An FAHP-based Quantitative Method for Risk Assessment of Debris-flow Hazards Using Different Fuzzy Numbers

Li-Jeng HUANG¹

¹ Dept. of Civil Engineering, National Kaohsiung University of Applied Science (Kaohsiung, 80778, Taiwan, R.O.C.)
E-mail: ljhuang@kuas.edu.tw

This paper presents the fuzzy analytic hierarchy process (FAHP) for risk assessment of debris-flow occurrence using three different fuzzy numbers. Three layers are involved in the structure of the FAHP: the goal layer, the criteria layer, and the sub-criteria layer. In the criteria and sub-criteria layers, nine major influence factors are grouped into three categories: (1) topological and geological conditions, which includes the influence factors of slope angle, type of deposit, grain size distribution, and surface plants; (2) watershed conditions, which includes effective watershed area and quantity of outflow of sediment; and (3) rainfall conditions, which includes rainfall intensity, duration, and accumulated rainfall. Judgment regarding the relative influence of these factors is based on a nine-level scale used to form the fuzzy reciprocal judgment matrices for evaluating the weighting vectors for each layer. Two cases of debris-flow disasters that occurred in eastern Taiwan were tested using the FAHP; one was a debris flow, and the other a mudslide. The results showed that the proposed FAHP models using the three kinds of fuzzy numbers as well as the associated influence factors and criteria can successfully predict the risk of debris-flow hazard occurrence. Furthermore, the predicted overall risk indices obtained from the FAHP using the three kinds of fuzzy numbers were smaller than those obtained from AHP, but more practical due to consideration of the uncertainty and vagueness involved in natural hazards.

Key words: case studies, debris flow, fuzzy analytic hierarchy process (FAHP), multi-criteria, risk assessment

1. INTRODUCTION

Natural disasters caused by debris flows have been frequent occurrences worldwide in recent years, and many natural and manmade factors have led to these tremendous accidents (Takahashi, 1991). Debris flow is a special type of hyper-concentrated flow composed of mud, clay, sand, gravel, water, air, and some other materials that moves downward due mainly to gravitational force. The development of preventive techniques must be based on the understanding and analysis of the mechanical behavior of debris flows. Thus, prediction and assessment of risk regarding the occurrence of debris flow hazards is becoming increasingly important for many countries.

The occurrence of debris flows greatly depends on local topographic, meteorological, geological, and hydrological conditions. In Taiwan, many disasters caused by debris flows and mud flows have been reported and studied (Jan and Shen, 1993; Lin, 2006; Wu et al., 2006), and the particular reasons for Taiwan's propensity for debris-flow hazards have been summarized in previous studies (Jan, 2000; Huang, 2001).

Extensive overviews of risk modeling and some quantitative methods for risk analysis can be found in relevant books (Haimes, 2009; Vose, 2009). The task of risk assessment is a multi-level, multi-criteria, complex process. Methods based on probability theory rely on a great deal of statistical data, prior probability, model evidence, and a likelihood function, and require long-time experiments. In many real-world cases, especially for debris-flow hazards, this is very difficult. Huang (2014) produced a review paper summarizing previous studies on disaster potential prediction techniques for debris-flow hazards.

In the field of operational research, Saaty (1980) proposed the so-called analytic hierarchy process (AHP) for multiple-criteria decision-making problems. It is also a good approach for risk assessment of problems with multiple criteria for influencing factors. The author attempted to apply AHP to the risk assessment of debris-flow hazards that occurred in Sen-Mu and Hua-San (Huang, 2014) and in Tung-Men and Tung-Shing (Huang et al, 2015), and the results proved useful.

Considering the vagueness involved in life and in engineering problems, Zadeh (1965) first introduced

the theory of fuzzy sets. Subsequently, some researchers attempted to combine fuzzy logic with the analytic hierarchy process, a combination that has been termed the fuzzy analytic hierarchy process (FAHP), and applied it to decision making and safety or risk assessment for commercial and engineering practices.

In the risk assessment of debris-flow hazards, engineers need methods that are easy to operate, adaptive for different criteria, verified by case studies, reliable for use in quantitative prediction, and capable of handling ambiguous risk factors. Integrating AHP and fuzzy set theory to construct a FAHP model for this purpose seems to be a valuable approach.

The present work aimed to apply FAHP to risk assessment of debris-flow hazards using case studies that occurred in eastern Taiwan. Three layers and nine influence factors (criteria) are involved in the structure of FAHP. These nine major influence factors are grouped into three categories: (1) topological and geological conditions, including critical slope, type of deposits, grain size distribution, surface plants; (2) effective watershed area, i.e., area of the watershed, sediment outflow; and (3) rainfall conditions, including intensity, duration, accumulated rainfall. The judgment of the relative influence of each factor is developed based on a nine-level scale to form the reciprocal judgment matrices for evaluating the weighting vectors for each layer. Two linear functions (triangular and trapezoidal) and one nonlinear function (generalized exponential) are included in the FAHP models to account for the vagueness and uncertainty of human judgment. Two real cases of debris-flow hazards that occurred in eastern Taiwan are used for verification of the proposed FAHP models. One was a debris flow that occurred in Tung-Men Village, and the other was a mud flow that occurred in Tung-Shing Village, both are located in Hua-lien County.

