Four-dimensional Space-time Visualization for Understanding Three-dimensional Motion

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Summary
We propose a novel visualization of moving or shape-changing 3-D objects. Events in 3-D space are represented as 4-D spatio-temporal information in 4-D space composed of three space dimensions and one time dimension. We have developed an interactive 4-D space-time visualization system that enables users to intuitively understand such events using an interactive interface that consists of a glasses-free 3-D display and a flight controller pad. While conventional systems visualize events in 3-D space by degenerating one dimension of space or time, the proposed system visualizes events including both 3-D space and time by projecting 4-D spatio-temporal information onto a 3-D viewing field. Using the proposed system, we can flexibly observe various dynamic events of a 3-D object from an arbitrary eye point and visual axis while changing the form of the viewing field in 4-D space-time. Consequently, we can visually perceive dynamic characteristics of a 3-D object that cannot generally be overviewed in 3-D space, such as 3-D velocity and acceleration, in 4-D space-time.

Key words: four-dimensional space-time, three-dimensional time-varying data visualization, three-dimensional motion recognition, computer graphics, virtual reality

1. Introduction

Time continuously flows in one direction, from the past to the future. The combination of 3-D space and time is referred to as 4-D space-time. This 4-D space-time is conceptually constructed by stacking 3-D space at different time steps along the time direction. Here, we assume 4-D space-time where the four axes are orthogonal to each other. The temporal axis is equivalent to the other three spatial axes in 4-D space-time. With a 4-D space-time representation, the real world can be described universally. However, it is difficult to visualize 4-D space-time in the 3-D space that we are able to perceive. There are two conventional ways to represent 4-D spatio-temporal information. The first one visualizes 4-D spatio-temporal information by overwriting a time-varying 3-D object in 3-D space such as a trajectory. The second one visualizes 4-D spatio-temporal information in 3-D space-time (composed of two space dimensions and one time dimension) by neglecting one spatial dimension. In these approaches, however, one dimension of time or space is degenerated in the representations. To understand a dynamic 3-D scene, it is desirable for both 3-D space and time information to be inclusively visualized.

Recently, 4-D space-time has been of interest in various scientific fields. For example, the data mining techniques of geoinformatics are mainly dealt with in 2-D and 3-D space. Considering temporal importance, 4-D spatio-temporal data mining techniques to unify 3-D space and time are currently required. In the database field, 3-D spatial data modeling with temporal information is required for modeling 3-D objects that change position and shape over time. In 3-D motion recognition, space-time invariants of 3-D motion are computed from 4-D spatio-temporal information. In one 3-D shape morphing technique...
of computer graphics, a 4-D object is generated from a shape-changing 3-D object over time. One 3-D object smoothly changes to another 3-D object using the key-frame operation along the time direction in 4-D space-time\(^4\).

Moreover, some studies on the visualization of 4-D space-time have already been reported. Conventional visualization approaches are divided into two categories: the first performs perspective projection for 4-D spatio-temporal information transformed by a 4-D rotation operation in 4-D space-time\(^5\), and the second slices 4-D spatio-temporal information with a hyperplane (subspace) in 4-D space-time\(^6\). In the former case, the eye point and the visual axis are fixed. In the latter case, a 3-D object is cut by a 2-D plane temporally and the obtained cross-sections are stacked. Therefore, in this approach, subspace generation is not directly assumed in 4-D space-time.

In this paper, we propose a visualization system to understand a variety of moving 3-D objects in 4-D space-time. The developed system regards 4-D space-time as 4-D orthogonal space. Using this system, we can observe 4-D spatio-temporal information in an arbitrary viewing field from various eye points and visual axes, while moving freely in 4-D space-time. We can also have visual contact with the existence of time. Moreover, the system can visualize the cross-section sliced with an arbitrary hyperplane (subspace) and generate the 3-D CG animation of a moving 3-D object by controlling the target viewing field in 4-D space-time. The proposed 4-D space-time display system makes it easy to observe various dynamic 3-D scenes with moving and shape-changing 3-D objects. Therefore, we can provide an interactive environment so that scientists, transportation engineers, meteorologists, and artists who deal with 3-D time-series data can observe and construct the relationship between 3-D space and time at a glance. Our proposed framework will be effective in various fields, not only in visualization for the understanding of 3-D motion, but also in representations of 3-D time-varying volumetric data, the walk-through technique including time in virtual 3-D space, 3-D time-varying feature tracking, and 4-D capture systems for dynamic 3-D events. In section 2, we first describe a framework for the visualization of a dynamic 3-D object in 4-D space-time with an arbitrary viewing field.

