Watermarking Using Wavelet Transform and Genetic Algorithm for Realizing High Tolerance to Image Compression

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Summary
Recently, several digital watermarking techniques have been proposed for hiding data in the frequency domain of image signals. However, little attention has been given to the optimal position in the frequency domain for embedding watermarks. We have attempted to improve the performance of both the extraction of watermarks and the quality on an image after compressed by JPEG technique by creating a multicriteria optimization problem for deciding the positions of watermarks in the frequency domain and obtaining an approximately optimum solution to this problem. The approximately optimum solution is obtained using the genetic algorithm. The experimental results show that the proposed method generates watermarked images of good quality and high tolerance to JPEG compression. In addition, the improvement of security has been achieved using the characteristic secret key to embed and extract watermark information. The reported method with the wavelet transforms is used to embed and extract watermark information.

Key words: watermark, wavelet transforms, secret key, genetic algorithm, optimization

1. Introduction

Recent progress in digital media and digital media distribution systems, such as the Internet and cellular phones, has enabled us to easily access, copy, and modify digital content, such as electric documents, images, sounds, and videos. Under these circumstances, techniques to protect the copyrights of digital data and prevent unauthorized duplication or tampering of this data are strongly desired.

Digital watermarking (DW) is a promising method for copyright protection of digital data. Several studies have investigated digital watermarking\(^1\)\(^-)\(^7\). These digital watermarking methods embed information in the frequency domain of an image using the discrete cosine transform (DCT)\(^1\)\(^)\(^5\), or the discrete wavelet transform (DWT)\(^2\)\(^-)\(^4\)\(^)\(^6\)\(^)\(^7\). According to Inoue et al.\(^2\), the DW must satisfy the following three properties. First, the embedded DW does not distort visually the original image and should be perceptually invisible. To make the embedded DW invisible on the image, a frequency domain based approach is thought to be better than spatial domain based techniques\(^6\).

Second, the DW should be difficult for an attacker to remove. Third, the DW should also be robust with respect to common signal processing and geometric distortions, such as digital-to-analog and analog-to-digital conversion, smoothing, compression, rotation, noise addition, geometric translation, and cropping.

In addition, there are a number of problems associated with techniques in which the DW is extracted by relying on comparison between the watermarked and original images, as follows\(^8\).

It is assumed that person G having the copyright for the original image \(O\) created the image \(O'\) by embedding a watermark \(W_G\) into \(O\), and distributed image \(O'\). Thereafter, pirate B could produce the image \(F\) by subtracting another watermark \(W_B\), which was made by pirate B, from the image \(O'\). Then, after extracting the watermark \(W_B\) from the image \(O'\), pirate B might be able to assert that the original image of image \(O'\) was image \(F\). Therefore, the copy-
right of person G might not necessarily be protected against pirate B. Accordingly, it is necessary that the DW can be extracted from the watermarked image without using the original image. Therefore, DW techniques\(^2\),\(^4\),\(^6\) have been developed in which the original images are not used in extracting the DW.

We have attempted to develop a method in which (1) the DW can be sufficiently extracted from the watermarked image, even after compression and (2) the quality of the image remains high after embedding the DW. However, there generally is a trade-off relation between these two properties. In this paper, we focus on the trade-off relation between these two properties and attempt to overcome this essential difficulty of the DW by optimizing the positions of the DW in the frequency domain.

Recently, digital images distributed over the Internet or cellular phone system are often modified by compression, which is one of the easiest and most effective ways to defeat a DW without significantly deteriorating the quality of image. At present, to our knowledge, there is no practical technique for protecting the copyright of a digital image that have been highly compressed.

We have attempted to improve both the performance of extraction of DW and the quality of the image by developing a multicriteria optimization problem for deciding the positions of DWs in the frequency domain and obtaining an approximately optimum solution for the problem. In this paper, a method for embedding a DW using DWT\(^4\) and a genetic algorithm (GA)\(^9\),\(^10\) for realizing high tolerance to compression developed by the Joint Photographic Experts Group (JPEG), which is the most popular compression techniques, has been proposed. In addition, an improvement in the security of the watermarked image has been attempted by using a characteristic secret key to embed and extract the DW.

