \(V_n\) is discussed as a geometric space, regarding an element of \(V_n\) as a geometric point. As, from 1), \(\mathfrak{F}\) is a two-dimensional cycle in the three-dimensional space (solid space), and as the dimension does not alter\(^5\) under the topological mapping-\(gf\), the isomorphic image \(V_n\) must be also a two-dimensional cycle.

"An isomorphic image of the electric field is a two-dimensional cycle."

As was seen in Chapter 3, the field \(\mathfrak{F}\) is decomposed to an arbitrary number of component fields, where formula (3-9) and (3-10) hold. The intuitive structure of these components will be discussed by means of their images. From formulas (3-9), (3-10) and (3-6, 7),
\[
\mathfrak{F} = \mathfrak{F}_1 + \mathfrak{F}_2
\]  
(4-6)
\[
d\mathfrak{F} = d\mathfrak{F}_1 + d\mathfrak{F}_2
\]  
(4-7)
\[
\mathfrak{F} = \mathfrak{F}_1 \cup \mathfrak{F}_2, \quad \mathfrak{F}_1 \cap \mathfrak{F}_2 = 0
\]  
(4-8)
\[
d\mathfrak{F} = d\mathfrak{F}_1 + d\mathfrak{F}_2
\]  
(4-9)

hold. And from formula (3-8),
\[
p = p_1 + p_2 \quad (p \in \mathfrak{F}, \ p_1 \in \mathfrak{F}_1, \ p_2 \in \mathfrak{F}_2)
\]  
(4-10)

holds uniquely for given \(p\). When these formulas are mapped by \(g\) into \(V_n\),
\[
V_n = V_{n1} \cup V_{n2}, \quad V_{n1} \cap V_{n2} = 0
\]  
(4-8)\(^{\prime}\)
\[
n = n_1 + n_2
\]  
(4-9)\(^{\prime}\)

hold from formulas (4-8) and (4-9). And from formula (4-10),
\[ v = v_1 + v_2 \]  \hspace{1cm} (v \in V_n, \ v_1 \in V_{n_1}, \ v_2 \in V_{n_2}) \hspace{1cm} (4-10)'

holds uniquely for given \( v \). \( V_{n_1} \) and \( V_{n_2} \) are \( n_1 \)-and \( n_2 \)-dimensional vector spaces respectively, will be called geometrically as "\( n_1 \)- and \( n_2 \)-dimensional planes", respectively (a plane in usual sense is here a two-dimensional plane and a straight line is a one-dimensional plane). They intersect themselves at only one point \( O \) (origin).

As the sets \( V_{n_1} \) and \( V_{n_2} \) consisting of \( v_1 \)'s and \( v_2 \)'s, respectively, corresponding to all \( v \)'s in \( V_n \) are of course isomorphic images of two component fields of the electric field, the problem is then reduced to find a geometric method to determine \( v_1 \) and \( v_2 \) from \( v \) given. If \( v_{n_1} \) is regarded as a variable in the relation derived from formula \((4-10)'\)

\[ v - v_{n_1} \in V_{n_2} \]  \hspace{1cm} (4-10)''

the intersection of the plane \( V_{n_1} \) with locus \( V_x \) of \( v_1 \) which satisfies this relation, is the point \( v_{n_1} \). And \( V_x \) is a plane \( V_{n_2} \) containing the point \( v \) and parallel to \( V_{n_2} \). Hence, the point \( v_{n_1} \) is determined as the intersection of \( V_{n_2} \) with \( V_{n_1} \).

Similarly, the point \( v_{n_2} \) is determined as the intersection of plane \( V_{n_2} \) with plane \( V_{n_1} \) containing the point \( v \) and parallel to \( V_{n_1} \).

When the field is decomposed to \( k \)-components, the method to determine their images is reduced, by the following manipulation, to the above-mentioned case where the components are only two. The formulas \((3-9)\) and \((3-6)\) are modified as 

\[ \mathfrak{P} = \mathfrak{P}_1 + \mathfrak{P}'_1 \]  \hspace{1cm} (4-12)

\[ \mathfrak{P}_i = \mathfrak{P}_{i+1} + \mathfrak{P}_{i+1} + \ldots + \mathfrak{P}_i + \mathfrak{P}_{i+1} + \ldots + \mathfrak{P}_{k_1}, \]  \hspace{1cm} (4-13)

Then we get the relation \( v = v_i + v_i' \). Thus, \( v_i \) is the intersection of plane \( V_i \) with (\( n-n_i \)) -dimensional plane \( V_i' \) containing the point \( v \) and parallel to \( V_i' \) or to \( V_1, V_2, \ldots, V_{n-1}, V_{n+1}, \ldots, V_n \).

