A Numerical Study on Surge Degeneration (Stall Stagnation) and Recovery therefrom in Axial Flow Compressors

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Abstract

Phenomena of surge degenerations and recoveries therefrom, which are related with so-called stall stagnation problems, are studied on the basis of numerical-experimental results by one-dimensional surge simulations on a single-stage axial flow compressor and a five-stage one. The phenomena are observed to show some different tendencies depending on the number of stages and the relative location of the compressor in the flowpath. The mass flow amplitudes, as the measure of the surge sizes, show behaviors of either continuous decrease in the amplitudes or discontinuous ones in the degeneration process. The latter is seen in the five-stage compressor and the former in the single-stage one. The difference in the tendency appears to be influenced by the levels of the compressor pressure-ratios and by the relative compressor locations also. Recoveries from seriously degenerated surge situations are observed to be achieved by opening the exit valve widely. It suggests that the stall margins available for alleviating significantly the pressure loads are indispensable in the situation. In the sense, it could be said that the stall stagnation or non-recoverable stall is the phenomenon related intimately with the surge degeneration tendency in the compressor-flowpath system characteristics, deteriorated furthermore by operational factors on-site, such as insufficient stall margins and limited flexibilities of operational procedures.

Keywords: Compressor, Surge, Surge degeneration, Stagnation stall, Reduced resonance frequency, and Reduced surge frequency parameter

1. Introduction

The fundamental principle concerning the occurrences and behaviors of surges in pumping systems, such as pumps and compressors, was made clear first by Fujii [1] (cf. Yamaguchi and Tsujimoto [2] for reference). Since then, surges in the pump systems ceased to be intimidating phenomena of unpredictable sudden rush of big waves, which could have been overcome by devising various preventive measures (Oyama [3]). Compressor surges have been studied and clarified by Greitzer [4] on the basis of a simplified model. In nearly the same times, more precise methods of mathematical descriptions on surge behaviors in multi-stage axial flow compressors were presented by Corbett and Elder [5]. Since then, details and features of the surge behaviors in compressors have been being investigated both analytically and experimentally. They were reviewed by Greitzer [6 and 7]. Some of relatively recent ones were introduced by Cumpsty [8].

The present author has been studying surge phenomena in axial flow compressors by use of SRGTRAN, a code for numerical surge simulation prepared by himself (Yamaguchi [9]). The code of the kind could be not only useful practically, but also, as a numerical-experimental tool, could help clarifying basic physical rules behind the complicated phenomena. Particularly in the environment of compressor surges where a wide range of experiments and measurements are difficult to conduct, it could be effective in shedding light on the outline by conducting numerical simulations over a wide range of possibly essential parameters.

During the numerical studies, the author have had a feeling that some important points of general nature remained unclear. One of such problems is the unestablished rule of surge frequencies. It could have some relation with the system resonance frequency, since a term “resonant surge” was used in some literature (for example, Katto [10]). However, surge frequencies lower than that were measured often in some practical situations, and most of the simulated results gave much lower frequencies for multi-stage compressor situations. The related tendency was clarified and established only recently by the author (Yamaguchi [19 and 20]) as summarized in the next section.

Another important problem is the phenomenon of so-called “stall stagnation”, which is a catastrophic phenomenon encountered occasionally in jet engines, called also as non-recoverable stall or hung stall. The phenomenon appears to have long been known in jet-
engine field. Some papers have referred to the phenomena and described some study results concerned in as early as 1980’s (Stetson [11], Lorenzo, et al. [12]). Drewes [13] described about the stall stagnation as follows:

“An engine stall is a momentary hesitation in engine operation caused by a disturbance in the air flow and resulting in an aerodynamic stall of the compressor blades. If the stall is not self-recovering through immediate adjustment of the air flow, it is called a stagnation. The pilot must then shut down the engine and restart it.”