2. Preliminaries

2.1 Wavelet Transform of Image Signals

Hierarchical decomposition of DWT has been widely used in image compression and other image processing techniques. The image is first decomposed into four subbands, 1LL, 1LH, 1HL, and 1HH. The subbands labeled 1LH, 1HL, and 1HH represent the finest scale wavelet coefficients. The subband 1LL is further decomposed in order to obtain a coarser scale of wavelet coefficients. The above process continues until the final scale wavelet coefficients are obtained. Fig. 1 describes level 3 decomposition, in which the image is decomposed into ten subbands for three scales.

In general, the wavelet coefficients on the three domains described as HH, HL and LH are called elements of multi-resolution representation (MRR), whereas the wavelet coefficients of LL are called elements of multi-resolution analysis (MRA). For further information on the DWT, see Appendix.

2.2 Digital Watermarking Method Used in the Present Research

In the present research, we use the method for embedding the DW using the DWT\(^4\). This method does not require the original image for extracting the DW and is robust against JPEG compression\(^4\). We can estimate the parameters used for the DW extraction from the watermarked image. The details of this method are described in Section 3..

2.3 Genetic Algorithm

The GA, which is based on biological evolution, has been applied for solving the mathematical programming problem. The solution of the mathematical programming problem is expressed as a genotype. In each generation, there is a population composed of several individuals identified by their genotypes. The basic idea of GA is that if the number of better individuals is increased by generation updating, an optimum or approximately optimum solution, as expressed by an individual, will eventually be obtained.
3. Wavelet Domain Digital Watermarking Based on Threshold-variable Decision

It is known that the histogram of the wavelet coefficients of each domain at MRR parts has a distribution with the center at the value of almost 0 when the DWT is performed on a natural image. In the present research, a technique that exploits the above phenomena is used for embedding the DW on the wavelet coefficients of the MRR domain. The procedure is described below.

3.1 Embedment of Watermark Information
3.1.1 Conversion from RGB to YCrCb Forms

In general, human eyes are more sensitive to changes in brightness than changes in color. Therefore, in compressing an original image, it is practically acceptable to approximate color elements in images more roughly than brightness elements. For image compression using certain techniques, such as JPEG, the three intensities related to red, green, and blue in the image are first converted into intensities of brightness (Y element) and color (Cr and Cb elements). The intensities of color are then quantized more roughly than that of brightness, and the symbols are then coded by a technique such as entropy coding.

The following equation is used when converting the elements of image from RGB to YCrCb elements:

\[
\begin{pmatrix}
Y \\
C_r \\
C_b
\end{pmatrix}
= \begin{pmatrix}
0.229 & 0.587 & 0.114 \\
-0.16875 & -0.33126 & 0.5 \\
0.5 & -0.41869 & -0.08131
\end{pmatrix}
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
\]

Conversely, when converting the elements of image from YCrCb to RGB elements, (2) is used.

\[
\begin{pmatrix}
R \\
G \\
B
\end{pmatrix}
= \begin{pmatrix}
1.0 & 0 & 1.402 \\
1.0 & -0.34413 & 0.5 \\
0.5 & 1.772 & 0
\end{pmatrix}
\begin{pmatrix}
Y \\
C_r \\
C_b
\end{pmatrix}
\]

3.1.2 Setting of Parameters

Since the image compression might have a greater influence on the values of Cr and Cb elements than that of the Y element, the DWT for preparation of DW is performed on the Y element, and then the histogram of coefficients \( V \) in each domain for DW is obtained. Fig. 2 shows a schematic diagram of the histogram of the wavelet coefficients \( V \).