2. RISK ASSESSMENT OF DEBRIS-FLOW HAZARDS USING FAHP: THEORETICAL DEVELOPMENT

2.1 Establish a hierarchical model

The top of a hierarchical model is the goal to be achieved by FAHP. The second layer (criteria layer) refers to the proposed major categories of influence factors. Factors in the third layer (sub-criteria layer) support the factors in the second layer.

Table 1 1–9 scaling method for risk assessment.

Risk	Scale	Relationship
Absolutely equal	1	$\tilde{a}_{ij} \quad w_i = w_j$
Equally risky	$\tilde{1}$	$\tilde{a}_{ij} \quad w_i = \tilde{1} w_j$
Slightly more risky	$\tilde{3}$	$\tilde{a}_{ij} \quad w_i = \tilde{3} w_j$
Obviously more risky	$\tilde{5}$	$\tilde{a}_{ij} \quad w_i = \tilde{5} w_j$
Strongly more risky	$\tilde{7}$	$\tilde{a}_{ij} \quad w_i = \tilde{7} w_j$
Extremely more risky	$\tilde{9}$	$\tilde{a}_{ij} \quad w_i = \tilde{9} w_j$
Intermediate values	$\tilde{2}$	$\tilde{a}_{ij} \quad w_i = \tilde{2} w_j$
	$\tilde{4}$	$\tilde{a}_{ij} \quad w_i = \tilde{4} w_j$
	$\tilde{6}$	$\tilde{a}_{ij} \quad w_i = \tilde{6} w_j$
	$\tilde{8}$	$\tilde{a}_{ij} \quad w_i = \tilde{8} w_j$

(a)

(b)

(c)

Fig. 1 Nine-point scales for FAHP using different fuzzy numbers: (a) triangular, (b) trapezoidal, and (c) generalized exponential.

2.2 Set up fuzzy pair-wise comparison matrices

In general, reciprocal comparison matrices are obtained by experts in the field completing a questionnaire. Here, we adopt a 9-level scale, as suggested by Saaty (1980), which is shown in Table 1 and Fig. 1.

A typical reciprocal comparison matrix can be expressed as

$$[\tilde{A}] = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{bmatrix}, \quad (1)$$

where each entry is a fuzzy number defined by the following:

(a) Triangular function:

$$\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij}), \quad (2a)$$

with

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 0, & x \leq l_{ij} \\ (x - l_{ij}) / (m_{ij} - l_{ij}), & l_{ij} \leq x \leq m_{ij} \\ (u_{ij} - x) / (u_{ij} - m_{ij}), & m_{ij} \leq x \leq u_{ij} \\ 0, & u_{ij} \leq x \end{cases}. \quad (2b)$$

(b) Trapezoidal function:

$$\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}), \quad (3a)$$

with

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 0, & x \leq a_{ij} \\ (x - a_{ij}) / (b_{ij} - a_{ij}), & a_{ij} \leq x \leq b_{ij} \\ 1, & b_{ij} \leq x \leq c_{ij} \\ (d_{ij} - x) / (d_{ij} - c_{ij}), & c_{ij} \leq x \leq d_{ij} \\ 0, & d_{ij} \leq x \end{cases}. \quad (3b)$$

(c) Generalized exponential function (Chen and Li, 2000; Kumar et al. 2010):

$$\tilde{a}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij}), \quad (4a)$$

with

$$\mu_{\tilde{a}_{ij}}(x) = \begin{cases} 0, & x \leq a_{ij} \\ \exp(-\frac{b_{ij} - x}{b_{ij} - a_{ij}}), & a_{ij} \leq x \leq b_{ij} \\ 1, & b_{ij} \leq x \leq c_{ij} \\ \exp(-\frac{x - c_{ij}}{d_{ij} - c_{ij}}), & c_{ij} \leq x \leq d_{ij} \\ 0, & d_{ij} \leq x \end{cases}. \quad (4b)$$

In Eq. (1) all entries satisfy reciprocal relationship

$$\tilde{a}_{ji} = 1 / \tilde{a}_{ij}. \quad (5)$$

An example of the definition of a triangular fuzzy number $\tilde{a} = \tilde{3} = (l, m, u) = (2, 3, 4)$ and its inverse

$$\tilde{a}^{-1} = \frac{1}{\tilde{3}} = (\frac{1}{u}, \frac{1}{m}, \frac{1}{l}) = (\frac{1}{4}, \frac{1}{3}, \frac{1}{2}) = (0.250, 0.333, 0.500)$$

are shown as Fig. 2(a), and the counterparts for trapezoidal and generalized exponential fuzzy numbers, $\tilde{a} = \tilde{3} = (a, b, c, d) = (1.5, 2.5, 3.5, 4.5)$ and inverse $\tilde{a}^{-1} = \frac{1}{\tilde{3}} = (\frac{1}{d}, \frac{1}{c}, \frac{1}{b}, \frac{1}{a}) = (\frac{1}{4.5}, \frac{1}{3.5}, \frac{1}{2.5}, \frac{1}{1.5})$, are shown in Figs. 2(b) and 2(c), respectively. In the equation, $\tilde{a} = \tilde{3}$ denotes the “slightly more risky” level in Fig. 1.

