Evaluation of turbulence models in rough-wall boundary layers for hydroelectric applications

Rabijit Dutta¹, Jonathan Nicolle², Anne-Marie Giroux² and Ugo Piomelli¹

¹Department of Mechanical and Materials Engineering, Queen's University
Kingston (ON) K7L 3N6, CANADA
rabijit13@gmail.com, ugo@queensu.ca

²Institut de Recherche d'Hydro-Québec,
Varennes (QC) J3X 1S1, CANADA
nicolle.jonathan@ireq.ca, giroux.anne-marie2@ireq.ca

Abstract

The accuracy of turbulence models for the Reynolds-Averaged Navier-Stokes (RANS) equations in rough-wall flows is evaluated using data from large-eddy simulations (LES) of boundary layers with favourable and adverse pressure gradients. Some features of the flow (such as flow reversal in the roughness sublayer) cannot be captured accurately by any model, due to the fundamental model formulation. In mild pressure gradients most RANS models are sufficiently accurate for engineering applications, but if strong favourable or adverse pressure gradients are applied (especially those leading to separation) the model performance rapidly degrades.

Keywords: turbulence modelling, large-eddy simulations, roughness, boundary layers, separation.

1. Introduction

Roughness plays an important role in many fields of study; in geophysical and environmental applications, for instance, most surfaces of interest are rough (hilly terrain, plant canopies, urban environments, etc.). In engineering, roughness is important in electronic cooling, turbomachinery, and duct and pipe flows. The last two areas of application are of interest to the power industry and to Hydro Québec. Roughness occurs in all parts of a hydro-electric turbine, and the roughness of hydraulic surfaces increases with time, affecting the overall performance of equipment in a significant way. The present work is motivated by the need to improve industrial predictive capabilities for flow configurations including roughness.

It is well known that the roughness produces a thicker boundary layer and increases surface drag. The first study of rough wall flows was performed by Nikuradse [1], who glued sandgrains on the wall of a pipe and studied the pressure drop and velocity profiles for different sizes of sandgrains. He categorized rough-wall flows into three regimes based on non-dimensional sandgrain height $k^+$ (where a superscript + denotes quantities normalized by the friction velocity $u_* = (τ_w/ρ)^{1/2}$ and the fluid kinematic viscosity $\nu$, $k$ is the sandgrain size, $τ_w$ the wall stress and $ρ$ the fluid density). He reported that in a hydrodynamically smooth wall flow ($k^+ < 5$), roughness effects are not important and the skin friction depends only on Reynolds number. In a transitionally rough flow ($5 < k^+ < 70$) both viscous and roughness effects are significant; for fully rough flows ($k^+ > 70$) the skin friction coefficient is only dependent upon the roughness height. He also observed that the momentum loss due to roughness (higher drag) can be quantified by a downward shift in the logarithmic velocity profile, called the roughness function, $ΔU^+$.

Since Nikuradse [1], many studies have focused on the effects of roughness on wall bounded flows. Reviews by Raupach and co-workers [2,3] and Finnigan [4] summarize the research on roughness in atmospheric applications, while [3,5] discuss engineering flows. Roughness increases near-wall mixing and the skin-friction coefficient; the velocity profile is altered, and so are the Reynolds stresses. Wall-normal fluctuations increase near the solid surface [6], while the anisotropy of the Reynolds stresses decreases [7]. The roughness effects do not propagate beyond a height of the order of $3−5k_s$ [3,8,9]. Here, $k_s$ is the equivalent sand-grain roughness height, defined as the mean height of the equivalent sand-grain roughness that produces the same deficit in the logarithmic region of the mean-velocity profile as the roughness in question. The roughness geometry plays a role [10], but most geometries can be related to the sandgrain roughness using $k_s$. Various correlations have been proposed to relate a real geometry to an equivalent sandgrain surface, and they are reviewed and discussed in [11].
Numerical methods have become useful tools for the studies of rough-wall boundary layers over the last twenty years. Techniques that resolve the turbulent eddies, such as the direct numerical simulation (DNS) and the large eddy simulation (LES) have historically been limited by the contrasting needs to have roughness elements that are much smaller than the boundary-layer thickness, Reynolds numbers high enough that the flow is in the fully rough regime (or transitionally rough) and a grid fine enough to resolve the flow around the roughness elements [5]. Only recently the available computational power has made it possible to perform calculations that satisfy all these requirements (for instance, Refs. [11–16]). Industrial calculations, however, still rely on the solution of the Reynolds-Averaged Navier-Stokes (RANS) equations, with turbulence models used to parameterize the Reynolds stresses. Therefore, it is important to develop robust RANS models that can accurately predict the surface roughness effects.

The roughness prediction with RANS-based models can be categorized into two approaches. The “discrete element method”, reviewed recently by Aupoix [17], models the blockage effects on the flow due to the roughness elements by adding extra terms in the governing equations. The “equivalent sandgrain roughness” approach employs turbulence modifications near the surface to predict the logarithmic velocity shift, based on experimental measurements of the roughness function for various $k^+$ [1,18,19]. One essential feature of these models is that the increase in friction, which is due in part (for transitional roughness) or entirely (for fully rough flows) to the form drag around each roughness element, is modelled by increasing the eddy viscosity. The mean flow equations are kept unchanged, while the boundary conditions for the turbulence quantities, the wall damping functions and the production and destruction terms in the turbulence transport equations are modified based on equivalent sandgrain roughness to give the desired roughness effect. The ease of implementation of the roughness modifications into an existing code makes this method the most commonly used in RANS solution codes, and we will consider this method only in this work. A distinction can also be made between methods that set the eddy viscosity to zero on the boundary, and those that allow a finite value for it. In the first case, conceptually, the computational domain extends to the bottom of the roughness elements, whereas in the second it reaches at least part of the way into the sublayer.