It could be caused by compressor stalls resulting from a variety of unfavorable situations related with, for examples, air-intake flow separations and inlet distortions in the aircraft maneuvers at high-angles of attack, or some strong shocks in the air flow, such as at the instant of afterburner ignition, and so on. The situations could often be difficult to recover to the stable engine conditions, which requires to shut off the engine and restart it even in flying (for example, Drewes [13], and Ishizawa [14]).

The situation could closely related with the post-stall zone, which is named “rotating stall zone” by Greitzer [4], into which the compressor falls and stays there. Although deep surges tend to describe loops encircling the stalling point and the wide area of the post-stall zone of the steady-state performance in the pressure-flow domain, the loops in the stagnation stalls tend to be confined within the post-stall zone and sometimes settle to a bottom point. The author would like to term the latter behavior “surge degeneration” in order to distinguish from the stagnation stalls, which possibly are peculiar to the field site of jet engine operations. The stagnation stall could be said to happen to occur in site, basically from the compressor surge degenerations but at the same time, affected severely by unfavorable operational environments, in a combined manner. The term particularly pays attentions to the recoverability of the engine to sound conditions. In the author’s prior papers, he has used the word “stagnation stall” in the sense of “degenerated surge”. He would like to exchange those words. If a better terminology from the aspect of fluid dynamics and engineering could be suggested, he would like to employ that.

It is to be added further that essential parameters governing them, which are crucially important for the consistent study in the above areas, were unknown also.

In the environments, the results by recent studies by the author (Yamaguchi [16-20]) could contribute to comprehension of the phenomena in the areas. They are based on considerations on many numerical-experimental results obtained by use of SRGTRAN. They are qualitative in the sense, but could help understanding the surge behaviors in the above areas up to the surge degenerations.

The present study aims to show the behaviors of surges in the degenerated zones and the possibility of recoveries thereof from, numerical-experimentally on the basis of simulated results. As typical examples, a single-stage fan, Comp11, and a five-stage compressor, Comp15, are selected for study on working conditions in low pressure-ratios and relatively high pressure-ratios, respectively.

2. General Behaviors of Compressor Surges

2.1 Whole Picture of Deep Surges and Degeneration Boundaries

A general picture of behaviors of deep surges in systems of axial compressors and flowpaths presently understood is shown before proceeding to the description on the present study. The compressor-flowpath system layout is shown schematically in Fig. 1. Here, $L_{c}^{*}$: length of the suction flowpath, including the compressor (m), $A_{c2}$: annulus area of the compressor exit (m$^2$), $L_{p}$: length of the delivery flowpath (m), and $A_{p}$: sectional area of the exit flowpath (plenum) (m$^2$). The following non-dimensional geometrical parameters are employed:

Flowpath length ratio:

$$LR = L_o / L_c^{*}$$

Sectional area ratio:

$$AR = A_p / A_{c2}$$

The distance between the compressor exit and the plenum inlet is assumed to be small, and the total length of the flowpath is defined as follows;

$$L_{tot} = L_{c}^{*} + L_o$$

In addition, the relative location of the compressor in the whole flowpath has a significant effect on the surge behaviors.

$$L_{c}^{*} / L_{tot} = 1 / (1 + LR)$$

Number of stages $z$ and pressure ratios $PR$ also affect the surge behaviors.

The essential parameter governing the surge frequencies and the surge degeneration boundaries is the following $M_c$-normalized flowpath-average reduced resonance frequency, as is presented in Yamaguchi [19 and 20];

$$f_{R_{ave}M_c} = f_1 \tau_{in} M_1 = f_1 (L_{tot} / V_{aver}) M_1$$

Here, $f_1$: resonance frequency of the first mode in the flowpath (Hz), $\tau_{in}$: time required for the fluid particle to pass through the whole flowpath (s), $M_1$: Mach number of the rotor tip peripheral speed for normalization purpose, $V_{aver}$: average velocity over the whole flowpath (m/s). $\tau_{in}$ is evaluated as the sum of the times required for passing through each control volume. The $f_{R_{ave}M_c}$ value is dependent on the flowpath configurations given by values of $LR$ and $AR$, and velocity distributions in the flowpath. It means times
of excitations on the fluid particle by the resonance frequency in the passing time, normalized by $M$, with respect to the typical Mach number; thus, it could be named $M$-normalized resonance excitation number, also (Yamaguchi [19]). The resonance frequency $f_0$ is here estimated for a still air column contained in the same flow path as that for the surge simulations, having the atmospheric pressure throughout, and the respective typical temperatures in the suction duct and in the delivery duct. The equations of sound are applied in an iterative manner together with the boundary conditions of an open inlet and a closed end.

