This paper presents a simulator that graphically presents the influence of rotary-axis geometric errors on the geometry of a finished workpiece. Commercial machining simulation software is employed for application to arbitrary five-axis tool paths. A five-axis kinematic model is implemented with the simulator to calculate the influence of rotary-axis geometric errors. The machining error simulation is demonstrated for 1) the cone frustum machining test described in ISO 10791-7:2015 [1], and 2) the pyramid-shaped machining test proposed by some of the authors in [2]. The influences of the possible geometric errors are simulated in advance. By comparing the measured geometry of the finished workpiece to the simulated profiles, major error causes are identified without numerical fitting to the machine’s kinematic model.

Keywords: five-axis machine tools, machining error, simulator, kinematic model, machining test

1. Introduction

All machine tools with two rotary axes that tilt and rotate a tool and/or the workpiece, in addition to the three orthogonal linear axes, are collectively called five-axis machine tools. In general, geometric errors in the five-axis kinematics are caused by the following contributors:

1. Error motions in each axis, e.g. the angular positioning and radial error motions of a rotary axis.

2. Assembly errors of each axis relative to the others, e.g. the position error of a rotary axis from its nominal position and the squareness error of a rotary axis relative to a linear axis.

3. Setup errors associated with a tool and workpiece, e.g. calibration error in tool length.


Contributors 1 and 2 can be minimized by good machine design, sufficient mechanical adjustment, or numerical compensation by machine tool builders. However, contributors 3 and 4 depend on the machine user’s environment and operation. The thermal deformation of the machine structure, typically caused by the heat generated in spindle and feed drive motors or by environmental temperature, is clearly a source of major error for any machine tools. As reviewed in [3–5], extensive studies have reported on the measurement, modelling and compensation of thermal errors.

When the finished workpiece does not meet the given dimensional or geometric tolerances, a machine user must find the cause of these deviations by investigating the machine’s geometric errors. Many “indirect” measurement methodologies have been proposed recently, as reviewed in [6, 7]. According to Schwenke et al. [6], “direct” measurement represents the analysis of a single error, such as the angular positioning or radial error motion of an individual rotary axis. “Indirect” schemes measure the location of the tool center point (TCP) as the superposition of these errors, and then separate each error motion from the TCP location. Typical indirect measurement methodologies for five-axis machines include the R-test [8] and probing-based schemes [9]. These schemes enable a machine tool user to measure the geometric errors of a machine in the working environment in an efficient, automated manner.

In conventional three-axis machining applications, the influences of linear axis geometric errors on the finished workpiece’s geometry are easily understood. For example, the squareness error between two linear axes is often apparent as the squareness error of two faces in the finished workpiece. In five-axis machining applications, this relationship is more difficult to understand. The five-axis kinematic model describes this relationship mathematically. It was introduced in the 1980s [10, 11] and has been essential in many previous works related to indirect error measurement for both three- and five-axis machine tools. The application of the five-axis kinematic model to the simulation of machining geometric accuracy is present
in many previous works, e.g. the analysis of the cone frustum machining test [12–15], the S-curve test [16], the pyramid-shaped five-axis test piece [2, 15], and a turbine blade [17]. Some works consider the location errors of the rotary axes, while others consider the dynamic errors of rotary axes. In both cases, the kinematic model is crucial.

The objective of this paper is the presentation of a simulator that graphically shows the influence of rotary axis geometric errors on the workpiece’s geometry, as created by arbitrarily given tool paths. It must be emphasized that the five-axis kinematic model itself is well developed in the literature. This paper’s contribution addresses its implementation in commercial machine simulation software, permitting its application to arbitrary numerical control (NC) programs. The paper also demonstrates the application of the error simulator to simple and practical error diagnoses using the finished workpiece’s geometric error. Sato et al. [18] presented a similar implementation of the kinematic five-axis model with commercial machine simulation software. They focused on the influence of rotary axis dynamic error motions on the finished workpiece’s surface quality. Hasebe et al. [19] presented 3D machining error simulator software, with consideration of the machine’s dynamic error motions caused by e.g. axis reversal or acceleration at different curvatures. Simulation of the influence of tool geometry [20] or vibration due to the cutting force [21] on the finished workpiece’s surface texture are also present in the literature. This paper focuses on the influence of static position and orientation errors of rotary axis average lines on the 3D geometry of the finished workpiece.

2. Machining Error Simulator

2.1. Five-Axis Kinematic Model

The machine’s kinematic model formulates the tool center point (TCP) position in the workpiece coordinate system, when either linear or rotary axes have geometric errors. The formulation of the kinematic model is explained in many previous publications, e.g. [2, 11, 13, 22]. However, it is briefly reviewed in this subsection, since it provides the basis for the simulator presented in this paper.