Roughness corrections have been developed for most commonly used models. We limit our attention here to three of those that are most popular in industrial practice, the Spalart-Allmaras (SA) one-equation model [20], the $K-\varepsilon$ model [21], and the $K-\omega$ model [22], in the Shear-Stress Transport (SST) formulation by Menter [23].

Roughness modifications for the one-equation SA model [20] were proposed by Lee and Paynter [24] and Aupoix and Spalart [25]. Both of them employ a wall offset, $\Delta y \propto k_+$, following Rotta’s [26] observation that the roughness effect on the logarithmic velocity profile can be obtained by moving the reference surface by a distance $\Delta y$ related to the mean roughness height. Lee and Paynter [24] used a rough-wall boundary condition for the eddy viscosity $\nu_l$ identical to the Cebeci and Chang [27] mixing-length model roughness correction, and a wall shift, $\Delta y$, in the transport equation terms; $\Delta y$ was also taken from Cebeci and Chang [27]. Their model, however, showed poor agreement with data in computations of zero, adverse and favorable pressure gradient boundary layers. Aupoix and Spalart [25], on the other hand, proposed two modifications based on physical arguments and using a direct approach. The Boeing modification used $\Delta y$ to define a Neumann boundary condition for the auxiliary eddy-viscosity-like variable, $\tilde{v}$, at the wall and to modify the destruction term that depends on the distance from the wall. The ONERA correction used a wall offset to develop a relation between the surface value of $\tilde{v}$ and $k_+$ based on the roughness function measured by Nikuradse [1]. They compared these extensions with various zero pressure gradient (ZPG) boundary layer experimental data. Both Boeing and ONERA yielded similar predictions and gave fair agreement with the data.

Numerous roughness corrections have been developed for $K-\varepsilon$ models. Zhang et al. [28] and Foti and Scandura [29] developed roughness corrections for the low-Reynolds number $K-\varepsilon$ model. This approach has not been very successful, however, due to the number of damping functions included in the low-Re $K-\varepsilon$ model, which must be modified one by one. The development of roughness corrections for the two-layer $K-\varepsilon$ model [30] is much simpler. In the two-layer $K-\varepsilon$ model, in the near-wall region the $\varepsilon$ equation is replaced by the specification of a mixing length [31]. Patel and Yoon [30] employed wall offset in definition of the mixing length, similar to the corrections developed by Cebeci and Chang [27] for a mixing length model. They used this model to study fully developed channel flow with rough walls and achieved poor agreement with the data. Durbin et al. [32] modified the wall boundary condition for $k$ by blending the solutions for smooth and fully rough conditions. They developed a correlation curve for the wall offset $\Delta y$ based on a numerical calibration procedure using the $\Delta U^+ \propto k_+$ correlation measured by Ligron and Moffat [19]. Their model gave good predictions of zero, favorable and adverse pressure gradient flows over rough walls. Separated flows over ramp and sand dunes were also predicted accurately.

Wilcox [22] modified the boundary condition for $\omega$ in his $K-\omega$ model to predict correctly the shift in logarithmic velocity profile. A reduction of $\omega$ (with surface roughness) increases the eddy viscosity near the surface and, eventually, the drag. He used Nikuradse’s [1] data to calibrate the variation of the wall value of $\omega$ with $k_+$, for both transitional and fully rough flows. Hellsten and Laine [35] reported that employing this correction in an SST $K-\omega$ model interferes with the definition of the eddy viscosity limiter close to wall, which becomes active for $k_+ > 30$, reducing the eddy viscosity and resulting in highly underpredicted drag. They instead proposed a roughness correction that keeps the limiter inactive. They tested their model for the flow past a roughened airfoil, and predicted lift and drag fairly well. Knopp et al. [36] also observed the numerical problems associated with the boundary conditions for Hellsten and Laine [35] model, and modified the boundary conditions for $\omega$ by introducing a shift $\Delta y$. They used this model to predict zero-pressure gradient boundary layers and roughened flow over airfoil. Their results were in fair agreement for fully rough flows, but underpredicted the friction in transitionally rough flows. Recently, Aupoix [37] reviewed various roughness corrections for the $K-\omega$ model and developed a new one by assuming the existence of a constant stress region, and shifting $k$ and $\omega$ based on the data by either Colebrook [18] or Nikuradse [1]. He used this model to predict zero pressure gradient and mild adverse and favorable pressure gradient boundary layers. The friction predictions were slightly better than those obtained by Hellsten and Laine [35].