The values of \( \text{Th(minus)} \) and \( \text{Th(plus)} \) shown in Fig. 2 are decided such that the wavelet coefficients \( V \) composing \( S_m \), which is the number of wavelet coefficients \( V \) having a negative or zero value, are equally divided into two groups at \( \text{Th(minus)} \), and the wavelet coefficients \( V \) composing \( S_p \), which is the number of wavelet coefficients \( V \) having a positive value, are equally divided into two groups at \( \text{Th(plus)} \). Next, the values of \( T1, T2, T3, \) and \( T4 \), which are parameters for controlling the embedment strength, are obtained as follows.

(a) \( T1 < \text{Th(minus)} < T2 < 0 < T3 < \text{Th(plus)} < T4 \)

(b) The value of \( ST_1 \), which is the number of wavelet coefficients \( V \) having values from \( T1 \) to \( \text{Th(minus)} \), is equal to \( ST_2 \), which is the number of wavelet coefficients \( V \) having values from \( \text{Th(minus)} \) to \( T2 \). This condition is abbreviated as \( ST_1 = ST_2 \).

(c) The value of \( ST_3 \), which is the number of wavelet coefficients \( V \) having values from \( T3 \) to \( \text{Th(plus)} \), is equal to \( ST_4 \), which is the number of wavelet coefficients \( V \) having values from \( \text{Th(plus)} \) to \( T4 \). This condition is abbreviated as \( ST_3 = ST_4 \).

(d) \( ST_1/S_m = ST_3/S_p \)

The values of both \( ST_1/S_m \) and \( ST_3/S_p \) are set to be 0.2 in the present study, according to Reference 5.

![Fig. 2 Schematic diagram of the histogram of MRR wavelet coefficients](image-url)
3.1.3 Embedment of Watermark Information

The wavelet coefficients of MRR are rewritten according to the following rules in embedding the DW. Here, $W_i$ denotes one of wavelet coefficients.

(a) In the case that the bit $W_i$ in DW $W$ is 0,
   - When $V_i < T_2$, $V_i$ is changed to be $T_2$.
   - When $V_i > T_3$, $V_i$ is changed to be $T_3$.
   - When $T_2 \leq V_i \leq T_3$, $V_i$ is kept.

(b) In the case that the bit $W_i$ in DW $W$ is 1,
   - When $T_1 < V_i \leq 0$, $V_i$ is changed to be $T_1$.
   - When $0 < V_i < T_4$, $V_i$ is changed to be $T_4$.
   - When $V_i \leq T_1$ or $V_i \geq T_4$, $V_i$ is kept.

The wavelet coefficient $V_i$ is set in the range of $T_2 \leq V_i \leq T_3$ when the bit $W_i$ in DW $W$ is 0, whereas the wavelet coefficient $V_i$ is set in the range of $V_i \leq T_1$ or $V_i \geq T_4$ when the bit $W_i$ in DW $W$ is 1. The frequency of change of $V_i$ toward the inside is expected to be approximately equal to that toward the outside when the number of 0 bits of DW is approximately the same as the number of 1 bits.

3.1.4 Generation of Watermarked Image

The inverse DWT (IDWT) is performed on wavelet coefficients $V'_i$ embedded with the watermark to obtain the brightness signal $Y'$ with the watermark, and the color image with the watermark is generated from $Y'$ and color elements (Cb, Cr).

3.2 Detection of Watermark

3.2.1 Conversion from RGB to YCrCb

The brightness element $Y'$ obtained from the RGB watermarked image is converted into wavelet coefficients. The wavelet coefficient in the region in which watermark information is embedded is described as $V''$.

3.2.2 Presumption of Parameters

For the histogram of $V''$, the two parameters of $Th'(\text{minus})$ and $Th'(\text{plus})$, which correspond to $Th(\text{minus})$ and $Th(\text{plus})$, respectively, for the histogram of $V$ before embedding the watermark, are obtained in the same manner as that for $Th(\text{minus})$ and $Th(\text{plus})$ mentioned in Section 3.1.2. $Th'(\text{minus})$ and $Th'(\text{plus})$ can be treated as presumptive values for $Th(\text{minus})$ and $Th(\text{plus})$, respectively, because it is expected that the distribution of the histogram of $V''$ might be approximately the same as that of $V$ before embedding the watermark.