In ISO 230-1 [23], the axis average line of a rotary axis is defined as “the straight line representing the mean location and orientation of its axis of rotation.” Position and orientation errors of a rotary axis average line, called location errors in ISO 230-7 [24], are clearly among the most fundamental error contributors in the five-axis kinematics. Table 1 shows location errors for the machine configuration in shown Fig. 1.

They only represent the “average” position or orientation of a rotary axis. The axis of rotation may also change its position and orientation during rotation. Such an error motion can be parameterized by position-dependent geometric errors [13]. This paper only addresses geometric errors of rotary axis average lines, as shown in Table 1, but the model can be extended easily to position-dependent geometric errors (this extended formulation can be found in [13]).

The workpiece coordinate system is the coordinate system attached to the rotary table, with an origin set at the nominal intersection of the B- and C-axes. Suppose that the nominal TCP in the workpiece coordinate system is given by \( \vec{w}_q \in \mathbb{R}^3 \). Throughout this paper, the left-side superscript \( w \) represents a vector in the workpiece coordinate system. When the B- and C-axes are nominally indexed at \( b_i \) and \( c_j \), the actual TCP position in the workpiece coordinate system, \( \vec{w}_q \in \mathbb{R}^3 \), under the geometric errors of the rotary axis average lines shown in Table 1, is given by:

\[
\begin{bmatrix}
\vec{w} \\
1
\end{bmatrix} = (T_w)^{-1} \tau_w \begin{bmatrix}
\vec{w}^*_q \\
1
\end{bmatrix} \quad \ldots \ldots (1)
\]

where \( T_w \in \mathbb{R}^{4 \times 4} \) is the homogeneous transformation matrix (HTM) representing the transformation from the workpiece coordinate system to the machine coordinate system:

\[
\tau_w = T_{bc} \quad \ldots \ldots \ldots \ldots \ldots (2)
\]
\( T_b = D_x(\delta_{xBR}^0)bRF \)
\( D_a(\alpha_{BR}^0)D_c(\gamma_{BR}^0)b(-b_i) \)
\( b_T = D_x(\delta_{xBR}^0)bRF \)
\( D_a(\alpha_{BR}^0)D_c(\gamma_{BR}^0)b(-c_j) \)

where \( D_x(*) \in R^{3 \times 4} \) denotes the HTM representing the translation in X, Y or Z and the rotation around X, Y, or Z (see e.g. [22] for their formulation). \( wT^r \in R^{3 \times 4} \) represents the nominal transformation, i.e.

\( r_{T_b} = D_b(-b_i)D_c(-c_j) \) \hspace{1cm} (3)

The linearized representation of the model (1) is given in [2].

The TCP trajectory in the workpiece coordinate system is copied to the finished workpiece’s geometry. Therefore, for the arbitrarily given nominal TCP trajectory, represented by \( wq^i \) (\( i = 1, \ldots, N \)), the influence of rotary axis geometric errors on the finished workpiece’s geometry can be simulated by using model (1).

2.2. Implementation of the Kinematic Model with Commercial Machining Simulation Software

The objective of the present software is to simulate and graphically present the geometric error of the finished workpiece generated by an arbitrarily given tool path. For the graphical presentation, commercial machining simulation software is adopted. In this study, VERICUT ver. 7.3.1 by CGTech [25] is used. This software simulates the material removal for any given NC programs.

It is widely used in manufacturing to detect errors in NC programs, potential collisions, and leftover or overcut material in the finished workpiece. To our knowledge, no commercial machining simulation software can incorporate with geometric errors or error motions of the machine tool. This paper presents the following procedure to use such a commercial software for the graphical presentation of the finished workpiece’s geometric error.

1. TCP trajectory in the machine coordinate system.

   For the given NC program, represent the nominal TCP trajectory as a set of points in the machine coordinate system, \( q^i \) (\( i = 1, \ldots, N \)), with the tool orientation represented by B- and C-axis angular positions, \( b_i \) and \( c_i \). Throughout this paper, the left-side superscript \( \tau \) represents a vector in the machine coordinate system. The origin of the machine coordinate system is fixed at the nominal intersection of the B- and C-axes, with orientation unchanged by B- and C-axis rotation.

2. Calculation of TCP trajectory in the workpiece coordinate system under given geometric errors.

   The nominal TCP position in the workpiece coordinate system is given by \( wq^i = (T_w^* )^{-1} q^i \), where \( T_w^* \) is given by Eq. (3). Suppose that geometric errors of the rotary axis average lines in Table 1 are given. For each \( wq^i \), calculate the actual TCP position in the workpiece coordinate system, \( wq^i_T \), using Eq. (1).