Compared with the comparison matrices employed in AHP, the entries in the comparison matrices employed in FAHP are fuzzy numbers, which allow for consideration of the vagueness of human judgment and assessment based on the theory of fuzzy sets. It should be noted that there are many different forms of fuzzy judgment matrices, with some taking the value of 0.5 as the entry for equal judgment. Here, we employ a manner that, similar to the original AHP approach, keeps the reciprocal relationship satisfied.

Fig. 2 Fuzzy numbers and inverses employed in FAHP: (a) trigonometric fuzzy numbers, (b) trapezoidal fuzzy numbers, and (c) generalized exponential fuzzy numbers.

2.3 Calculate fuzzy relative weights

The weights for each influence factor in each layer can be calculated using the root method:

$$[\tilde{A}] \{\tilde{W}\}_i = \tilde{\lambda}_i \{\tilde{W}\}_i, \quad (5)$$

where $\tilde{\lambda}_i$ are eigenvalues, and $\tilde{W}_i$ are eigenvectors calculated as

$$\tilde{W}_i = \tilde{Z}_1 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \dots \oplus \tilde{Z}_n)^{-1}, \quad i=1,2,\dots,n \quad (6a)$$

$$\tilde{Z}_i = (\tilde{a}_{i1} \otimes \tilde{a}_{i2} \otimes \dots \otimes \tilde{a}_{in})^{1/n}, \quad i=1,2,\dots,n \quad (6b)$$

where $\oplus$ and $\otimes$ denote the addition and product of fuzzy numbers, respectively, and obey the rules of operation:

$$\begin{aligned} \tilde{a}_1 \oplus \tilde{a}_2 &= (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2), \\ \tilde{a}_1 \otimes \tilde{a}_2 &= (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2), \\ \tilde{a}_1^{-1} &= (d_1^{-1}, c_1^{-1}, b_1^{-1}, a_1^{-1}) \\ \tilde{a}_1^{1/n} &= (a_1^{1/n}, b_1^{1/n}, c_1^{1/n}, d_1^{1/n}) \end{aligned} \quad (7)$$

2.4 Defuzzify the relative weights to create crisp weights

In practical applications, the relative weights can be transformed into crisp values for convenient use. In addition, the calculation of relative weights does not involve vague human thought, and fuzzy logic need not be considered. There are many approaches for ranking (sorting) fuzzy numbers, and these can be classified into four types, as reported by Chen and Hwang (1992).

There are also many modified and extended methods for scaling fuzzy numbers based on the previous list of approaches. However, for the current use of linear and nonlinear membership functions, the centroid method is the same as the fuzzy mean and can be calculated as

$$w_i = \frac{\int_{l_i}^{u_i} x \mu_{w_i}(x) dx}{\int_{l_i}^{u_i} \mu_{w_i}(x) dx} = \frac{Q_x}{A}, \quad (8a)$$

with the results being expressed explicitly as follows:

(a) Triangular fuzzy numbers:

$$w_i = \frac{l_i + m_i + u_i}{3}, \quad i=1,2,\dots,n \quad (8b)$$

(b) Trapezoidal fuzzy numbers:

$$w_i = \frac{d_i^2 + c_i^2 + d_i c_i - b_i^2 - a_i^2 - b_i a_i}{3(d_i + c_i - b_i - a_i)}, \quad i=1,2,\dots,n \quad (8c)$$

(c) Generalized exponential fuzzy numbers:

$$\begin{aligned} w_i &= \{e^{-1}[(b-a)(b-2a+ae^1) + (d-c)(c-2d+de^1)] \\ &\quad + (c^2 - b^2)/2\} / \{(1-e^{-1})(d-c+b-a) + (c-b)\} \end{aligned} \quad (8d)$$

from which we can calculate the crisp relative weights for each level.

2.5 Obtain the crisp relative risk impacts (RRI)

The relative weights for each influence factor in both layers can be furthermore normalized as

$$w_i^* = \frac{w_i}{\sum_{i=1}^n w_i}, \quad i=1,2,\dots,n, \quad (9)$$

which satisfies

$$\sum_{i=1}^n w_i^* = 1, \quad i=1,2,\dots,n. \quad (10)$$

The consistency should be checked as employed in AHP (Huang et al. 2015).

2.6 Evaluate the final assessed risk index

Calculate the relative risk impact (RRI) of each influence factor on the overall risk:

$$\{RRI\}_{C \rightarrow A} = [M]_{C \rightarrow B} \bullet \{w_0^*\}_{B \rightarrow A}, \quad (11)$$

where $\{RRI\}$ is a $n \times 1$ crisp vector denoting the relative risk impact factor for each influence factor (C layer) on the global risk (A layer), $[M]$ is a $n \times m$ crisp transfer matrix that converts each sub-criteria layer (C layer) to the criteria layer (B layer), and $\{w_0^*\}$ is a $m \times 1$ crisp transfer vector that converts the criteria layer (B layer) to the goal layer (A layer).

Once the RRI of an FAHP (or AHP) model is obtained, we can plot a bar chart to observe the relative importance of each sub-criteria-layer influence factor on the overall risk, and the assessment model as constructed can be applied to practical assessments.