By virtue of the isomorphic image, the meaning of the expression "component" or "projection" may have become more familiar. The image of component is a geometric projection of the image of the electric field. Later, these will be explained more concretely in case of \( n = 3 \).

Next, the image of "sub-field" will be demonstrated. Locus \( \mathfrak{S}' \) of a point, on \( \mathfrak{S} \), of which field-position-coordinate \( A \) satisfies the equation 

\[ p_T = A \cdot P_T = 0 \]  \hspace{1cm} (4-11)

for a given time-position \( T \), is known empirically to exist and to be a closed curve on \( \mathfrak{S} \) (an one-dimensional cycle). It is the "transitional pathway" called by Grant, R. P. \(^{18}\) or the "straight line" in the previous report. \(^{18}\) Hence, in the following formulas due to formulas \((2-2)\) and \((4-5)\),

\[ \mathfrak{S}' = f(\mathfrak{S}') \]  \hspace{1cm} (4-12)

\[ \mathfrak{V}'_n = g_f(\mathfrak{S}') \]  \hspace{1cm} (4-13)

\( \mathfrak{S}' \) and \( \mathfrak{V}'_n \) are not void. \( \mathfrak{S}' \) is named as a "sub-field" of \( \mathfrak{P} \), and \( \mathfrak{V}'_n \), an one-dimensional cycle, is an isomorphic image of the sub-field.

"An isomorphic image of the sub-field is an one-dimensional cycle."

And the formula \((4-11)\) determines a \((n-1)\)-dimensional sub-space (or hyperplane)
$V_{n-1}$ in $V_n$. Hence,
\[ \tilde{V}_n = \tilde{V}_n \cap V_{n-1} \]

(4-14)

"An isomorphic image of the sub-field is intersection of the image of the electric field with hyperplane."

The reason why $\varnothing'$ was named as straight line in the previous report\(^{18}\) is due to the fact that it is an "one-dimensional" cycle and that it has the similar characters with straight line in primary geometry because its image lies on a hyperplane.

By applying the above-mentioned general arguments to the practically most important three-dimensional field, more concrete explanation will be demonstrated here.

As the image $\tilde{V}_3$ of the three-dimensional field $\mathfrak{B}$ is a two-dimensional cycle, it is a closed surface in the solid space $V_3$. Then, when the $\pi$-image is adopted as the isomorphic one, it can be drawn really. And according to the previous report, $L = (L')_A$, where $L$ is the lead vector of $p \in \mathfrak{B}$. So an isomorphic mapping is obtained by making $p$ correspond to $L$. Then, the image by such mapping is no other than the "image surface" called by Frank, E\(^{16}\). But it is almost impossible to be drawn in reality.

As to the components, there are only two ways to decompose the three-dimensional field.

\[ \mathfrak{B} = \mathfrak{B}_1 + \mathfrak{B}_2 \quad d\mathfrak{B}_1 = 1, \quad d\mathfrak{B}_2 = 2 \]

and
\[ \mathfrak{B} = \mathfrak{B}_1 + \mathfrak{B}_2 + \mathfrak{B}_3 \quad d\mathfrak{B}_1 = d\mathfrak{B}_2 = d\mathfrak{B}_3 = 1 \]

At first, the former case is explained. $V_{31}$ and $V_{32}$ are a straight line and a plane in the solid space $V_3$, which intersect with each other only at origin $O$ as shown in Fig. 1. Plane $V_{32}$ and straight line $V_{31}^*$, which is parallel to $V_{31}$ and contains

![Fig. 1. Isomorphic images of component fields (Illustration-1).](image-url)

a point $v$ on the closed surface $\tilde{V}_3$, determine the point $v_2$ as their intersection. Locus of the point $v_2$ when the point $v$ moves on all over the surface $\tilde{V}_3$, in other words, the projection of $\tilde{V}_3$ on the plane $V_{32}$, is the image of the two-dimensional component

Note: Here, the correspondence of $p$ to its field-position-coordinate $A$ was used as $g$, but it is same the matter if another one was used.
field $\mathbf{\Psi}$ of $\mathbf{\Phi}$. It is a closed curve on the plane $V_{32}$. The intersection of straight line $V_{31}$ with plane $V_{32}^*$, which is parallel to plane $V_{32}$ and contains the point $v$, is point $v_1$. Locus of $v_1$ or projection of $\tilde{V}_3$ on the straight line $V_{31}$, is the image of the one-dimensional component field $\mathbf{\Psi}_1$ of $\mathbf{\Phi}$. $\tilde{V}_3$ is the sum of $\tilde{V}_{31}$ and $\tilde{V}_{32}$.