Critical comparisons of various roughness corrections for industrial problems have not been reported. Furthermore, it is difficult to comment on the performance of these roughness corrections for even simple flows, as these models are not tested in a unified framework. However, most of the models perform reasonably well for zero pressure gradient boundary layers as they were generally
calibrated using this type of flows. In the present study we assess the accuracy of some of the roughness modifications described above in boundary layers subjected to strong favourable and adverse pressure gradients. Pressure gradients occur in most engineering flows (airfoils, variable-section ducts, turbine blades etc.) and may cause significant modifications to the turbulence. Favourable pressure gradients cause turbulence to become more anisotropic and, if strong enough, cause the flow to revert to a quasi-laminar state. Adverse pressure gradients, on the other hand, tend to amplify the turbulence, and may cause flow separation. To separate the effects of the pressure gradient from those of geometry (in particular, curvature) all the simulations are performed using a flat-plate boundary-layer flow, in which the pressure gradient is imposed through blowing and suction at the freestream [39, 40]. In all cases a wall-resolved large-eddy simulation (LES) is also performed, at the same Reynolds number and with the same boundary conditions, in which the roughness elements are resolved by the grid using an Immersed-Boundary Method (IBM) [11, 41]. These calculations will yield not only data for comparison, but also an understanding of the physical phenomena that affect the models.

The next section presents the governing equations for RANS and LES, describes the RANS models and roughness corrections used. Then, the numerical procedure used is described. Results, including grid convergence studies, are discussed, and concluding remarks end the paper.

2. Problem formulation

2.1 Governing equations

In this work we solve either the filtered or the Reynolds-Averaged forms of the incompressible equations of motion. They have the same form:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0; \quad \frac{\partial \bar{p}}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \nabla^2 \bar{u}_i + \frac{\partial \tau_{ij}}{\partial x_j}$$

(1)

an overline denotes either a filtered or Reynolds-Averaged variable and \(\tau_{ij}\) are either the subgrid-scale (SGS) stresses, or the Reynolds stresses. The equations of motion, both in RANS or LES mode, were solved using a finite-difference code that employs second-order central differences on all terms and a staggered grid [43], and a second-order accurate time advancement scheme.

2.2 Large-eddy simulations (LES)

In the large-eddy simulations (LES) Eq. (1) was solved using the Lagrangian Dynamic Eddy Viscosity model [42] to parameterize the SGS stresses. The inflow conditions were based on the recycling method by Lund et al. [44]; at the freestream either the streamwise or the wall-normal velocity was specified to achieve a desired acceleration parameter

$$K_p = \frac{\nu}{U_\infty^2} \frac{dU_\infty}{dx}$$

(2)

Here, \(U_\infty\) is the freestream velocity. The other velocity components were specified by requiring either global mass conservation, or zero vorticity [43]. Periodic boundary conditions were used in the spanwise direction and convective boundary conditions [45] used at the outflow. At the wall, no-slip conditions were used for the smooth cases, while an Immersed Boundary Method (IBM) was employed to represent the roughness [40]. The rough surface was modelled as a random distribution of ellipsoids with axes \(k_x\), 1.5\(k_x\) and 2.0\(k_x\); the volume-of-fluid in each cell was used to reduce the value of the velocity in grid cells partially or entirely occupied by the roughness elements. This method results in a sandgrain-like surface, in which the roughness height is equal to the smallest axis of the ellipsoid. This methodology has been extensively validated in rough-wall flows [11, 13–16].

In all the simulations the first part of the domain had a smooth wall (see figure 1); this section included the recycling plane. Roughness was then initiated, and the flow was allowed to achieve a fully developed state over the rough wall. A reference plane was chosen inside the fully developed region, and coordinates were measured from this location (\(x = 0\) in the figure). Quantities at this plane were used as reference length scales and velocity scale (momentum or displacement thickness and freestream velocity) and are denoted by a subscript \(o\). A summary of the simulation parameters can be found in Table I.

Grid convergence studies were performed for all simulations. Details can be found in Ref. [48]. Each roughness element was resolved using between 5×5 and 10×10 points in the \(xz\)-plane, and between 50 and 100 points in the \(y\) direction, to ensure accurate description of the flow in the roughness sublayer.

<table>
<thead>
<tr>
<th>Case</th>
<th>Reynolds number</th>
<th>Grid</th>
<th>Domain size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separating boundary layer</td>
<td>(U_\infty \theta_o/\nu=2, 300)</td>
<td>2530×432×384</td>
<td>8000o×960o×750o</td>
</tr>
<tr>
<td>Realistic acceleration, matched (K_p)</td>
<td>(U_\infty \delta_o/\nu=2, 420)</td>
<td>2560×219×384</td>
<td>620δ_o×27 δ_o×96 δ_o</td>
</tr>
<tr>
<td>Realistic acceleration, matched (\Lambda)</td>
<td>(U_\infty \delta_o/\nu=2, 105)</td>
<td>2560×205×384</td>
<td>684δ_o×27 δ_o×96 δ_o</td>
</tr>
</tbody>
</table>

Table 1 Summary of large-eddy simulation parameters. The case description is presented later in the text.
2.3 Turbulence Models

The RANS calculations employed three eddy-viscosity models, with various roughness corrections. The Reynolds stresses are given by

\[ \tau_{ij} = -u_i' u_j' = 2\nu_t S_{ij} - \frac{2}{3} \delta_{ij} K; \quad S_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \]  

(3)

The first model tested is the one-equation Spalart-Allmaras (SA) model [20], with the roughness modification developed by Aupoix and Spalart [25] (we use the Boeing correction). Second, we used the two-layer \( K-\epsilon \) model of Chen and Patel [46], extended to rough wall flows by Durbin et al. [32]. Lastly, we employed the SST \( K-\omega \) model [23]. Three roughness corrections were implemented: first the one proposed by Hellsten and Laine [35], second a new roughness correction we developed to extend the approach of Durbin et al. [32] to the Knopp et al. [36] modifications and finally, the roughness correction of Aupoix [37]. In the following, the details of each model (with focus on the roughness corrections) are given.