In section 3, we discuss the outline and interactivity of the developed system, and examine the effectiveness of 4-D space-time observation. In section 4, we observe some motions of 3-D objects by controlling the viewing field in 4-D space-time to achieve new understanding, different from usual 3-D perception. Finally, conclusions are given in section 5.

### 2. Framework of 4-D Space-time Visualization

Fig. 1 shows various spatial representations of a moving 3-D object. When we spatially represent a 3-D object with motion, 3-D space or 3-D space-time is generally used (see Figs. 1 (a) and (b)). The former can visualize the trajectory of a moving 3-D object and the latter can illustrate the relation between time and location for a moving 3-D object, although visible motion is limited to 2-D space. In order to perfectly represent events that occur in 3-D space, we should deal with them in 4-D space-time instead of 3-D space or 3-D space-time limited to three dimensions (see Fig. 1 (c)).

As shown in Fig. 2, 4-D spatio-temporal information...
Information is displayed on the 3-D screen by perspective projection to steer the visual axis in various directions from the eye point (the look-from point) \( p_f (x_{p_f}, y_{p_f}, z_{p_f}, t_{p_f}) \) to the look-at point \( p_a (x_{p_a}, y_{p_a}, z_{p_a}, t_{p_a}) \) in 4-D space-time. The center of the 3-D screen and the center of the background hyperplane are located on the visual axis at the distances \( h \) and \( f \) (> \( h \)) from the eye point, respectively. The volume of the 3-D screen is \( 2k \times 2k \times 2k \). The 4-D viewing field is defined as a truncated pyramid which is formed by the eye point, the 3-D screen and the background hyperplane. Only 4-D spatio-temporal information inside the 4-D viewing field is visible. This framework enables one to observe a variety of 4-D spatio-temporal information from an arbitrary 4-D eye point and visual axis.

We can take a picture of a 3-D scene with a camera by using the wide-field and telescopic functions. The proposed visualization method can perform similar functions in 4-D space-time by controlling the parameters \( k, h \) and \( f \) regarding the 4-D viewing field. For example, the wide-field operation in 4-D space-time coincides with changing the scale \( k \) of the 3-D screen. The telescopic operation is performed by scaling the distance \( h \) from the eye point \( p_f \) to the 3-D screen. The control of the distance \( f \) from the eye point \( p_f \) to the background hyperplane corresponds to the depth operation in the direction of the visual axis in 4-D space-time.

The proposed visualization model can represent various projections, not only the perspective projection but also the parallel projection and the slice operation. The parallel projection is performed by locating the eye point extremely far from the 3-D screen. This operation approximates the size of the background hyperplane to that of the 3-D screen (see Fig. 3). The slice operation is realized by allocating the 3-D screen nearby the background hyperplane (see Fig. 4).

With the visualization method mentioned above, we can freely walk in 4-D space-time and overview 4-D spatio-temporal information with an arbitrary 4-D viewing field.

3. An Interactive 4-D Space-time Visualization System

We have developed an interactive 4-D space-time visualization system based on the former 4-D space visualization system\(^7\),\(^8\). The former system dealt with 4-D orthogonal space where four axes are defined as four spatial dimensions.

3.1 An Approach to 4-D Space Visualization and 4-D Space Interaction

Conventional 4-D space visualization algorithms are divided into two categories. The first performs 4-D perspective projection for a 4-D object transformed by 4-D geometric operations such as rotation, translation and scaling\(^9\)–\(^11\). The second visualizes various 3-D perspective drawings of a 4-D object from different eye points in 4-D space\(^12\). In the former case, the eye point is fixed. In the latter case, the visual axis is fixed or the eye point change is lim-
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ited. Therefore, these 4-D visualization algorithms cannot be generalized and systematized to construct a 4-D display system. There are a few reports concerning interaction with 4-D objects. In existing interactive environments, keyboard input or a joystick associated with human actions are often used for geometric operations of a 4-D object\(^{13}\), which enable a user to experience the rotation of the 4-D object in 3-D space. Some papers have introduced haptic interfaces for data exploration in higher-dimensional space to recognize the distribution of higher-dimensional data\(^{14,15}\). However, in these methods, the user’s 4-D viewing position is fixed and the user is not allowed to move around the 4-D object in 4-D space.