3. Representation of actual TCP trajectory as the modified NC program.

   Modify the TCP position in the machine coordinate system from \( q^i \) to \( q^i_T \) by:

\[ r_{q^i} = r_{q^i_T} + \epsilon \cdot T_w^* (wq^i - wq^i_T) \] \hspace{1cm} (4)

where \( \epsilon \in R \) is a magnifier constant for graphical presentation in Step 6. Make an NC program using a set of modified TCP points, \( q^i_T \) (\( i = 1, \ldots, N \)).

4. Influence on measurement coordinate system.

   The geometric accuracy of the finished workpiece is defined with respect to the measurement coordinate system. The measurement coordinate system is typically defined based on the position and the orientation of a part of the finished workpiece. Therefore, the influence of rotary axis geometric errors on the measurement coordinate system must also be considered.

   Suppose that the measurement coordinate system is defined from a set of measured points on the workpiece, whose nominal position is represented by \( wq^i_b \) (\( i = 1, \ldots, N_m \)). Calculate their actual positions, \( wq^i_b \), under the given rotary axis geometric errors using Eq. (1). Then, calculate the influence of the actual positions on the position and the orientation of the measurement coordinate system.

5. 3D CAD model.

   Construct a 3D-CAD model representing the nominal geometry of the finished workpiece. Its position and orientation are determined by the position and the orientation of the measurement coordinate system calculated above.

6. Machining simulation:

   On commercial machining simulation software, perform the machining simulation by using the modified NC program from Step 3. Many commercial machining simulation software suits can compare simulated workpiece geometry with the nominal geometry, enabling a user to check the overcut or the left-over in the finished workpiece geometry. In VERICUT ver. 7.3.1 by CGTech, this function is called “AUTO-DIFF” [25]. Import the 3D-CAD model from Step 5 to highlight the geometric error between the simulated workpiece and its nominal geometry. Try a larger value of \( \epsilon \) in Eq. (4) to visually enlarge this geometric difference.

Remark #1: In a commercial machining simulation software, the geometric resolution of the graphical representation of the machined workpiece is determined by the discretized voxel size or computational truncation. The influence of this truncation can be minimized by increasing the magnifier constant, \( \epsilon \), in Eq. (4). To our knowledge, the graphical resolution in VERICUT 7.3.1 is not disclosed.

Remark #2: The influence of rotary axis geometric errors is calculated only at discretized TCP positions, \( q^i_T \), (\( i = 1, \ldots, N \)). Their influence on the trajectory between each discretized point is linearized. When a rotary axis
continuously rotates between the points, a simulation error may be introduced. Careful monitoring of the discretized segment length in Step 1 is necessary.

2.3. Error Diagnosis Using the Present Simulator

In five-axis machining applications, the influence of each rotary axis geometric error on the finished workpiece’s geometry is difficult to intuitively understand. The present simulator helps machine tool users to understand this relationship. This study presents the following scheme to diagnose major error contributors using the measured geometry of the finished workpiece:

- A set of geometric errors of rotary and linear axes that “typically” exist in the machine structure is selected in advance. For the given arbitrary tool path, the influence of every geometric error on the finished workpiece’s geometry is simulated.
- When the actual geometry of the finished workpiece is measured, it is compared with a set of simulated error profiles. By finding the simulated profile that matches the measured profile most closely, the most influential error contributor can be found.

3. Case Studies

3.1. Case Study #1: Cone Frustum Machining Test

ISO 10791-7:2015 [1], as well as NAS (National Aerospace Standard) 979 [26], describes a machining test of a cone frustum test piece. This test has been adopted by many machine tool builders as a final acceptance test for five-axis machine tools. In the literature, many researchers have studied the influence of rotary axis geometric errors on the geometry of the finished test piece, e.g. [12–15]. This subsection demonstrates the application of the present simulator to the cone frustum machining test. The simulation results are similar to those reported in the previous works.

**Figure 2** depicts the nominal geometry of the finished cone frustum test piece. The nominal dimensional parameters are set as follows: the diameter of the bottom surface, \( D = 259.8 \) mm, the half-apex angle, \( \theta = 30^\circ \), the tilt angle, \( \beta = 15^\circ \), the offset of the axis of the cone from the C-axis centerline, \( d = -101.2 \) mm, and its offset from the B-axis centerline, \( p = 261.8 \) mm. The circular disk at the top surface of the test piece is machined at \( B = C = 0^\circ \) with the circular interpolation of the X- and Y-axes, and is used as a reference to define the measurement coordinate system.