2.7 Build up the evaluation criteria for each influence factor

When we want to assess a hazard, we first analyze the data of the case obtained from field investigation based on the influence factor from well-established evaluation criteria, and then assign evaluation values $\tilde{E}_i$ ($i=1,2,\dots,n$), where the fuzzy numbers are also considered to include the vagueness of human judgment.

$$\tilde{E}_i = (E_a^i, E_b^i, E_c^i, E_d^i) \quad (i=1,2,\dots,n) \quad (12)$$

The evaluation values can be assigned based on the judgment method, as depicted in Table 1.

2.8 Calculate the overall risk index (ORI)

Next, we can calculate the fuzzy overall risk index (ORI) from the linear combination of the nine relative risk impacts $\{RRI\}$ and the nine components of the evaluation vector $\{\tilde{E}\}$:

$$\tilde{\Gamma} = (\Gamma_a, \Gamma_b, \Gamma_c, \Gamma_d) = \{RRI\}^T \bullet \{\tilde{E}\} = \sum_{i=1}^n RRI_i \bullet \tilde{E}_i. \quad (13)$$

Using the method of centroid for defuzzification of membership functions, the final crisp overall risk index is

$$ORI = \Gamma = \frac{\Gamma_{Qx}}{\Gamma_A}, \quad (14)$$

with the results being expressed explicitly as:

(a) Triangular fuzzy numbers:

$$ORI = \Gamma = \frac{\Gamma_l + \Gamma_m + \Gamma_u}{3} \quad (15a)$$

(b) Trapezoidal fuzzy numbers:

$$ORI = \Gamma = \frac{\Gamma_d^2 + \Gamma_c^2 + \Gamma_d \Gamma_c - \Gamma_b^2 - \Gamma_a^2 - \Gamma_b \Gamma_a}{3(\Gamma_d + \Gamma_c - \Gamma_b - \Gamma_a)} \quad (15b)$$

(c) Generalized exponential fuzzy numbers:

$$ORI = \Gamma = \{e^{-1}[(\Gamma_b - \Gamma_a)(\Gamma_b - 2\Gamma_a + \Gamma_a e^1) + (\Gamma_d - \Gamma_c)(\Gamma_c - 2\Gamma_d + \Gamma_d e^1)] + (\Gamma_c^2 - \Gamma_b^2)/2\} / \{(1 - e^{-1})(\Gamma_d - \Gamma_c + \Gamma_b - \Gamma_a) + (\Gamma_c - \Gamma_b)\} \quad (15c)$$

3. RISK ASSESSMENT OF DEBRIS-FLOW HAZARDS—FAHP MODEL

3.1 Establish a hierarchical model

In this research, the focus of the assessment of risk of debris flow is mainly on evaluation of the quantified degree (potential) of hazard in a region in addition to the qualitative judgment of safety or danger. This is not the same as the methods based on probability theory, in which many events are required to build up the rules so that the possibility of occurrence may be predicted.

To compare the risk assessment of debris-flow hazards using FAHP with the results using AHP (Huang et al. 2015), the same hierarchical model is considered. The top layer of a hierarchical model is the goal to be achieved, i.e., the risk assessment of debris-flow hazards caused by rainfall. The second layer (criteria layer) refers to the proposed three major categories of influence factors: (B1) topological and geological conditions, (B2) effective watershed area conditions, and (B3) rainfall conditions. The factors in the third layer (sub-criteria layer) support the factors in the second layer and are denoted by $C_{11}, C_{12}, \dots, C_{33}$, as shown in Fig. 3.

Spatial factors such as slope and geology are based mainly on on-site investigation data obtained from previous studies. Although the proposed AHP and FAHP methods can be referenced, when applied to different areas, researchers should be aware of the possibility of selecting different factors.

Fig. 3 Hierarchical model for risk assessment of debris-flow hazards using AHP and FAHP.

3.2 Setup comparison matrices

Here, the reciprocal comparison matrices were built up as

$$[B] = \begin{bmatrix} 1 & \tilde{2} & \frac{1}{\tilde{2}} \\ \frac{1}{\tilde{2}} & 1 & \frac{1}{\tilde{4}} \\ \tilde{2} & \tilde{4} & 1 \end{bmatrix}, \quad [C_1] = \begin{bmatrix} 1 & \tilde{2} & \frac{3}{\tilde{2}} & \tilde{4} \\ \frac{1}{\tilde{2}} & 1 & \frac{3}{\tilde{2}} & \tilde{2} \\ \frac{1}{\tilde{3}} & \frac{2}{\tilde{3}} & 1 & \tilde{2} \\ \frac{1}{\tilde{4}} & \frac{1}{\tilde{2}} & \frac{1}{\tilde{2}} & 1 \end{bmatrix}$$

$$[C_2] = \begin{bmatrix} 1 & \tilde{3} \\ \tilde{1} & 1 \end{bmatrix}, \quad [C_3] = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{\tilde{3}} \\ \tilde{2} & 1 & \frac{\tilde{2}}{\tilde{3}} \\ \tilde{3} & \frac{\tilde{3}}{2} & 1 \end{bmatrix}.$$

Compared with the comparison matrices employed in AHP (Huang, 2014; Huang et al. 2015), entries in the judgment matrices employed in FAHP are fuzzy numbers by which the vagueness of human consideration and assessment can be included based on the theory of fuzzy sets.