We constructed the 4-D visualization algorithm to observe 4-D space from an arbitrary 4-D eye point, visual axis and viewing field. However, we had some difficulties in moving around in 4-D space, since action in 3-D space is not directly related to movement in 4-D space. In order to overcome these difficulties, by extending our 4-D visualization algorithm, we developed a real-time and interactive 4-D space visualization system that makes human actions in 3-D space correspond to movement and rotation in 4-D space bounded on a 4-D spherical surface. The developed system consists of a glasses-free 3-D display, a flight controller pad and a personal computer (see Fig. 5). We associated human actions in 3-D space with an intuitive interface, in this case, a flight controller pad\(^{7,8}\). Using this system, we could smoothly observe any 4-D object and data from the desired 4-D position, direction and distance, extending the experience of 3-D space. As mentioned above, although some attempts at 4-D visualization and interactive 4-D interaction have been reported, our system differs in the generality of 4-D data processing and in having a user interface that allows real-time 4-D space travel.

The developed system is an application of our 4-D space visualization system, and has basically the same visualization functions and high interactivity. Consequently, the system enables us to intuitively move in 4-D space-time and observe 4-D spatio-temporal information in real-time.

3.2 The Effectiveness of the Interactive 4-D Space-time Visualization System

To examine the effectiveness of our system, we conducted evaluation experiments in which we interviewed several users who had knowledge of physics about the 4-D spatio-temporal information obtained with the proposed system. We explained how to use the flight controller pad for interactive movements in 3-D space, and outlined the correspondence between actions in 3-D space and actions on the 4-D spherical surface in 4-D space-time. Throughout the real-time interaction, most users realized how the user interface works without an explanation of the details, and intuitively traveled in 4-D space-time using the flight controller pad. We presented the trajectory of a mutually-perpendicular motion in 3-D space to users (see Fig. 6). As is well known, we cannot learn much about the details of 3-D motion, such as the number of moving 3-D objects, direction of movement and time-sensitive 3-D vector quantities, in the trajectory in 3-D space. Fig. 7 shows snapshots of the users’ observational results in 4-D space-time. Users observed 4-D spatio-temporal information from an initial position on the time axis, and searched around 4-

![Fig. 5](image1) Setup of the interactive 4-D space visualization system

![Fig. 6](image2) Trajectory of a mutually-perpendicular motion in 3-D space
D spatio-temporal information from various positions and directions in 4-D space-time. The experimental results are summarized in Table 1. Users achieved some new insights that could not be obtained from a trajectory in 3-D space.

User A discovered some 3-D lines from the 4-D position. By observing the 3-D image in the direction of the \( t_w \)-axis and rotating at the position in 4-D space-time, user A understood that the trajectory in 3-D space shown in Fig. 6 was constructed by six different 3-D motions (see Fig. 7 (a)). Four 3-D motions move in the positive and negative directions on \( x_w \) - and \( z_w \)-axes from the origin, respectively. From the relationship between the spatial axes and the time axis, user A intuitively realized that straight-line 3-D motions have constant velocity and that zigzag 3-D motion has non-constant velocity. Furthermore, user A noted that there are two types of 3-D motion on the \( y_w \)-axis (see Fig. 7 (b)). The first is a zigzag 3-D motion moving in the positive direction on the \( y_w \)-axis from the origin. The second is the straight-line 3-D motion moving in the direction of the origin from the negative position on the \( y_w \)-axis. User A understood the movements on the \( y_w \)-axis of the trajectory in 3-D space shown in Fig. 6.

User B realized many of the 3-D motions of the trajectory in 3-D space shown in Fig. 6. The 3-D trajectory was constructed by six different 3-D motions (see Figs. 7 (c) and (d)). Four 3-D motions are uniform and linear in the positive and negative directions on the \( x_w \) - and \( z_w \)-axes from the origin, respectively. One 3-D motion is uniform and linear in the direction of the origin from the negative position on the \( y_w \)-axis.