For single-stage compressors, surge degenerations tend to occur at the $f_{R\text{iaM}}$ value of roughly 1.5 (Yamaguchi [19, 20]). The boundary between the zones for deep surges and degenerated surges are named” surge degeneration boundary”. For larger or smaller $f_{R\text{iaM}}$ values than the boundary value, deep surges or degenerated surges tend to occur. It means that the oscillation will grow to deep surges for sufficiently large $f_{R\text{iaM}}$ values, and decay for sufficiently small value.

Compressor surge frequencies and surge degeneration boundaries are shown as a basic framework in Fig. 2 (Yamaguchi [20]). In the figure, the ordinate is the $M$-normalized reduced surge frequency $f_{R\text{iaM}}$ defined as follows;

$$f_{R\text{iaM}} = \frac{f_{s0} \tau_{av} M_1}{f_{R\text{iaM}}(f_{s0}/f_1)} \quad (6)$$

Here, $f_{s0}$ is the surge frequency (Hz).

A series of data points connected together in Fig. 2 have conditions for a particular compressor at a particular rpm, housed in flow paths having a specific value of length ratio $LR$ with varying AR values; thus it shows behaviors of $f_{R\text{iaM}}$ vs. $f_{R\text{iaM}}$ for variations in the $f_{R\text{iaM}}$ values. Different data point group refer to different conditions. The data points for the larger $f_{R\text{iaM}}$ values in the same group are seen to be positioned approximately horizontally, i.e., to have nearly the same level of the $f_{R\text{iaM}}$ values. On the other hand, the data points for the smaller $f_{R\text{iaM}}$ values in the same group tend to show a complicated behavior and finally vanish or change abruptly. The left-most point is located immediately before the surge degeneration boundary. Abrupt reductions in the $f_{R\text{iaM}}$ value close to the degeneration boundary tend to indicate occurrences of subharmonic surges, which could often be a precursor of imminent surge degeneration.

In Fig. 2, number of stages and rmps of compressors are specified by colors and shapes of the data points, respectively. As indicated in the explanatory note, for example, “15-7” stands for the five-stage compressor at 7000rpm. The identical compressor is shown by the same color data. The relative locations of the compressors are not specifically indicated, although the effects are important also.

Characteristic zones of importance are encircled by dotted lines in Fig. 2. Behaviors of $f_{R\text{iaM}}$ vs. $f_{R\text{iaM}}$ in each zone have the following features.

1. Data points for multi-stage compressors away from the respective surge-degeneration boundaries tend to be in this zone. The behaviors are nearly horizontal. It appears to show the convective character of surges of filling and emptying type.

2. Data points for single-stage compressors of low pressure ratios tend to be in this zone. The behaviors show tendencies upward to right, in which surge frequencies tend to approach the resonance line, $f_{s0} = f_0$. It appears to show the surge character of near-resonant type where acoustic resonances appear to have some significant effects.

3. This zone contains the behaviors in approaching the surge degeneration boundaries, showing various manners. In many situations, especially, low pressure-ratio compressors, the degenerations tend to occur gathered around the $f_{R\text{iaM}}$ value of 1.5. In multi-stage compressors, some of the degeneration boundaries could reduce to much lower values of the $f_{R\text{iaM}}$ parameter. In some situations, the behaviors show either upward or downward movements.