3.3 Calculate the relative weights

The fuzzy weights for $\tilde{w}_{B1}$, $\tilde{w}_{B2}$, ..., $\tilde{w}_{C33}$ can be calculated. For example, using triangular fuzzy numbers:

$$\tilde{w}_{B1} = (0.693, 1.000, 1.442) \otimes \left(\frac{1}{4.602}, \frac{1}{3.500}, \frac{1}{2.541} \right) \\ = (0.151, 0.286, 0.568)$$

$$\tilde{w}_{B2} = (0.406, 0.500, 0.693) \otimes \left(\frac{1}{4.602}, \frac{1}{3.500}, \frac{1}{2.541} \right) \\ = (0.088, 0.143, 0.273)$$

$$\tilde{w}_{B3} = (1.442, 2.000, 2.466) \otimes \left(\frac{1}{4.602}, \frac{1}{3.500}, \frac{1}{2.541} \right) \\ = (0.313, 0.571, 0.971)$$

$$\tilde{w}_{C11} = (1.565, 2.213, 2.783) \otimes \left(\frac{1}{6.215}, \frac{1}{4.637}, \frac{1}{3.306} \right) \\ = (0.252, 0.477, 0.842)$$

$$\tilde{w}_{C12} = (0.760, 1.107, 1.565) \otimes \left(\frac{1}{6.215}, \frac{1}{4.637}, \frac{1}{3.306} \right) \\ = (0.122, 0.239, 0.473)$$

$$\tilde{w}_{C13} = (0.595, 0.817, 1.107) \otimes \left(\frac{1}{6.215}, \frac{1}{4.637}, \frac{1}{3.306} \right) \\ = (0.096, 0.176, 0.335)$$

$$\tilde{w}_{C14} = (0.386, 0.500, 0.760) \otimes \left(\frac{1}{6.215}, \frac{1}{4.637}, \frac{1}{3.306} \right) \\ = (0.062, 0.108, 0.230)$$

$$\tilde{w}_{C21} = (1.22, 1.58, 1.87, 2.12) \otimes \left(\frac{1}{2.707}, \frac{1}{2.309}, \frac{1}{1.914} \right) \\ = (0.522, 0.750, 1.045)$$

$$\tilde{w}_{C22} = (0.500, 0.577, 0.707) \otimes \left(\frac{1}{2.707}, \frac{1}{2.309}, \frac{1}{1.914} \right) \\ = (0.185, 0.250, 0.369)$$

$$\tilde{w}_{C31} = (0.437, 0.550, 0.794) \otimes \left(\frac{1}{4.236}, \frac{1}{3.302}, \frac{1}{2.490} \right) \\ = (0.103, 0.167, 0.319)$$

$$\tilde{w}_{C32} = (0.794, 1.100, 1.442) \otimes \left(\frac{1}{4.236}, \frac{1}{3.302}, \frac{1}{2.490} \right) \\ = (0.187, 0.333, 0.579)$$

$$\tilde{w}_{C33} = (1.260, 1.651, 2.000) \otimes \left(\frac{1}{4.236}, \frac{1}{3.302}, \frac{1}{2.490} \right) \\ = (0.297, 0.500, 0.803)$$

3.4 Defuzzify the relative weights

Using the example of employing triangular fuzzy numbers and using Eq. (8b), we can calculate

$$w_{B1} = \frac{l_i + m_i + u_i}{3} = \frac{0.151 + 0.286 + 0.568}{3} = 0.3347.$$

Similarly, we can calculate w_{B2} , w_{B3} , ..., w_{C33} .

3.5 Normalize the relative weights

Using the case of triangular fuzzy numbers as an example, after normalization using Eq. (9), we have

$$w_{B1}^* = 0.3347 / 1.1211 = 0.2985$$

$$w_{B2}^* = 0.1679 / 1.1211 = 0.1498$$

$$w_{B3}^* = 0.6185 / 1.1211 = 0.5517$$

$$w_{C11}^* = 0.5237 / 1.1373 = 0.4605$$

$$w_{C12}^* = 0.2781 / 1.1373 = 0.2446$$

$$w_{C13}^* = 0.2022 / 1.1373 = 0.1778$$

$$w_{C14}^* = 0.1333 / 1.1373 = 0.1172$$

$$w_{C21}^* = 0.7724 / 1.0404 = 0.7424$$

$$w_{C22}^* = 0.2680 / 1.0404 = 0.2576$$

$$w_{C31}^* = 0.1962 / 1.0963 = 0.1789$$

$$w_{C32}^* = 0.3666 / 1.0963 = 0.3344$$

$$w_{C33}^* = 0.5335 / 1.0963 = 0.4867$$

3.6 Evaluate the final assessed risk index

The calculated results are

(a) AHP (Huang et al., 2015):

$$\{RRI\} = [M]_{C \rightarrow B} \bullet \{w_0^*\}_{B \rightarrow A}$$

$$= \begin{bmatrix} 0.4774 & 0 & 0 \\ 0.2387 & 0 & 0 \\ 0.1761 & 0 & 0 \\ 0.1078 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0.1667 \\ 0 & 0 & 0.3333 \\ 0 & 0 & 0.5 \end{bmatrix} \bullet \begin{Bmatrix} 0.2857 \\ 0.1429 \\ 0.5714 \end{Bmatrix} = \begin{Bmatrix} 0.1364 \\ 0.0682 \\ 0.0503 \\ 0.0308 \\ 0.1071 \\ 0.0357 \\ 0.0952 \\ 0.1905 \\ 0.2857 \end{Bmatrix}$$