As shown in Figs. 7 (e) and (f), user C discovered that four 3-D motions move in the direction of the positive and negative \( x_w \) - and \( z_w \)-axes over time from the origin. For the one gradual zigzag 3-D motion in the positive direction of the \( y_w \)-axis, it was difficult for user C to understand the motion in the 3-D scene. However, user C gradually realized that constant and non-constant gradients express the velocity.

![Fig. 7 Observation of mutually-perpendicular motion in 4-D space-time](image)

**Table 1** Recognition of moving 3-D objects in 4-D space-time

<table>
<thead>
<tr>
<th>Characteristics of moving 3-D objects</th>
<th>User A</th>
<th>User B</th>
<th>User C</th>
<th>User D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of moving 3-D objects</td>
<td>⃝</td>
<td>⃝</td>
<td>⃝</td>
<td>⃝</td>
</tr>
<tr>
<td>Velocity of moving 3-D objects</td>
<td>⃝</td>
<td>×</td>
<td>⃝</td>
<td>×</td>
</tr>
<tr>
<td>Direction of movement of 3-D objects</td>
<td>⃝</td>
<td>⃝</td>
<td>×</td>
<td>⃝</td>
</tr>
<tr>
<td>Number of correct answers</td>
<td>3/3</td>
<td>2/3</td>
<td>2/3</td>
<td>2/3</td>
</tr>
</tbody>
</table>

(○⇒understand, ×⇒not understand)
of 3-D motion.

By moving along the $t_w$-axis, user D understood that five 3-D motions except for the negative direction of the $y_w$-axis move away from the origin and one 3-D motion approaches the origin from the negative direction of the $y_w$-axis (see Figs. 7 (g) and (h)).

Although we can see the final trajectory of 3-D motion in 3-D space, we cannot judge the start position, the intermediate step and the final position of 3-D motion. According to the visualization of 3-D CG animation with the key-frame function, we can see 3-D spatial information over time. However, we need to spend more time in understanding dynamic 3-D scenes, and have to rely on our memory. Moreover, for the representation in 3-D space-time, one spatial dimension is perfectly degenerated. Therefore, it is difficult to instantaneously understand the characteristics of dynamic 3-D scenes in 3-D space. As we can statically observe dynamic 3-D events in 4-D space-time with the proposed system, we can chase and obtain information regarding multiple 3-D motions that occur together in 3-D space at the same time, while experiencing 4-D interactions such as movement and rotation in 4-D space-time. In order to perfectly understand an event in 4-D space-time, however, users will need to train themselves repeatedly and acquire knowledge and experience in 4-D space-time.

4. Observation of 4-D Spatio-temporal Information

Using the proposed system, we observed 4-D spatio-temporal information. It is expected that the user will be able to understand through interactions with various 4-D spatio-temporal information in our 4-D interactive environment. In the following, we specify various observational results.

4.1 Observation of a Moving 3-D Object by Perspective Projection

Fig. 8 shows the figure eight motion of a 3-D sphere in 3-D space. The start and end points are the origin $o$ of 3-D space and the whole motion corresponds to one cycle. Consequently, the 3-D sphere passes through the origin $o$ three times and the path is closed. We visualized this motion in 4-D space-time (see Fig. 9 (a)). Although the path in 3-D space is closed at the origin $o$, the origin in 4-D space-time is separated into three origins $o$, $o'$ and $o''$ (see Fig. 9 (b)). The path visualized in 4-D space-time is described as a single open curve, which is called a world line. A cyclic figure eight motion in 3-D space is visualized as shown in Figs. 9 (c) and (d). The path in 4-D space-time expands in the direction of the $t_w$-axis and is distinguished from the path in 3-D space. Cyclic motion is effectively visualized in 4-D space-time, while we cannot recognize this cyclicity in 3-D space, since the
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Fig. 10  Piecewise-linear motions of a 3-D cube in 3-D space

(a) Stereoscopic image of 3-D motion  
(b) Front view of the path

(c) Top view of the path  
(d) Side view of the path

Fig. 11  Observation of the piecewise-linear motions of a 3-D cube in 4-D space-time

same paths cd and dc in 3-D space are inseparable, as shown in Fig. 10. On the other hand, the path in 4-D space-time is raveled out as aa'bedc'a'' (see Fig. 11 (b)). We can observe the different paths cd and dc' in 4-D space-time (see Fig. 11 (c)). Furthermore, as shown in Fig. 11 (d), the initial position a is separated into the three positions a, a' and a'' in 4-D space-time. The stationary state at the initial position a is represented as the path aa' steered in the direction of the t_w-axis in 4-D space-time.