2.3.1 Spalart-Allmaras (SA) model

In the SA model [20] a transport equation for a modified eddy viscosity, \( \bar{\nu} \), is solved. The eddy viscosity is then given by

\[ \nu_t = \frac{\chi^3}{\chi^3 + 7.1^3} \bar{\nu}, \quad \chi = \frac{\bar{\nu}}{\nu} \]  

(4)

The transport equation for the model and the value of the constants are those used in [20]. For smooth wall calculations, a Dirichlet boundary condition, \( \bar{\nu} = 0 \), is prescribed. The Boeing correction for roughness [25] uses a wall offset and prescribes a Neumann boundary condition for \( \bar{\nu} \); \( \chi \) is modified to account for transitional roughness [25]:

\[ \frac{\partial \bar{\nu}}{\partial n} \bigg|_w = \frac{\bar{\nu}}{\Delta y}, \quad \chi = \frac{\bar{\nu}}{\nu} + \frac{k_s}{(d + \Delta y)} \]  

(5)

with \( \Delta y = 0.03k_s \).

2.3.2 \( K-\epsilon \) Model

In the two-layer \( K-\epsilon \) model of Chen and Patel [46], transport equations for the turbulent kinetic energy (TKE), \( K \), and the dissipation rate, \( \epsilon \), are solved away from the wall to prescribe the eddy viscosity \( \nu_t = C_\mu K^2/\epsilon \), with \( C_\mu = 0.09 \). In the inner part of the boundary layer the dissipation is given by

\[ \epsilon = K^{3/2}/\ell_\epsilon, \]  

(6)

where \( \ell_\epsilon = C_\ell y \left[ 1 - \exp\left( -R_y/A_\epsilon \right) \right] \)

is a mixing length. Here, \( R_y = y K^{1/2}/\nu, \) \( C_\ell = 2.5, A_\epsilon = 5.0 \) and \( y \) is the distance from the wall. The model was solved using the equations and constant in [46].

In smooth-wall cases, the TKE is set to zero at \( y = 0 \). When the wall is rough, on the other hand, Durbin et al. [32] propose using an effective origin for the turbulence at \( y = -\Delta y \). Thus, the TKE at \( y = 0 \) is not zero, but can be obtained by requiring logarithmic-law behaviour:

\[ K(0) = \frac{u_*^2}{C_{\mu}^{1/2}} \min \left[ 1, \left( \frac{k_s}{90} \right)^2 \right] \]  

(7)
In the damping function in (6) the distance from the wall is replaced by an effective distance \( y_{\text{eff}} = y + \Delta y \), and the constant \( A_\varepsilon \) is replaced by

\[
A_\varepsilon = \max \left[ 1, A_\nu (1 - k_+^4 / 90) \right]
\]

(8)

\( \Delta y = 0.03 k_+^4 \) for fully rough flows, but is calibrated using an iterative procedure for transitionally rough ones. Computations of a zero-pressure-gradient boundary layer are performed using a prescribed \( k_+^4 \) and an assumed \( \Delta y^* = 0.03 k_+^4 \) at \( Re_\theta = 4,000 \). The L2 norm of the difference between the computed velocity profile and the experimental data of Ligrani and Moffat [19] is then calculated. This procedure is repeated for \( \Delta y^* - \epsilon \) and \( \Delta y^* + \epsilon \), and an optimal value of \( \Delta y^* \) is obtained for a particular \( k_+^4 \) using Newton iteration to minimize the error norm. The calibration curve for \( \Delta y^* \) vs \( k_+^4 \) reported by Durbin et al. [32] is presented in Figure 2.

![Fig. 2 Calibration of \( \Delta y \) for the zero-pressure-gradient boundary layer.](image)

### 2.3.3 SST K - \( \omega \) Model

The \( K - \omega \) model solves governing equations for \( K \) and an inverse time scale \( \omega \); the eddy viscosity is given by \( \nu_t = K / \omega \). The SST \( K - \omega \) model [23] uses the \( K - \omega \) model close to the wall and the \( K - \varepsilon \) model (written in \( K - \omega \) form) in the outer layer. A blending function \( F_1 \) is used to bridge the \( K - \omega \) model coefficients close to the wall to \( K - \varepsilon \) model ones away from it. The definition of the eddy viscosity is also modified:

\[
\nu_t = \frac{a_1 K}{\max(a_1 \omega, SF_2)}
\]

(9)

where \( a_1 = 0.31 \) and \( S \) is the magnitude of mean strain rate, \( S = \left( 2S_y \right)^{1/2} \). The max function ensures that the Reynolds shear stress does not change more rapidly than \( a_1 K \), and the function \( F_2 \) assumes a value of 1 in the boundary layer and swiftly changes to zero outside the boundary layer to guarantee that the limiter is deactivated in free shear flows.