(b) FAHP using triangular fuzzy numbers:

$$\{RRI\} = [M]_{C \rightarrow B} \bullet \{w_0^*\}_{B \rightarrow A}$$

$$= \begin{bmatrix} 0.4605 & 0 & 0 \\ 0.2446 & 0 & 0 \\ 0.1778 & 0 & 0 \\ 0.1172 & 0 & 0 \\ 0 & 0.7424 & 0 \\ 0 & 0.2576 & 0 \\ 0 & 0 & 0.1789 \\ 0 & 0 & 0.3344 \\ 0 & 0 & 0.4866 \end{bmatrix} \bullet \begin{Bmatrix} 0.2985 \\ 0.1498 \\ 0.5517 \end{Bmatrix} = \begin{Bmatrix} 0.1375 \\ 0.0730 \\ 0.0531 \\ 0.0350 \\ 0.1112 \\ 0.0386 \\ 0.0987 \\ 0.1845 \\ 0.2685 \end{Bmatrix}$$

(c) FAHP using trapezoidal fuzzy numbers:

$$\{RRI\} = [M]_{C \rightarrow B} \bullet \{w_0^*\}_{B \rightarrow A}$$

$$= \begin{bmatrix} 0.4595 & 0 & 0 \\ 0.2444 & 0 & 0 \\ 0.1810 & 0 & 0 \\ 0.1150 & 0 & 0 \\ 0 & 0.7314 & 0 \\ 0 & 0.2686 & 0 \\ 0 & 0 & 0.1817 \\ 0 & 0 & 0.3330 \\ 0 & 0 & 0.4853 \end{bmatrix} \bullet \begin{Bmatrix} 0.2972 \\ 0.1487 \\ 0.5541 \end{Bmatrix} = \begin{Bmatrix} 0.1366 \\ 0.0726 \\ 0.0538 \\ 0.0342 \\ 0.1087 \\ 0.0399 \\ 0.1007 \\ 0.1845 \\ 0.2690 \end{Bmatrix}$$

(d) FAHP using generalized exponential fuzzy numbers:

$$\{RRI\} = [M]_{C \rightarrow B} \bullet \{w_0^*\}_{B \rightarrow A}$$

$$= \begin{bmatrix} 0.4567 & 0 & 0 \\ 0.2453 & 0 & 0 \\ 0.1820 & 0 & 0 \\ 0.1160 & 0 & 0 \\ 0 & 0.7274 & 0 \\ 0 & 0.2726 & 0 \\ 0 & 0 & 0.1844 \\ 0 & 0 & 0.3328 \\ 0 & 0 & 0.4829 \end{bmatrix} \bullet \begin{Bmatrix} 0.2989 \\ 0.1495 \\ 0.5515 \end{Bmatrix} = \begin{Bmatrix} 0.1365 \\ 0.0733 \\ 0.0544 \\ 0.0347 \\ 0.1088 \\ 0.0408 \\ 0.1017 \\ 0.1835 \\ 0.2663 \end{Bmatrix}$$

These results are plotted in Fig. 4 and shown in Table 2; AHP results from a previous study (Huang et al., 2015) are also shown for comparison. Only small differences exist between the AHP and the three FAHP results. Furthermore, the importance ranking of the influence factors to the overall risk obtained from the AHP and the three FAHP models is kept the same. In this study, it was found that accumulated rainfall (AR), duration of rainfall (DR), and averaged slope angle (SA) were the three most important influence factors in debris-flow hazards. From these inspections, we find that the fuzzy weighting employed in this study is adequate and may suggest wider implementation by researchers and risk-assessment engineers.

Fig. 4 Diagram of relative risk impacts (RRIs) for influence factors of debris-flow hazards using AHP and three FAHPs.

Table 2 Comparison of relative risk impact of each influence factor for risk assessment of debris flow based on the AHP and the FAHPs.

Influence Factor	RRI % (AHP)	RRI % (FAHP) (Triangular)	RRI % (FAHP) (Trapezoidal)	RRI % (FAHP) (Generalized exponential)	Rank
C11 (SA)	13.64	13.75	13.66	13.65	3
C12 (TD)	6.82	7.30	7.26	7.33	6
C13 (SD)	5.03	5.31	5.38	5.44	7
C14 (SP)	3.08	3.50	3.42	3.47	9
C21 (EA)	10.71	11.12	10.87	10.88	4
C22 (OS)	3.57	3.86	3.99	4.08	8
C31 (IR)	9.52	9.87	10.07	10.17	5
C32 (DR)	19.04	18.45	18.45	18.35	2
C33 (AR)	28.57	26.85	26.90	26.63	1
SUM	100	100	100	100	

3.7 Develop the evaluation criteria for each influence factor

The evaluation values and criteria for debris flow occurring in Taiwan are proposed as listed in Table 3.

Table 3 Criteria and sub-criteria for risk assessment of debris flow hazards occurring in Taiwan.