4. In this zone mainly for multi-stage compressors, the behaviors tend to show abrupt and complicated changes in the $f_{R\text{iaM}}$ values, sensitively affected by small changes in the $f_{R\text{iaM}}$ values. Changes in the stage-matching conditions caused by the off-design speeds and possibly interacted with the local modes along the flowpath could result in the phenomena. Relative compressor locations could also affect the phenomena.

It could be said that the basic structure of the surge behaviors has been visualized generally in this manner in terms of the parameters of $f_{R\text{iaM}}$ and $f_{R\text{iaM}}$. Some portion of the two problems mentioned above in the introduction, i.e., the surge frequency and the surge degeneration boundary, appear to be solved. Further details are described in Yamaguchi [19 and 20].

The present study pays attention qualitatively to the phenomena in the area from the degeneration boundaries in Zone 3 to further degenerated surges at further reduced levels of the $f_{R\text{iaM}}$ parameter.

![Fig. 2 Essential framework of surge behaviors in terms of basic surge parameters, $f_{R\text{iaM}}$ vs. $f_{R\text{iaM}}$, together with surge degeneration boundaries (Yamaguchi [20])](image)
Fig. 3 Examples of tendency of surge loops in $P_r^2$ vs. $W_c^2$ domain for changes in values of $AR$ and $f_{klm+bt}$ from deep surges to seriously-degenerated ones. The condition is for Comp11, 10000rpm, $L_{lwc}^e = 0.342$. (a) Deep surges to degeneration boundary for $f_{klm+bt}$ values of 2.54 to 1.56 ($AR$ values of 6.5 to 2.1), including a degenerated one for $f_{klm+bt}$ of 1.53 ($AR$ of 2.0), and (b) Degeneration boundary to seriously-degenerated ones for $f_{klm+bt}$ values of 1.56 to 1.19 ($AR$ values of 2.1 to 1.0)

2.2 Surge Loops in Deep Surges and Degenerated Surges

Some examples of changing behaviors of surge loops affected by changes in the $f_{klm+bt}$ values are shown in Fig. 3 in the domain of the compressor exit pressure $P_r^2$ vs. mass flow $W_c^2$. The compressor is a single-stage one, Comp11 at 10000rpm, whose main data are listed in Table 1 given later and the flowpath is shown schematically in Fig. 1. The suction flowpath is the same for all the situations, and the length of the delivery flowpath $L_d$ is kept the same with the ratio $LR$ of 1.923, and the relative compressor location ($L_{lwc}^e$/$L_{lwc}^m$) of 0.342, at about a third of the whole flowpath. In the circumstances, only the sectional area ratios $AR$ are changed from 6.5 to 1.0, thus the $f_{klm+bt}$ values are changed. The calculation procedure is outlined in Section 4.

Figure 3(a) supersedes the surge loops varying for $AR$ values of 6.5 to 2.0, mainly for deep surges. The loops are seen to be lessening in the internal area, and shrink abruptly between $AR$ values of 2.1 and 2.0, where the surge degeneration occurs. Meanwhile the $f_{klm+bt}$ value reduces from 2.5 to 1.56. In the process, both the top pressure area in the low to negative mass flow range and the bottom pressure area in the maximum mass flow range are seen to be vanishing little by little. In other words, the surge loops metamorphose from fat ones to lean ones, and finally to the core deep loop for $f_{klm+bt}$ value of 1.56 immediately before the degeneration. The core loop is shown covered roughly by the green-colored shaded zone.

On the other hand, Fig. 3(b) shows the loops for the further lowered $AR$ values. The outermost one is the core deep loop for $AR$ value of 2.1, which is shown in Fig. 3(a) also. Further decreases in the $AR$ values make the loops smaller within the core deep surge loop, and enhance the degree of degenerations. These are named degenerated surges or minor-loop surges. The loops finally reach the condition of very small ones containing the bottom zone, as shown by the one for $AR$ value of 1.0 in Fig. 3(b). It could often reduce to nearly a point. These finally observed ones could be named seriously-degenerated surges.