The watermarked image might suffer from certain types of image processing, such as compression, so that the difference between the distribution of the histogram of $V''$ after image processing and that of $V$ before embedding the watermark might not be negligible. In such a case, it may not be persuasive that $Th'(\text{minus})$ and $Th'(\text{plus})$ can be treated as presumptive values for $Th(\text{minus})$ and $Th(\text{plus})$, respectively.

3.2.3 Detection of Watermark Information

When the wavelet coefficient $V''_i$ is in the range of $Th'(\text{minus}) \leq V''_i \leq Th'(\text{plus})$, the corresponding bit $W''_i$ in the measured watermark $W''$ is judged to be 0. When the wavelet coefficient $V''_i$ is in the range of $V''_i < Th'(\text{minus})$ or $V''_i > Th'(\text{plus})$, the corresponding bit $W''_i$ in the measured watermark $W''$ is judged to be 1.

4. USE of Secret Key

In the present research, the DW is produced using a secret key $S(N)$, which is composed of a row of $N$ integers selected randomly in the integer range of from 0 to $2^M - 1$, as shown by the example below ($N = 100$, $M = 10$). Both $N$ and $M$ are integers.

$$S(100) = \{82, 34, \ldots, 324\}$$ (3)

Here, $N$ is the number of bits of the watermark, and the number of wavelet coefficients in the region for embedding the DW is equal to $2^M$. Each value and order of numbers in $S(N)$ indicate the position of each bit of DW in the region for embedding the DW. Here, the position in the region is expressed as a one-dimensional coordinate generated by raster scan, as described in Fig. 3, where the size of the region is $2^3 \times 2^5$. For example, the first number, 82, in $S(100)$ means that the first bit of DW is set for the wavelet coefficient at the coordinate of 82 in the DW region.

The selected wavelet coefficient in the target region is changed according to the value of each bit of DW, the secret key, and the shift value of coordinate, which is described later, as described in Sec-

![Fig. 3](image-url) Position for embedding DW in the selected DWT region when the number in the secret key is 80

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In Section 3.1.3, the value of each bit of DW and the secret key decide an initial bit pattern of the DW on the positions in the region for DW. The positions of wavelet coefficients decided by the above mentioned secret key \( S(100) \) is simply demonstrated in Fig. 4, where the size of the region is \( 2^5 \times 2^5 \), then numbers \((1, 2, 3, 4, 5, 6, 98, 99, 100)\) denote the order of bits of DW and the numbers from 7 to 97 and their corresponding positions are not described for the sake of simplicity.

The coordinate shift is performed by generating \( S'(N) \) such that the difference between the minimum value among all numbers in \( S(N) \) and the shift value is added to all values in \( S(N) \). For example, it is assumed that the minimum value among all numbers in \( S(100) \) above is 34 of the second number and the shift value is 166. As a result,

\[
S'(100) = \{214, 166, \ldots, 456\}
\]  

Position 166 is then translated into two-dimensional coordinates \((5, 5)\), where \((0, 0)\) is the top-left corner. All positions for DW are shifted in a similar manner to that demonstrated in Fig. 5.

5. Optimal Watermarking Problem

To minimize the error \( e(x, y) \) caused by watermarking and compression by the JPEG technique and to maximize the detection rate \( d(x, y) \) of DW after compressed by JPEG technique under the restriction condition on \( d(x, y) \), an optimum watermarking problem is formulated as follows.