In the latter case, in which the electric field is decomposed to three one-dimensional fields, the same is the matter (Fig. 2). Intersection of straight line $V_{31}$ with plane $V_{31}^*$, which is parallel to straight lines $V_{32}$ and $V_{33}$ and contains $v$, is $v_1$. Points $v_2$ and $v_3$ are determined in the same way. Thus, $\tilde{V}_3$ is decomposed to the images of three one-dimensional fields $\mathbf{\Psi}_1$, $\mathbf{\Psi}_2$ and $\mathbf{\Psi}_3$ of the field $\mathbf{\Phi}$.

![Fig. 2 (Left). Isomorphic images of component fields (Illustration-2).]

![Fig. 3 (Right). Isomorphic image of sub-field.]

The image of the sub-field is, of course, the intersection of closed surface $V_3$ with plane $V_2$ (Fig. 3). This is the image of the transitional pathway or of the straight line in the previous report.

Finally, some of the important characters of the mapping-"$\pi$" or the $\pi$-image will be demonstrated. This is a sort of isomorphic mapping and plays an important rôle in the present study because of the following characters. In general, $\pi$ is used most commonly as an isomorphic mapping. Firstly, $\pi$ makes potential $p$ correspond to $P(p_1, p_2, \ldots, p_n)$, an ordered set of potentials at $n$-basic time-positions, which is possible and easy to be measured. So, $\pi$ is the most practical among all isomorphic mappings. Secondly, $P$, image of $p$ by $\pi$, may be regarded as an approximation of $p$. Being discrete values at $n$-basic time-positions of a continuous function $p$ concerning variable time-position $T$, the components of $p$, $p_1$, $p_2$, $\ldots$, $p_n$, are able to represent approximately the function, if the basic time-positions are adequately selected. $P(p_1, p_2, \ldots, p_n)$ is called as the "base of a potential $p$" or as the "base of an electrocardiogram $p". Note Then, the base $P$ of an electrocardiogram $p$ represents an approximation of $p$. In the previous reports, the ratio $p_1 : p_2 : \ldots : p_n$ was called as the "pattern of an electrocardiogram $p" , and two electrocardiograms $p$ and $p'$ were "similar" if their patterns were equal $p_1 : p_2 : \ldots : p_n = p'_1 : p'_2 : \ldots : p'_n$. This was due to the idea that the ratio approximately represented the configuration of electrocardiogram. In such mean-

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Note: The "base field matrix" in the previous report is called also as the "base of the electric field" corresponding to the base of an electrocardiogram (see the next chapter).
ings, the mapping-π makes approximation of p. Thirdly, the image P plays an equivalent rôle to the lead vector (see the previous report). In other words, P is an indicator of the site from which the electrocardiogram is recorded. So, P is named also as a “field-position-vector” from this point of view. P is named as the "base of electrocardiogram" and as the "field-position-vector" according to its rôles.

SUMMARY AND CONCLUSION

(I) DECOMPOSITION OF THE ELECTRIC FIELD (Chapter 3)

(1) The concept “component (field) of the electric field” was introduced. The field has its own component in any direction and of any dimension.

(2) The electric field is decomposed to given-dimensional components, or it is reduced to the sum of the lower-dimensional (component) fields.

(3) From (2), the electric field is reduced to the sum of the three-dimensional fields or to the sum of the three-dimensional fields and a one- or two-dimensional one. The three-dimensional field is regarded as equivalent to that with a single electric dipole (Chapter 2), and the one- or two-dimensional field is considered to be a special field with a dipole. Thus, the electric field may be regarded as equivalent to that with electric dipoles.

(4) A scalar electrocardiogram represents a one-dimensional component (field) of the electric field. Or, in catchy expression, ELECTROCARDIOGRAM IS SCALAR.

(5) The scalar electrocardiogram has hitherto been thought, as literally shown, as a component or a projection of some vectorial quantity. The above (4) is the corresponding result to this conventional view. The result (4) is appeared to be inconsistent with the result in Part I that an electrocardiogram is a vector but this is not so.

(6) A vectorcardiogram represents a three-dimensional component (field) of the electric field, and does no more than this. This is the law which sets limits quantitatively to the vectorcardiography. Various methods of deriving vectorcardiogram can only determine the direction of the component field represented by the vectorcardiogram.

(II) ISOMORPHIC IMAGE OF THE ELECTRIC FIELD (Chapter 4)

(1) A geometric image of which structure is the same with that of the electric field, an “isomorphic image”, is conceivable.

(2) An isomorphic image of the field is a two-dimensional cycle in the n-dimensional geometric space. The “image surface” called by Frank, E. is a sort of isomorphic image of the three-dimensional field.

(3) The isomorphic image of a component field is a geometric projection of the image of the electric field.

(4) The isomorphic image of so-called transitional pathway, a closed
curve on the body surface, is a one-dimensional cycle, and lies on a hyperplane. This determines the linear nature of the potentials on the pathway, and is the reason why the transitional pathway was named as straight line in the previous report.18)

(The remaining part of this study will be reported in the succeeding issue.)

References