With the visualization mentioned above, as a dynamic scene in 3-D space is transformed to a static scene in 4-D space-time, we can overview 3-D motions that include time in one glance. The framework of the proposed system will be effective not only for the feature tracking of a moving 3-D object, but also for the path planning of public transportation systems and interference problems among moving 3-D objects.

4.2 Observation of a Moving 3-D Object by Parallel Projection and a Slice Operation

The proposed visualization system can perform parallel projection by controlling the 4-D viewing field in 4-D space-time as described in section 2. Through parallel projection, we can visualize the trajectory of a moving 3-D object in 3-D space without deforming the 3-D object’s shape. As shown in Fig. 12, we performed the parallel projection of a moving 3-D cube (defined in section 4.1) by setting the eye point on the t_w-axis in 4-D space-time. Consequently, the time component is invisible. Figs. 12 (a) and (b) represent the trajectory of the moving 3-D cube and its path from the front view, respectively. Figs. 12 (c) and (d) are the top and side views of the path obtained by rotating the visualized 3-D space. Figs. 12 (b), (c) and (d) coincide with the paths shown in Figs. 10 (b), (c) and (d), respectively.

Moreover, by using the slice operation, we can take snapshots of a moving 3-D object and generate a 3-D CG animation of a moving 3-D object over time. We performed a slice operation on the moving 3-D cube with changes in the eye point along the t_w-axis in 4-D space-time (see Fig. 13). The sliced 3-D space shown in Figs. 13 (a), (b), (c) and (d), respectively, corresponds to the original 3-D cubes a, b, c and d in 3-D space shown in Fig. 10 (a).
As the proposed visualization of 4-D space-time includes the degeneration of the time component, the system also effectively generates a 3-D CG animation and visualizes the trajectory of the moving 3-D object in 3-D space. By moving in the radial direction of the 4-D sphere and rotating at the position on the time axis in 4-D space-time, we can apply the framework of the proposed system to computer graphics techniques such as 3-D CG animation with the key-frame function and the walk-through in virtual 3-D space.

4.3 Observation of Motion Features of a Moving 3-D Object with the Slice Operation and Perspective Projection

The proposed system can visualize not only 3-D space-time degenerating one dimension of 3-D space, but also an arbitrary subspace in 4-D space-time. Therefore, the past, present and future scenes of the

Fig. 12 Trajectory of the piecewise-linear motions of a 3-D cube in 4-D space-time

Fig. 13 The 3-D CG animation of the piecewise-linear motions of a 3-D cube in 4-D space-time

Fig. 14 Observation of piecewise-linear motions of a 3-D cube from different visual axes in 4-D space-time
moving 3-D object are simultaneously visualized on the 3-D screen. This enables users to observe motion features such as moving velocity and acceleration.

Let us assume the motion of the 3-D cube defined in section 4.1. Figs. 14 (a) and (e) correspond to the 3-D scenes when the 3-D cube is moving on the paths \( ab \) and \( bc \). When we turn the visual axis a little out from the look-at point, we can see the appearance of the past, the present and the future of the moving 3-D object on the 3-D screen. Figs. 14 (b), (c) and (d) correspond to the front, top and side views of the projected 3-D object regarding Fig. 14 (a). These are obtained by rotating the projected 3-D object in 3-D space. Figs. 14 (f), (g) and (h) have the same correspondence regarding Fig. 14 (e). As shown in Figs. 14 (b), (c) and (d), the moving 3-D object is described as an assembly of the cross-sections of the sliced moving 3-D object from the past to the present and the future. We can observe the past motion in the direction from the position \( a \) to the positions \( e \) and \( b \) in the left half-space in the 3-D screen and the future motion in the direction from position \( e \) to the positions \( b, c \) and \( d \) in the right half-space in Fig. 14 (b). Figs. 14 (f), (g) and (h) also indicate the motion feature as well as the cases mentioned above. In this environment, past, present and future 3-D scenes of the moving 3-D cube are simultaneously visualized on the 3-D screen by controlling the visual axis in 4-D space-time.