For smooth wall calculations, a Dirichlet boundary condition with a value of zero is assumed for \( K \). Wilcox [22] enforced \( \omega = 6 \nu / \beta y^2 \) a small distance into the viscous sublayer (\( \beta = 0.0750 \) is a constant), whereas Menter [23] proposed \( \omega(0) = 60 \nu / \beta y^2 \), which is easier to apply numerically.

The roughness correction for the \( K - \omega \) model proposed by Wilcox [22] keeps the homogeneous Dirichlet boundary condition for \( K \) at the wall (which implies that \( \nu_t \to 0 \)) and modifies the boundary condition for \( \omega \) to increase the eddy viscosity close to the wall. At a rough wall \( \omega \) is given by

\[
\omega(0) = \frac{\nu^2}{\nu} S_r (k_+^4),
\]

(10)

and \( S_r \) is calibrated using the data of Schlichting [46], to give:

\[
S_r = \begin{cases} 
(50/k_+^4)^2 & k_+^4 < 25 \\
100/k_+^4 & k_+^4 \geq 25.
\end{cases}
\]

(11)

Hellsten and Laine [35] reported that this roughness correction, when applied to the SST \( K - \omega \) model, causes the limiter in (9) to be activated close to wall, which is undesirable. Therefore, they included an additional blending function \( F_3 \) to correct this behaviour;

\[
\nu_t = \frac{a_1 K}{\max(a_1 \omega, SF_2 F_3)}; \quad F_3 = \frac{1}{1 - \tanh \left( \frac{150 \nu}{\nu y^2} \right)^4}
\]

(12)

An alternative roughness model was proposed by Knopp et al. [36], who assign the eddy viscosity to satisfy logarithmic-law behaviour: the new definition of eddy viscosity is

\[
\nu_t(0) = u_r \kappa (y + \Delta y)
\]

(13)

The TKE \( K \) is given by (7). Given \( \nu_t \) and \( K \) at the wall, the resulting boundary condition for \( \omega \) at the wall is:

\[
\omega(0) = \frac{u_r}{C \kappa (y + \Delta y)}
\]

(14)
Knopp et al. [36] assign a fully rough value for $\Delta y$, which is multiplied by a blending function to reproduce the transitional roughness effects. However, their model under-predicts the drag in transitional roughness [37]. We followed the procedure proposed in [32] for the $K-\epsilon$ model, to determine $\Delta y$, and obtained the calibration curve shown in Figure 2. It is to be mentioned that Seo [38] also developed a similar calibration curve for the standard $K-\omega$ model, however, since SST $K-\omega$ performs better than the standard model for adverse pressure gradient flows, we decided to extend the calibration to the SST $K-\omega$ model. We will later see that the modified Knopp correction, using the present calibration, improves prediction in the transitional roughness region.

The simulations of the Reynolds-Averaged Navier-Stokes (RANS) equations were performed using the same numerical scheme and grid used in the LES. At the freestream and outlet, the same boundary conditions used for the LES were employed. At the inlet, we specified Dirichlet conditions on all quantities (velocity, turbulent kinetic energy and eddy). Turbulent kinetic energy and eddy viscosity were calculated from their definitions as

$$K = \frac{1}{2} \left( u_1'^2 + u_2'^2 + u_3'^2 \right); \quad \nu_t = -\frac{\tau_{12}}{S_{12}} \quad (15)$$

Since these calculations do not include the roughness sublayer, the solid surface is replaced by a virtual one in which the velocity vanishes, but the turbulent kinetic energy (TKE) and, in most cases, the eddy viscosity $\nu_t$ at $y=0$ are not zero. The logarithmic law is generally used to assign the values of TKE and $\nu_t$ at the virtual wall, but the wall distance is offset by an amount (generally taken equal to $0.03k_\tau$) to account for the presence of turbulence in the roughness sublayer. At the inflow, the mean velocity and turbulence profiles were drawn from LES data. The dissipation rate, $\varepsilon$, and turbulence frequency, $\omega$, were derived from $K$ and $\nu_t$ using the definition of eddy viscosity. Since, the above specification of turbulence parameters are not exact because of the differences in model specific definition of the turbulence parameters, we performed a sensitivity analysis by varying the input parameters and found that the results are insensitive (within a 2% difference) to the input parameters. This could be attributed to the fact that the ZPG region of the computational domain is long enough before the blowing/suction boundary condition is applied at the top surface to allow turbulence to reach equilibrium with the mean flow.

3. Results and discussion

We considered two test cases. In both of them, the boundary layer evolves on a flat plate, and a pressure gradient is imposed by suction and blowing on the top boundary of the computational domain. The first case is a boundary layer with separation induced by a strong adverse pressure gradient. We consider the case studied by Na and Moin [38], at a higher Reynolds number, suitable for turbulence modelling. The second test case has a realistic pressure gradient profile, much milder than in the first case, obtained from the velocity distribution outside the boundary layer in a hydraulic turbine distributor.