When one of the geometric errors of rotary axis average lines shown in **Table 1** is set as 0.01 mm (for position errors) or 0.01° (for orientation errors) and all other errors are zero, the geometric error of the finished test piece’s conical face is simulated by the procedure presented in Section 2.2. **Fig. 3** shows the simulated geometries of the test piece for (a) \( \delta x_{BR} = 0.01 \) mm, (b) \( \delta y_{CB} = 0.01 \) mm, (c) \( \delta z_{BR} = 0.01 \) mm, (d) \( \alpha_{BR} = 0.01^\circ \), and (e) \( \beta_{BR} = 0.01^\circ \). The “AUTO-DIFF” function of VERICUT 7.3.1 depicts the geometric error of the finished surface using the colors on the conical face; the correspondences to the geometric error amounts are shown in the color bar in **Fig. 3(f)**. Red represents the largest overcut, while magenta represents the largest leftover. For clearer representation, the red line (“Simulated conical face trajectory (error magnified)” also shows the simulated trajectory of the simulated conical surface, with the error magnified.

It can be observed that \( \delta y_{CB} \) (Fig. 3(a)) and \( \beta_{BR} \) (Fig. 3(e)) have the same influence on the finished test piece’s geometry. This also applies to \( \delta y_{CB} \) (Fig. 3(b)) and \( \alpha_{BR} \) (Fig. 3(d)) (in the opposite direction). Furthermore, \( \delta z_{BR} \) (Fig. 3(c)) has no effect on the finished test piece’s geometry. Therefore, it is clearly not possible to separately identify all geometric errors of the rotary axis average lines using the geometry of the finished cone frustum test piece.

The previous works [12–14] presented similar error profile simulations. The contribution of this paper is the development of software that performs this simulation for an arbitrarily given NC program. To highlight this development, the following subsection demonstrates a different application example.

![Fig. 2. Geometry of a cone frustum test piece in ISO 10791-7:2015 [1].](image-url)
3.2. Case Study #2: Pyramid-Shaped Five-Axis Machining Test

In [2], some of the authors proposed a five-axis machining test in which rotary axis geometric errors could be separately identified by measuring the geometric error of the machined test piece. Analogous tests are found in [1, 15]. This subsection presents the application of the present simulator to this machining test.

(1) Machining test procedure

The machining test proposed in [2] is briefly reviewed.

The machine configuration shown in Fig. 1 is considered. Fig. 4 illustrates the machining test procedure. At $B = C = 0^\circ$, a square-shaped step is machined by a straight end mill with driving of the X- or Y-axis only, as the reference step. Then, the square step is machined at different Z heights for $C = 90^\circ$, 180$^\circ$, and 270$^\circ$. Similarly the square step is machined at every side face at $B = 90^\circ$ and $C = 0, 90, 180, 270^\circ$. This is repeated at $B = -90^\circ$.

Fig. 5 shows the nominal geometry of the finished test piece and the measurement coordinate system, defined using a five-axis machining error simulator.
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\[ \delta x^{0}_{BR} = 1 \mu m \]

\[ \delta y^{0}_{CB} = 1 \mu m \]

\[ \delta z^{0}_{BR} = 1 \mu m \]

\[ \alpha^{0}_{BR} = 0.0005^\circ \]

\[ \gamma^{0}_{BR} = 0.0005^\circ \]

Fig. 6. The pyramid test piece’s simulated geometric error for each geometric error of rotary axis average lines shown in Table 1.

(2) Machining error simulation

When one of the geometric errors of the rotary axis average lines shown in Table 1 is set to 1 \( \mu m \) (for position errors) or 0.0005\(^\circ\) (for orientation errors) and all other errors are zero, the finished test piece’s geometric error is simulated by the procedure presented in Section 2.2. The origin of the test piece’s measurement coordinate system is located at \((X, Y, Z) = (-58.8, -134.6, 205.0) \) mm in the machine coordinate system with an origin at the nominal intersection of the B- and C-axes.

Figure 6 shows simulated geometries for (a) \( \delta x^{0}_{BR} = 1 \mu m \), (b) \( \delta y^{0}_{CB} = 1 \mu m \), (c) \( \delta z^{0}_{BR} = 1 \mu m \), (d) \( \alpha^{0}_{BR} = 0.0005^\circ \), and (e) \( \gamma^{0}_{BR} = 0.0005^\circ \). The geometric error of the machined faces is represented by the color, and its correspondence is shown in Fig. 6(f). Red represents the largest overcut, while magenta represents the largest left-over.