With the visualization mentioned above, the proposed system can represent Cubist art styles\(^{16,17} \) such as redefinition and the co-mingling of 3-D space and time. In the near future, we will try to explore 4-D space-time motifs inspired by painters and poets.

Next, we visualized the motion of 3-D objects with constant or varying velocities, as shown in Fig. 15. The straight line indicates the trajectory. Consider three motions of a 3-D cube at different velocities. The velocity values are described as \( v_1, v_2 \) and \( v_3 \), where the velocities \( v_1 \) and \( v_2 \) (\(< v_1 \)) are constant, and the velocity \( v_3 \) gradually decreases from the velocity \( v_1 \) to the velocity \( v_2 \), or in other words decelerates. This deceleration is arranged such that the 3-D cube starts from the initial position \( a \) and reaches the destination position \( b \) at the same time as in the case of velocity \( v_2 \). Although the moving 3-D cube takes the same path in 3-D space, we can observe the different motion patterns in 4-D space-time (see Fig. 16 (a)). As shown in Fig. 16 (b), the 3-D cube moving at constant velocities \( v_1 \) and \( v_2 \) draws straight world lines as the paths \( ab \) and \( ab' \), respectively. The length difference in the world lines indicates the velocity difference: \( v_1 > v_2 \). In case of the velocity \( v_3 \), the curved world line, the path \( ab' \), suggests deceleration. Moreover, we can confirm that its arrival time at the destination is same as in the case of the velocity \( v_2 \), since the two cubes finally meet together in 4-D space-time.

The proposed system makes motion features such as velocity and acceleration visible. This system could be effective for optimum path planning and design that includes the velocity of moving 3-D objects.
4.4 Observation of a Changing 3-D Object Using Perspective Projection

When we visualize a shape-changing 3-D sphere with contraction and expansion through time in 3-D space, we generally represent it as a 3-D CG animation as shown in Fig. 17. However, it is difficult to visualize a shape-changing 3-D sphere in 3-D space at the same time, since future 3-D spheres will occlude or be occluded by past 3-D spheres. Figs. 17 (a) to (e) show the contractive state when the future 3-D sphere is overlapped by the past 3-D sphere. Conversely, Figs. 17 (f) to (j) show the expanding state when a future 3-D sphere overlaps a past 3-D sphere. In either case, the past or future 3-D sphere is not visible.

Fig. 18 shows the observation of the shape-changing motion of a 3-D sphere in 4-D space-time. As shown in Fig. 18 (a), when we can have visual contact with the shape-changing 3-D sphere over time from an eye point on the spatial axis, the displayed 3-D scene can be expressed as an assembly of shape-changing 3-D spheres along the time direction. Therefore, we can understand that the 3-D sphere gradually contracts and newly expands. Moreover, by turning out the visual axis of Fig. 18 (a), the sequential 3-D spheres are cut off by the 4-D viewing field and the occluded past 3-D spheres appear as shown in Fig. 18 (b).

As the proposed system can slice 4-D spatio-temporal information from various positions and directions in 4-D space-time, not only the position on the time axis but also the position including 3-D space and time, we can expand the framework of 3-D shape morphing techniques for computer graphics.

5. Conclusions

We have constructed an interactive environment to travel in 4-D space-time via human actions for the intuitive understanding of 4-D space-time, and proposed various visualizations of 4-D spatio-temporal information to induce a novel understanding of moving 3-D objects in 4-D space-time. In this environment, we were able to flexibly observe a variety of 4-D spatio-temporal information and the world line from an arbitrary 4-D eye point and visual axis by controlling the 4-D viewing field. Moreover, in 4-D space-time, we had visual contact with the dynamic characteristics of a 3-D object that cannot generally be seen in 3-D space, such as 3-D motion, 3-D velocity and 3-D acceleration. We could also trace a shape-changing 3-D object which was occluded by itself through time. We expect that the developed 4-D space-time visualization system will be effective for 3-D time-series data analysis, robot motion planning, flight management and guidance systems, and interference design for moving 3-D objects.

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