In both cases the RANS simulations are accompanied by LES of the same geometry, in the same conditions. These tests allow us to assess the accuracy of turbulence models in rough-wall boundary layers both in extreme cases (including separation) and in milder ones, in which the adverse and favourable pressure gradients do not perturb the flow far from its equilibrium state. The use of a flat-plate boundary layer as the basis, moreover, allows us to concentrate on the modelling errors and on the effects of flow acceleration alone, separate from those due to curvature. The fact that the numerical scheme is the same for LES and RANS calculations results in a consistent evaluation of the modelling errors. A grid convergence study was carried out by performing calculations on coarser and finer grids than the production one shown here. In all cases, the results presented did not differ by more than two percent from the ones obtained on finer meshes.

3.1 Separating boundary layer

Figures 1 and 3 show the computational domain used for the separating boundary layer calculations, and the freestream streamwise velocity and acceleration parameter. Only the part of the domain in which the freestream velocity is changed is shown in this and subsequent figures.

![Fig. 3 Separating boundary layer. Streamwise velocity distribution at the top surface (circles) and acceleration parameter $K$ (crosses).](image)

The Reynolds number, based on momentum thickness and freestream velocity at the reference location is $Re_\theta = U_\infty \theta_\theta / \nu = 2300$. Two roughness heights were considered, in addition to a case with smooth wall: $k_z/\theta_\theta = 0.5$ and 1; $\theta_\theta$ is the momentum thickness at the reference plane. $k_z/\theta_\theta = 0.5$ and 1 corresponds to $k_z^*$ at inlet of 65 and 170 respectively. Almost 400 million grid points...
were used in the LES. The LES model, in this configuration, has been validated by comparison with the DNS results of Na and Moin [39], as reported in [49]. For the RANS simulations, 110,000 grid points were used to discretize the 2D domain.

Figure 4 shows the skin-friction distribution for the smooth and rough cases obtained from LES and RANS models. The skin-friction coefficient is defined as

$$C_f = \frac{\tau_w}{\rho U_s^2/2}$$

$$\tau_w = 2 \frac{d C_D}{dx}$$

(16)

where $C_D(x) = 2D(x)/\rho U_s^2$ is the local drag coefficient (the integral of viscous and pressure force); $D(x)$ is the drag force per unit streamwise and spanwise length (for more details on the method used to evaluate $\tau_w$ see [14]), and $U_s$ is the freestream velocity at the reference location. Thus, the wall stress includes the contributions of both friction and the form drag around the roughness elements. The prediction of $C_f$ is insensitive to the detail of the correction used for SST methods (Figure 5) for fully rough case, $k_x/\theta_o = 1$. However, for the lower roughness case, which is in the transitionally rough regime ($k_x^+ = 65$ at the inlet) in the ZPG region, the Knopp et al. [36] roughness correction underpredicts the skin friction compared to other models. From here on, only the modified Knopp correction for the SST $K$-ω model (i.e., the present calibration) will be reported.

![Fig. 4 Separating boundary layer. Skin friction coefficients $C_f$ and $C'_f$. The $k_x/\theta_o=1.0$ data are shifted upwards by 4 units.](image)

![Fig. 5 Separating boundary layer. Comparison of SST modifications. The $k_x/\theta_o=1.0$ data are shifted upwards by 4 units.](image)

When the wall is rough, the wall stress becomes zero much earlier than in the smooth-wall case, at $x/\theta_o \approx 142$ rather than 207. This phenomenon, however, is not associated with separation of the flow from the solid surface, but rather with flow reversal inside the roughness sublayer [48]: below the roughness crest, a region of reversed flow is observed in the LES; this region becomes larger in the APG zone, until the average velocity in the roughness sublayer and the total stress (sum of friction and form drag) becomes
zero. Unlike the case of smooth walls, the condition \( \tau_w = 0 \) does not require that the flow separate from the surface: the flow reversal remains confined to the sublayer, and the fluid velocity above the roughness crest remains parallel to the wall. Only when the entire sublayer is experiencing reversed flow the fluid above the crest separates. This can be observed by comparing the contours of streamwise velocity (figure 6). In the rough-wall case the streamline dividing the outer flow from the recirculation region separates much downstream of the point where \( C_f = 0 \), at \( x/\theta_o \approx 190 \); the zero-velocity contour also emerges from the roughness sublayer around the same location. The real separation (as opposed to the flow reversal) correlates with the location where the total stress becomes negative: we can define a normalized stress at the roughness crest as

\[
C_f' = \frac{\tau_{crea}}{\rho U_o^2/2} = \frac{\tau_{crea}}{\rho} = \left( \frac{\partial U}{\partial y} - \langle u'v' \rangle \right)_{y=k_{max}}
\]

where \( <> \) represents time averaging. In turbulence models for rough surfaces the eddy viscosity at the virtual wall is non zero; the stress at the virtual wall, therefore, is

\[
\frac{\tau_w}{\rho} = \left[ (\nu + \nu_t) \frac{\partial U}{\partial y} \right]_{y=0}
\]

The first term in (18) is the viscous stress at the virtual wall (approximately the roughness crest), while the second the Reynolds stress at that location. \( C_f' \), therefore, represents the skin friction coefficient that would be calculated by a RANS; one cannot expect a turbulence model to predict \( C_f' \) correctly, but only to approximate \( C_f' \) which is also shown in Figures 4, 5 and 6, and approaches zero near \( x/\theta_o = 210 \), very close to the separation point. Thus, one can expect that, in APG boundary layers, RANS turbulence models would significantly overpredict the friction drag (as in the initial region of the flow in Figure 4), but would be more accurate in the estimation of the form drag, since the separation point is better captured; thus, in massively separated flow, in which pressure drag is larger than the friction component, the accuracy could be expected to be higher than in mildly separated ones, in which the errors in the prediction of the frictional component of drag would affect the results more. Notice that this is a feature present in any technique that takes the present modelling approach, using a virtual wall and bypassing the roughness sublayer entirely; it cannot be easily corrected by changing either the model coefficients or the modelling ansatz.