	Qualitative Ranking Levels	Quantitative Ranking Levels	(1-9)
B1 Topological & Geological Conditions			
C11	Very small	$\theta < 10^\circ$	1
	Small	$10^\circ < \theta < 15^\circ$	3
	Intermediate	$15^\circ < \theta < 20^\circ$	5
	Large	$20^\circ < \theta < 25^\circ$	7
	Very large	$25^\circ < \theta$	9
C12	Very tight	Very tight sand, gravel, rocks	1
	Tight	Tight sand, gravel, rocks	3
	Intermediate	General sand, gravel, rocks	5
	Loose	Loose sand, gravel, rocks	7
	Very loose	Very loose sand, gravel, rocks	9
C13	Very wide	W (Well graded)	1
	Wide	P (Poorly graded)	3
	Intermediate	L (Low plasticity)	5
	Narrow	H (High plasticity)	7
	Very narrow	Fluidized	9
C14	Frost	Dense frost	1
	Deep rooted	Trees	3
	Intermediate rooted	Arca, Fruits	5
	Shallow rooted	Vegetables	7
	No plants	None	9
B2 Watershed Conditions			
C21	Very small	$A < 50,000 \text{ m}^2$	1
	Small	$50,000 \text{ m}^2 < A < 100,000 \text{ m}^2$	3
	Intermediate	$100,000 \text{ m}^2 < A < 150,000 \text{ m}^2$	5
	Big	$150,000 \text{ m}^2 < A < 200,000 \text{ m}^2$	7
	Very big	$200,000 \text{ m}^2 < A$	9
C22	Very small	$V < 1,000 \text{ m}^3$	1
	Small	$1,000 \text{ m}^3 < V < 5,000 \text{ m}^3$	3
	Intermediate	$5,000 \text{ m}^3 < V < 10,000 \text{ m}^3$	5
	Big	$10,000 \text{ m}^3 < V < 50,000 \text{ m}^3$	7
	Very big	$50,000 \text{ m}^3 < V$	9
B3 Rainfall Conditions			
C31	Very low	$I < 5 \text{ mm/hr}$	1
	Low	$5 \text{ mm/hr} < I < 10 \text{ mm/hr}$	3
	Intermediate	$10 \text{ mm/hr} < I < 15 \text{ mm/hr}$	5
	High	$15 \text{ mm/hr} < I < 20 \text{ mm/hr}$	7
	Very high	$20 \text{ mm/hr} < I$	9
C32	Very short	$T < 1 \text{ hr}$	1
	Short	$1 \text{ hr} < T < 5 \text{ hr}$	3
	Intermediate	$5 \text{ hr} < T < 10 \text{ hr}$	5
	Long	$10 \text{ hr} < T < 20 \text{ hr}$	7
	Very long	$20 \text{ hr} < T$	9
C33	Very low	$A < 100 \text{ mm}$	1
	Low	$100 \text{ mm} < A < 200 \text{ mm}$	3
	Intermediate	$200 \text{ mm} < A < 300 \text{ mm}$	5
	High	$300 \text{ mm} < A < 400 \text{ mm}$	7
	Very high	$400 \text{ mm} < A$	9

In Table 3, the qualitative scaling levels are similar to those employed in fuzzy logic or fuzzy inference systems (FIS) using fuzzy linguistics; however, the corresponding quantitative ranking levels are based on the nine scale values from Table 1. The interval values for each influence factor refer to statistical data obtained from previous on-site investigational research in the area. Among these items, the formula for estimating the volume of outflow sediment (C22) is from Jan (2000) and Huang (2001).

4. CASE STUDY OF RISK ASSESSMENT OF DEBRIS-FLOW HAZARDS

4.1 Description of debris-flow disasters that occurred in eastern Taiwan

(1) Case 1: Debris flow that occurred in Tung-Men Village (Fig. 5)

The first debris-flow disaster case that employed to check the validity of the risk assessment model was the disaster that occurred on June 23, 1990, in Tung-Men Village, Sho-Lin Township, Hua-Lien County, Taiwan, during landfall of a typhoon with strong storms. This debris flow resulted in 24 houses fully destroyed and 11 almost destroyed, 29 deaths, 6 missing persons, and 7 injured persons. Yu (1990) and Chao and Kuo (1995) conducted thorough investigations of this hazard disaster. Furthermore, Shieh et al. (1992) reported on a field investigation of this disaster, Chen et al. (1993) reported the material properties of the deposit, and Fang and Mau (1997) investigated the rainfall characteristics.

(2) Case 2: Mud flow that occurred in Tung-Shing Village (Fig. 5)

The second case of debris-flow disaster employed for checking the validity of the risk assessment model was a mudslide that occurred July 10–11, 1994, in Tung-Shing Village, Fung-Bin Township, Hua-Lien County, Taiwan, during the landfall of a typhoon with strong storms. This muddy flow caused more than 50 deaths. Chang (1995), Chen and Shiek (1996) studied and reported about this muddy flow and the resulting disasters.

Fig. 5 Locations of two cases of debris-flow hazards that occurred in eastern Taiwan.

Table 4 Judgment vector values for Tung-Men village and Tung-Shing village.

Judgment vector	E1	E2	E3	E4	E5	E6	E7	E8	E9
Case\ Factors	C11	C12	C13	C14	C21	C22	C31	C32	C33
Tung-Men	9	7	5	7	9	9	9	9	9
Tung-Shing	5	7	7	7	9	9	9	7	9

Table 5 Comparison of overall risk index (ORI) of risk assessment using the AHP and the three FAHPs.