(3) Experiment

The test piece shown in Fig. 5 was machined. Table 2 shows major machining conditions. Fig. 7 shows the experimental setup. The “probed points” shown in Fig. 5 are probed by a coordinate measuring machine (CMM).

Figure 8 shows the geometry of the finished test piece measured by the CMM. The difference between the nominal point (red dot) and the measured point (green dot) is magnified by 200 times. The gray-colored polygon represents the surfaces calculated by using the least-square fit to the measured points. Fig. 9 summarizes the position and orientation errors of each step from the nominal position and orientation. The index numbers corresponds to the numbers shown in Fig. 8. Steps 1 to 4 are on the
Fig. 7. Experimental setup (at $B = -90^\circ$).

top side of the machined test piece and are machined at $B = 0^\circ$ (1: $C = 0^\circ$, 2: $90^\circ$, 3: $180^\circ$, 4: $270^\circ$). Steps 5, 7, 9, and 11 are on side faces, machined at $B = -90^\circ$ (5: $C = 90^\circ$, 7: $180^\circ$, 9: $270^\circ$, 11: $0^\circ$). Steps 6, 8, 10 and 12 are machined at $B = 90^\circ$ (6: $C = 270^\circ$, 8: $0^\circ$, 10: $90^\circ$, 12: $180^\circ$). Step 1, machined at $B = C = 0^\circ$, is the reference step to define the measurement coordinate system and is thus without error.

(4) Observation – error contributor diagnosis

By comparing the measured test piece geometry in Fig. 8 with the set of simulated geometries in Fig. 6, the major error contributors are identified. The observations include:

- In Fig. 8(a), Steps 5 to 12, all machined at $B = 90^\circ$ or $-90^\circ$, have position errors of approximately 30 $\mu$m in the positive Z-direction. This is quantified in Fig. 9(a) (see $\delta_z$ for Steps 5 to 12). Analogous geometry occurs in Fig. 6(c). This indicates that the Z-position error of Steps 5 to 12 is mostly caused by $\delta_z^{0BR}$, the Z-position error of the B-axis average line.

- In Fig. 8(a), Step 2 is displaced in the negative Y-direction by approximately 30 $\mu$m, Step 3 is displaced in the negative Y-direction by approximately 50 $\mu$m, and Step 4 is displaced in the negative Y-direction by approximately 20 $\mu$m. This is also seen in Fig. 9(a) (see $\delta_y$ for Steps 2 to 4). Analogous geometry is found in Fig. 6(b). This indicates that the Y-position error of the C-axis from its nominal position, $\delta_y^{0CB}$, contributes to the Y-position error of Steps 2 to 4.

Some of the authors [2] previously presented an algorithm to numerically identify the geometric errors of rotary axis average lines (Table 1) from the measured geometry of the finished test piece. By applying this algorithm, the rotary axis geometric errors are identified as shown in Table 3. The table shows that $\delta_z^{0BR}$ and $\delta_y^{0CB}$, observed above, are relatively large.

(5) Contribution of the present simulator

The algorithm presented in [2] is based on the numerical fitting of the five-axis kinematic model (Section 2.1) to the test piece’s measured geometry. To apply this algorithm to workpieces of arbitrary geometry, an analytical or numerical parameterization of the relationship between rotary axis geometric errors and the finished workpiece geometry is required. For machine tool users, developing the mathematical formulations for a variety of machining applications is not practically feasible. Instead of this numerical fitting approach, this paper presented a crude diagnosis scheme by simply comparing the finished workpiece’s measured geometry to a set of simulated geometries. It may be difficult to quantitatively estimate error causes, particularly when there are many error causes. However, in helping a user to identify one or two major error causes, the present machining error simulator may be useful.
4. Conclusion

The five-axis kinematic model describing the influence of a machine’s geometric errors on the TCP position has been well known. This paper contributes to the implementation of the model with a commercial machining simulation software to perform a machining error simulation with an arbitrarily given tool paths. As application examples, the present simulator was demonstrated for 1) the cone frustum machining test described in ISO 10791-7:2015 [1], and 2) the pyramid-shaped machining test proposed by some of the authors in [2]. In the case study 2), by comparing the measured geometry of the finished cone frustum with an arbitrarily given tool paths. As application examples, the present simulator was demonstrated for 1) the cone frustum machining test described in ISO 10791-7:2015 [1], and 2) the pyramid-shaped machining test proposed by some of the authors in [2]. In the case study 2), by comparing the measured geometry of the finished cone frustum with an arbitrarily given tool paths.

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