3.2 Boundary layer with realistic acceleration

Next, we considered a case with a realistic acceleration of the type that could occur, for instance, in the hydraulic turbine distributor shown in Figure 7. To separate the errors due to the effect of the pressure gradient from others such as streamline curvature or interblade crossflow, we extracted the velocity along a streamline on the pressure side of the stay vane and guide vane, and used that to generate the pressure gradient at the freestream over the flat plate. We then performed both LES and simulations that solved the RANS equations, with roughness corrected turbulence models. For the comparisons, we have considered SA with Boeing correction, two-layer \( K-\epsilon \) with Durbin correction and SST \( K-\omega \) with modified Knopp correction (present calibration). The roughness height chosen was \( k_s/\delta_0^* = 0.32 \), which corresponds to \( k_\ast \) at inlet of 45.

![Figure 6](imageurl) Separating boundary layer, LES. Streamwise velocity contours. The thick white line shows the separation streamline, the dotted line the \( u = 0 \) contour, the solid black line the point where \( C_f = 0 \), the dashed line \( C_f' = 0 \).
Since the LES could not be performed at a Reynolds number comparable to that in actual turbines, determining the pressure gradient at the freestream was somewhat arbitrary. In addition to $K_p$, Eqn. (2), another dimensionless acceleration parameter can be defined:

$$\Lambda = \frac{2}{C_f,0} \frac{\delta^*}{U_\infty} \frac{dU_\infty}{dx}$$  \hspace{1cm} (18)$$

Here the subscript $0$ refers to the reference station. $K_p$ is purely related to the freestream velocity and carries no information about the underlying boundary layer, whereas $\Lambda$ is a ratio of pressure forces to the viscous forces at the beginning of the pressure gradient region. At low Reynolds number, matching $K_p$ will result in a much lower change in the freestream velocity $U_\infty$ than observed in the real turbine, and might lead to underestimation of the pressure-gradient effects. Matching $\Lambda$, on the other hand, will result in more significant changes in $U_\infty$ than observed in the real case, and overestimation of the pressure-gradient effects. We compare simulations in which the product $\Lambda C_{f,0}$ is matched with those in which $K_p$ is matched, and expect these cases to be extrema, bracketing the real configuration.

Figure 8 shows the acceleration parameters and skin-friction coefficient in the two cases. As expected, when $\Lambda$ is matched, a significantly stronger acceleration is achieved, and the skin-friction coefficient is much larger (note that $C_f$ is normalized by the reference velocity). It is to be observed that when $\Lambda C_{f,0}$ is matched, the $K-\varepsilon$ model is able to predict the peak $C_f$ correctly. It can be observed that the peak $C_f$ is under-predicted by the SST $K-\omega$ model for the rough case. For the smooth, matched $-\Lambda C_{f,0}$ case, the LES data shows the tendency of the flow to revert to a quasi-laminar state, which none of the RANS models was able to predict. Mean velocity profiles (Figure 9) are also in reasonable agreement for the matched $-\Lambda C_{f,0}$ case. The matched $K_p$ corresponds to mild FPG and $k_\varepsilon^+ \approx 30-50$ within the computational domain; none of the model was able to predict $C_f$ correctly.

![Flow with realistic freestream velocity profile](image)

**Fig. 7** RANS prediction of the flow over a stay-vane, guide-vane combination.

*(a) Freestream velocity $U_\infty$ (circles) and acceleration parameter $K_p$ (crosses). (b) Skin-friction coefficient.*
An additional source of error, in industrial applications in which the Reynolds number is high, and the grid cannot be refined to resolve the wall layer, is in the use of wall functions, which relate the wall stress to the velocity when the first grid point is in the logarithmic region. Wall functions assume that the wall layer is in equilibrium (i.e., production equals dissipation) so that a logarithmic region exists. Such is not the case in flows with strong pressure gradients, three-dimensionality and separation. In this flow, in the region of strong acceleration, the accuracy of the wall functions decreases, and the error in the skin friction prediction (Figure 10) increases. The general behaviour is, however, still captured.

Fig. 9 Flow with realistic freestream velocity profile, case with matched $\Lambda, \frac{k_z}{\delta_o} = 0.32$. Mean velocity profiles.

Fig. 10 Flow with realistic freestream velocity profile, case with matched $\Lambda, \frac{k_z}{\delta_o} = 0.32$. Skin-friction coefficient obtained using ANSYS CFX with wall functions.