\[
P \text{ Minimize } e(x, y) \tag{5}
\]

\[
\text{Maximize } d(x, y) \tag{6}
\]

\[
\text{Subject to } d(x, y) \geq a \tag{7}
\]

\[
e(x, y) = e_R(x, y) + e_G(x, y) + e_B(x, y) \tag{8}
\]

\[
e_I(x, y) = \sum_{k=1}^{m} \sum_{i=0}^{7} \sum_{j=0}^{7} (f_{l,k}(i, j) - f_{l,k}^*(x, y, i, j))^2 \frac{64}{m} \tag{9}
\]

where \( x \) and \( y \) denote the integer variables indicating the element number on the entire matrix obtained by DWT, as shown in Fig. 1, an original image having RGB components is transformed into an image having YCrCb components, followed by DWT on the Y component. A secret key, described in Section 4, is randomly generated for each image. The DW that started at the \((x, y)\) position on the entire matrix is embedded in the selected region for the DW. As described in Section 4, \((x, y)\) is decided by the secret key and the shift value. As decision variables, we use the coordinates \( x, y \) rather than the shift value of the one-dimensional coordinate. An image having YCrCb components obtained by IDWT to the wavelet coefficient matrix of watermarked Y components is then transformed into an image having RGB components, which is then compressed by the JPEG technique using the discrete cosine transformation (DCT). Here, \( f_{l,k} \) and \( f_{l,k}^* \) denote the \( l \) \((l = R, G, B)\) component of the original partial image at block \( k \) and the corresponding decoded JPEG partial image, respectively. Here, \( m \) denotes the number of blocks having 8 x 8 pixels, and \( a \) is the user-specified constant. The extraction of DW is performed on the Y component of the watermarked image compressed by the JPEG technique.

6. Experiment

Five standard images (Earth, Couple, Mandrill,
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Lena, and Aerial) in 24-bit BMP form having 256 × 256 pixels are prepared for evaluation of the proposed method. The images used in this experiment are selected from Standard Image Data-BAse (SIDBA)\(^1\). We use the Daubechies wavelet for DWT.

Moreover, a string composed of 100 randomly generated bits is used as watermark information. The approximately optimum solution for \(P\) is obtained for each image. The DWT for each image is performed to level 3. Moreover, since the JPEG compression causes a larger error on the higher-frequency elements, six areas described as 1HL, 1LH, 1HH, 2HL, 2LH, and 2HH are not used as the region for DW. The region described as 3LL is also not used as the region for DW, because DW for the 3LL region causes the image to have poor quality. Accordingly, the candidate regions for DW are limited to 3HL, 3LH, and 3HH.

Two components of the coordinates, which are those obtained by the shift, as described in Fig. 5, are treated as decision variables for each image. In addition, the integer indicating one of three regions of wavelet coefficients is also treated as a decision variable. The decision variables, \(x, y\) in \(P\) are given by the integer indicating one of three regions of wavelet coefficients and the two components of coordinate on the region.

6.1 GA Coding

The coordinate \((x, y)\) of the starting position of DW is expressed by \((x_i, y_i)\), where \(i = 0, 1, 2\) corresponds to regions 3HL, 3LH, and 3HH, respectively, and the origins of the coordinate system are located at the top-left corner of each region.

A gene is expressed by a bit having a value of 0 or 1. Accordingly, each chromosome is composed of the row of bits. The total number of bits is 12, among which the two bits of the highest and second highest ranks are associated with the region of wavelet coefficients, and the ten subordinate bits are associated with the starting position of DW in the region of wavelet coefficients in the binary expression (Fig. 6).

When an individual associated with the region that does not exist as one of wavelet coefficients is generated in the GA process, the individual is considered to have a fatal gene and therefore is deleted. Another individual is then newly generated.

6.2 Procedure

The procedure used in the experiment is as follows.

**Step 1** An initial population consisting of several individuals is generated. Each individual has genes consisting of 12 bits. In the process of generating an initial population having a given number of individuals, the individual that does not meet the restriction or that has a fatal gene is killed as soon as it is produced, and another individual is newly generated.

**Step 2** The embedment of DW according to the condition decided by each individual, the image compression with the JPEG technique and the detection of DW are performed. The fitness, described later herein, is then calculated.

**Step 3** Roulette wheel selection is performed. Two-point crossover is then performed with a pair of parent-individuals with a certain probability. The division point is selected randomly. At the division points, the chromosome is divided into three parts, and each part of the chromosome is then combined with that from a different parent in order to obtain a pair of child-individuals. A one-point mutation is then performed with a certain probability. The mutation gene is randomly selected. The allele of the selected gene is exchanged for another.