Debris-flow Hazards	Tung-Men Village	Tung-Shing Village
<i>ORI-AHP</i> (Huang et al., 2015)	8.6007	7.7748
<i>ORI-FAHP</i> (Triangular fuzzy numbers)	8.2921 (3.59 %)*	7.5867 (2.42 %)*
<i>ORI-FAHP</i> (Trapezoidal fuzzy numbers)	8.1165 (5.63 %)*	7.4828 (3.76 %)*
<i>ORI-FAHP</i> (Generalized exponential fuzzy numbers)	8.0446 (6.47 %)*	7.4396 (4.31 %)*
*Deviation (%) = $\frac{AHP - FAHP}{AHP} \times 100\%$		

4.2 Recorded data and judgment vector {E}

The recorded data of the debris flows at Tung-Men and Tung-Shing Villages can be found in Huang et al. (2015). From the criteria in Table 3 and the recorded data, judgment vectors can be obtained as shown in Table 4.

4.3 FAHP risk assessment results

From Eq. (15a), using triangular fuzzy numbers, we can calculate the overall risk index of the debris-flow hazards:

(1) Tung-Men village:

$$\tilde{\Gamma} = \sum_{i=1}^9 RRI_i \bullet E_i = (7.5718, 8.5718, 8.7328)$$

$$ORI = \Gamma = \frac{7.5718 + 8.5718 + 8.7328}{3} = 8.2921$$

(2) Tung-Shing village:

$$\tilde{\Gamma} = \sum_{i=1}^9 RRI_i \bullet E_i = (6.7591, 7.7591, 8.2421)$$

$$ORI = \Gamma = \frac{6.7591 + 7.7591 + 8.2421}{3} = 7.5867$$

Similarly, the results using trapezoidal and generalized exponential fuzzy numbers can be calculated from Eq. (15b) and Eq. (15c). All assessment results using the FAHPs and the AHP (Huang et al., 2015) are shown in Table 5 and Fig. 6. Figure 6 shows an attempt to classify the debris-flow hazards predicted by ORI into five risk levels that are nearly in accordance with the nine scale levels employed in AHP:

Fig. 6 Diagram of overall risk index (ORI) based on risk assessment of the tested debris-flow hazards at Tung-Men Village and Tung-Shing Village, Taiwan, using AHP and three FAHPs.

- $1 \leq ORI < 2$: Very safe
 $2 \leq ORI < 4$: Safe
 $4 \leq ORI < 6$: Intermediate
 $6 \leq ORI < 8$: Dangerous
 $8 \leq ORI \leq 9$: Very dangerous

It is obvious that only a small deviation (within 7%) exists between the results of the AHP and those of the three FAHPs. Furthermore, the results of the risk indices predicted by the AHP model are higher than those obtained from the three FAHP models. This means that the results of the three FAHP models are somewhat conservative. In addition, using nonlinear (generalized exponential) fuzzy numbers leads to greater vagueness of judgment and therefore results in smaller overall risk indices than using linear (triangular and trapezoidal) fuzzy numbers. However, the predictions for these two real debris-flow hazards are nearly the same from the different prediction models, and all are above or within the “dangerous” range assessed using the AHP and the three FAHP models.

In practical applications for risk assessment of debris-flow hazards, the results can be reported using crisp or fuzzy values. Taking Tung-Men Village as an example, the overall risk of debris-flow hazard based on the FAHP using triangular fuzzy numbers can be reported either as (1) crisp mode: ORI is 8.2921, “very dangerous” or as (2) fuzzy mode: ORI is within 7.5718 to 8.7328, “dangerous to very dangerous.” We can see that the FAHP models provide more practical risk assessment approaches than does AHP for natural hazards inherently involving uncertainty and vagueness.

When AHP is employed, all of the weighting

matrices are crisp sets (definitely without vagueness), whereas in FAHP, the entries in these matrices are fuzzy numbers within which the vagueness and imprecision of measurement in the real world can be included and modeled.

5. CONCLUDING REMARKS

An FAHP-based quantitative method in which three different kinds of membership functions are employed to describe the fuzzy numbers was developed for risk assessment of debris-flow hazards. The FAHP model integrates the advantages of AHP and fuzzy set theory and is capable of accommodating uncertainty and vagueness in setting up comparison matrices and evaluation vectors. The FAHP models were verified successfully by using two actual cases from eastern Taiwan. The findings follow.

- (1) The importance of the influence factors to the overall risk obtained from the AHP and the FAHP models was the same for each model. In this study, accumulated rainfall (AR), duration of rainfall (DR), and average slope angle (SA) were the three most important factors for the assessment of debris-flow hazard.
- (2) The FAHP using generalized exponential fuzzy numbers yielded smaller overall risk indices than did the FAHPs using triangular and trapezoidal fuzzy numbers. The reason for this difference was the greater vagueness involved in the nonlinear (generalized exponential) fuzzy numbers than in the linear (triangular and trapezoidal) fuzzy numbers.
- (3) All three kinds of FAHP resulted in smaller overall risk indices than did the AHP model, which used crisp comparison matrices. In general, the FAHP models provided more conservative indices than those produced by the AHP model.

Importantly, the FAHP models can be modified or adjusted (especially the influence factors and evaluation criteria) for application in other countries.

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