(The LES and $k$-$\varepsilon$ simulations were performed using the in-house code)
3.3. Flow in a hydraulic turbine

Having determined the response of turbulence models in two simplified applications, we then applied them in a more realistic configuration. ANSYS CFX is used to study the flow. No experimental or numerical data is available for this configuration; however, the simplified geometries indicate that the numerical model can predict the behaviour of the skin-friction coefficient, except in the regions of very high acceleration. We performed three simulations, two at the Reynolds number encountered in realistic applications, one at the Reynolds number of the LES and RANS described above, one tenth of the realistic one. At the high Reynolds number, two values of sandgrain roughness were used, $k_y/\delta^*_y = 0.03$ and 0.21. The first results in transitional roughness, the second in fully rough flow.

The skin-friction coefficient is shown in Figure 11. Several features are noteworthy. First, the general trends match those of the flat-plate boundary layer with realistic acceleration, notably the mild increase of $C_f$ on the first blade, the peak at the beginning of the second one with a subsequent decrease. Second, as expected, the magnitude of the $C_f$ is intermediate between the cases with matched $K_p$ and matched $A$. Finally, the Reynolds number dependence is weak. This is due to the implementation of the wall functions, that assume fully rough flow in the range of roughness heights used here.

![Fig. 11 Flow over the stay-vane, guide-vane combination. Skin-friction coefficient obtained using ANSYS CFX with wall functions ($K-\epsilon$ model).](image)

4. Conclusions

We have carried out large-eddy simulations (LES) of the flow in boundary layers with favourable and adverse pressure gradients, and compared the results with those obtained using turbulence models for the Reynolds-Averaged Navier-Stokes (RANS) equations.

The LES were used in simple geometries, and provided both data for model validation, and also allowed us to determine the intrinsic limitations of the turbulence models. For instance, the LES data showed that flow reversal occurs first in the roughness sublayer. This is due to the fact that the recirculation regions behind roughness elements widen as an effect of the adverse pressure gradient, until they occupy most of the span of the boundary layer. At this point the wall stress $t_w$ becomes negative, but the flow remains attached. Only further downstream the zero-velocity contour separates from the wall, as does the dividing streamline, characterizing the inception of real separation. The change of sign of the total stress (sum of Reynolds and viscous stresses) above the roughness crest correlates better, for the cases examined, with the true separation. The total stress is the quantity actually predicted by RANS turbulence models, and the models tested were able to calculate the real separation reasonably well. The skin-friction coefficient, however, was significantly overestimated because the roughness sublayer was bypassed.

More significant errors, also due to the fundamental eddy viscosity assumption, appear above the recirculation bubble and in the reattachment region. These errors are due to the fact that turbulence models are only sensitive to the mean shear, and do not account for the contribution of the eddies generated at the separation point and advected over the recirculation bubble, which eventually impinge on the wall near the reattachment point. This issue is present in smooth-wall boundary layers as well, and in rough-wall cases the model prediction is slightly more accurate, as the turbulence level near the wall, upstream of separation, is higher. These errors are difficult to correct within the eddy-viscosity framework, and will probably need to be included in the error bars of the calculations.

For acceleration levels of the type encountered in a distributor the errors are lower, since the flow remains attached. In rough-wall cases, however, the model accuracy suffers. The use of wall functions further degrades the model accuracy. The trends are, however, predicted correctly.

Application of the turbulence models in an industrial code, with a realistic geometry, shows the same trends observed in the simplified cases. This indicates that the approach taken here, to perform careful validations of the model in simple geometries, but with realistic freestream velocity distribution, can be an effective tool to evaluate the validity and accuracy of RANS solutions in hydraulic turbines.
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$U_\infty$</td>
<td>freestream velocity [m/s]</td>
</tr>
<tr>
<td>$u$, $v$, $w$</td>
<td>instantaneous velocity components [m/s]</td>
</tr>
<tr>
<td>$u'$, $v'$, $w'$</td>
<td>fluctuating velocity components [m/s]</td>
</tr>
<tr>
<td>$(u'v')$</td>
<td>Reynolds stress per unit mass ([m^2/s^2])</td>
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<tr>
<td>$x$, $y$, $z$</td>
<td>cartesian coordinates [m]</td>
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<tr>
<td>$\beta$</td>
<td>turbulence modelling constant</td>
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<tr>
<td>$\Delta U^+$</td>
<td>roughness function [dimensionless]</td>
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<tr>
<td>$\Delta y$</td>
<td>wall offset [m]</td>
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<tr>
<td>$\delta^*$</td>
<td>displacement thickness ([m^2/s^2])</td>
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<td>$\epsilon$</td>
<td>dissipation of TKE ([m^2/s^2])</td>
</tr>
<tr>
<td>$\theta$</td>
<td>momentum thickness ([m^2/s^2])</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>von Karman constant (=0.41)</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>acceleration parameter</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity ([m^2/s])</td>
</tr>
<tr>
<td>$\nu_t$</td>
<td>eddy viscosity ([m^2/s])</td>
</tr>
<tr>
<td>$\bar{\nu}$</td>
<td>auxiliary eddy viscosity ([m^2/s])</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density ([Kg/m^3])</td>
</tr>
<tr>
<td>$\rho(u'v')$</td>
<td>Reynolds stress [Pa]</td>
</tr>
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<td>$\tau_{ij}$</td>
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</tr>
<tr>
<td>$\omega$</td>
<td>turbulence frequency [1/s]</td>
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References