The approximately optimum solution, which is defined as the solution having the highest fitness through all generations, is obtained by repeating the process from Steps 2 to 3 until reaching the final generation. However, when the highest fitness is not updated for 10 generations, the processing is terminated and the solution having the highest fitness up to this generation is treated as the approximately optimum solution.

6.3 Condition

As the GA condition, 20 generations, 30 individuals, two-points crossover with the rate of 0.6 and one-point mutation with a rate of 0.1 are used. The GA calculation is terminated when the highest value
of fitness function is not renewed for 10 generations. The JPEG compression is performed on the condition of quality 90 using the open-source software\textsuperscript{12} of Independent JPEG Group. Moreover, the lower bound of the detection rate $a$ described for restriction condition (7) is set to be 90, and the fitness function $f$ is defined as

$$f = \frac{d(x, y) - a + 1}{e(x, y)},$$

(10)

where $d(x, y)$ and $e(x, y)$ are introduced in Section 5. Moreover, for the performance evaluation of the watermarked image obtained by the proposed method, the following four tests are performed.

(a) JPEG compression with quality from 5 to 100.

(b) JPEG2000 compression with various compression levels using Jasc PaintShop Pro8.

(c) The comparison with the case (described as non-optimized), in which the positions for DW are decided by the region used in Reference\textsuperscript{4} and the secret key used in this experiment.

(d) The comparison with the performance of watermarked images produced under the condition that the DW embedment positions are decided by generating at random the individual while neglecting the restriction (7).

Condition (a) is used for evaluation under the condition that the watermarked image is distributed and received compression of varying degrees. Moreover, condition (b) is adopted because the JPEG2000 compression is expected as the image format at the next generation.

6.4 Calculation Environment

The experiment is performed in the following environment for computation: personal computer; Dell Dimension8400 (CPU: Pentium IV 3.4 GHz, main memory: 2 GB), OS; Microsoft WindowsXP, Development language; Microsoft Visual C++6.0.

6.5 Results

The experimental results for image Lena are described in this section. Figs. 7 (a) and (b) show the images before and after DW embedment, respectively. The DW is embedded according to the condition given by the approximately optimum solution. The DW is embedded according to the condition given by the approximately optimum solution. The difference between Figs. 7 (a) and (b) is perceptually invisible. Fig. 8, in which the white square indicates the relation between the compression condition used in this experiment and the detection rate of DW, shows the DW tolerance on image Lena with respect to the compression by JPEG and JPEG2000. The watermarked image produced by the proposed method has high tolerance against the compression by not only JPEG but also JPEG2000. Fig. 9 compares the proposed method and the method without optimization for DW tolerance on image Lena with respect to compression by JPEG. As shown in Fig. 9, the proposed method is very effective for DW tolerance with respect to JPEG compression. As shown
Fig. 9 Comparison of the proposed method with and without optimization for image Lena with respect to the relation between compression and detection rates.

Fig. 10 Comparison between the proposed method (white square) and random embedment (others) for image Lena with respect to detection rate and error.

in Fig. 10, the performance of the proposed method, which is indicated by the white square, is better than the performance obtained by embedding DW randomly, which is described by 100 dark circles. The DW starting position obtained by this experiment is $x_1 = 31$, $y_1 = 30$ of the 3LH region, and the detection rate is 95% under the compression condition of quality 90. Moreover, the value of the mean square error $e(x^*, y^*)$ expressed simply as error in Fig. 10, where $x^*$, $y^*$ indicates the approximately optimum solution, is 67.75. Under the compression condition of quality 90, the average detection rate of DW for these four images are 99.25%. In the case of JPEG quality 90, for Earth, Couple, Mandrill, and Aerial, the mean square errors $e(x^*, y^*)$ are 57.44, 54.50, 228.2, and 182.7, respectively, where $x^*$, $y^*$ indicates the approximately optimum solution. In addition, results similar to those shown in Figs. 9 and 10 are obtained for these four images. The average processing time for obtaining the approximately optimum solution for five images is 23.56s per image.

7. Conclusions

A method for embedding DW using DWT and GA for realizing high tolerance to compression by JPEG has been proposed. The proposed method enables us to embed DW in the approximately optimum manner for each image. Moreover, the approximately optimum solution and the initial pattern of DW are used as a secret key in extracting DW. The performance of the proposed method was demonstrated to be better than that of the conventional method.

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12) ftp://ftp.uu.net/graphics/jpeg/

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Appendix

A. Wavelet Transform

The DWT and the IDWT are performed according to the following procedure.
A.1 Calculation of the Wavelet Decomposition Coefficient

The original \(s_{u,v}^{(0)}\) image, which is used as the wavelet decomposition coefficient matrix at the level 0, is decomposed into the MRR and the coarsest approximation by applying DWT at several times, as shown in Fig. 1, where H and L denote the high- and low-frequency elements, respectively. The wavelet decomposition coefficient matrix \(s_{u,v}^{(j)}\) at the level \(j\) is decomposed into four wavelet decomposition coefficient matrices at the level \(j + 1\) by using (A–1), (A–2), (A–3), and (A–4).

\[
s_{u,v}^{(j+1)} = \sum_{k} \sum_{l} p_{k-2u} q_{l-2v} s_{k,l}^{(j)} \quad (A-1)
\]

\[
w_{u,v}^{(j+1,h)} = \sum_{k} \sum_{l} p_{k-2u} q_{l-2v} s_{k,l}^{(j)} \quad (A-2)
\]

\[
w_{u,v}^{(j+1,v)} = \sum_{k} \sum_{l} q_{k-2u} p_{l-2v} s_{k,l}^{(j)} \quad (A-3)
\]

\[
w_{u,v}^{(j+1,d)} = \sum_{k} \sum_{l} q_{k-2u} q_{l-2v} s_{k,l}^{(j)} \quad (A-4)
\]

where \(u\) and \(v\) denote the horizontal and vertical directions, respectively, and \(p_k\) and \(q_k\) denote the scaling and wavelet sequences, respectively. \(s_{u,v}^{(j+1,h)}\) denotes the development coefficient obtained by operating the scaling function in the direction of the horizontal axis and the wavelet in the vertical axis. \(w_{u,v}^{(j+1,v)}\) denotes the development coefficient obtained by operating the wavelet in the direction of horizontal axis and the scaling function in the vertical axis. Moreover, \(w_{u,v}^{(j+1,d)}\) denotes the development coefficient obtained by operating the wavelet in the direction of both the horizontal and vertical axes. The development coefficients at the level \(J\) are obtained by repeatedly using (A–1), (A–2), (A–3), and (A–4) from \(j = 0\) to \(j = J - 1\). In this method, the image must be a square composed of \(2^L \times 2^L\) \((J \leq L)\) pixels. In the present study, the image is decomposed by the DWT to level 3, as shown in Fig. 1, where the original image \(s_{u,v}^{(0)}\) is decomposed into \(s_{u,v}^{(1)}\) expressed by 1LL, \(w_{u,v}^{(1,v)}\) expressed by 1HL, \(w_{u,v}^{(1,h)}\) expressed by 1LH, and \(w_{u,v}^{(1,d)}\) expressed by 1HH. Here, \(H\) and \(L\) denote the high- and low-frequency components, respectively.

A.2 Re-composition of Signal

The signal is re-composed repeatedly using (A–5) from \(j = J - 1\) to \(j = 0\).

\[
s_{u,v}^{(j)} = \sum_{k} \sum_{l} (p_{u-2k} q_{v-2l} s_{k,l}^{(j+1)}) + p_{u-2k} q_{v-2l} w_{k,l}^{(j+1,h)} + q_{u-2k} v_{l-2v} w_{k,l}^{(j+1,v)} + q_{u-2k} q_{v-2l} w_{k,l}^{(j+1,d)}) \quad (A-5)
\]