As can be seen, the surges change the tendency of behaviors across the surge degeneration boundary. In addition to the above, in the neighborhood of the boundary, subharmonic surges could often occur, in which a complete surge cycle is made up of cycles of core deep loops and cycles of degenerated loops in an alternative manner (Fig. 4). In the situations, core deep loops and the degenerated loops tend to have nearly the same loop-cycle period. Thus the complete surge cycle tends to have a subharmonic frequency in comparison with the core condition.

The name “surge degeneration” is applied to both the degenerated surges and the subharmonic surges in the sense of the inclusion of surges other than the purely deep one.

When the $f_{klm+bt}$ values decrease, the surge conditions change in a manner as follows:

Deep surge (fat $\rightarrow$ lean $\rightarrow$ core) $\rightarrow$ Surge degeneration boundary $\rightarrow$ Subharmonic deep surge $\rightarrow$ Degenerated surge (minor-loop surge) $\rightarrow$ Seriously degenerated surge (degenerated to bottom-zone)

In the foregoing studies, the author has paid attention mainly to the phenomena of surge frequencies of deep surges and occurrences of surge degeneration boundaries, as seen in Fig. 2. In the present study, the phenomena seen in Fig. 3(b) is going to be studied.
3. Data for Analysis of Surge Behaviors in Axial Flow Compressors

Table 1 Main numerical figures related with the compressors for study

<table>
<thead>
<tr>
<th>Compressor Name</th>
<th>Comp15</th>
<th>Comp11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stages</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Inlet annulus area of stage 1: $A_{st1}$ ($m^2$)</td>
<td>0.10314</td>
<td>0.10314</td>
</tr>
<tr>
<td>Exit annulus area of the last stage: $A_{st5}$ ($m^2$)</td>
<td>0.07688</td>
<td>0.0975</td>
</tr>
<tr>
<td>Reference radius: $r_{st}$ (m)</td>
<td>0.254</td>
<td>0.254</td>
</tr>
<tr>
<td>Design rpm (rpm)</td>
<td>11300</td>
<td>11300</td>
</tr>
<tr>
<td>Design tip speed $u_{st}$ (m/s)</td>
<td>300.6</td>
<td>300.6</td>
</tr>
<tr>
<td>Suction temperature (K)</td>
<td>288.2</td>
<td>288.2</td>
</tr>
<tr>
<td>Suction pressure (Pa)</td>
<td>101300</td>
<td>101300</td>
</tr>
<tr>
<td>Suction duct length: $L_{pt}^{st}$ (m)</td>
<td>4.329</td>
<td>4.093</td>
</tr>
<tr>
<td>Stalling pressure ratio: $PR$</td>
<td>2.87 (12000rpm)</td>
<td>1.187 (10000rpm)</td>
</tr>
</tbody>
</table>
The two-step Lax-Wendroff procedure and the method of characteristics are applied to the compressor stage CVs and the flowpath CVs respectively. The stage behaviors are evaluated on the basis of the stage characteristics given in Fig. 5. The procedure does not consider swirl component including rotating stall phenomena, but only the axial component.

**d) Procedure of the study** First of all, a value of LR or a relative compressor location \((L^*/L_{eo})\) is fixed, and then a series of values of area ratio AR are given; then, the corresponding surge behaviors are surveyed for the situation of the values of LA and AR by use of SRGTRAN, so as to cover a wide range of surge conditions from deep surges to seriously-degenerated surges. Out of the results is selected a flowpath configuration that yields a seriously-degenerated surge behavior, for the purpose of the present investigation to study its recoverability to the sound performance. For the specified flowpath condition, the situations of the recovery are studied by opening the exit valve.

In the simulation, time is given as follows;

\[
t = k\Delta T
\]

(10)

Here, \(t\) time (s), \(\Delta T\) time step of calculation (s), and \(k\) integer expressing the time. \(\Delta T\) is determined so as to satisfy the Courant-Friedrichs-Lewy condition. \(k=0\) is the time when the initial steady-state calculation first arrives at the stalling condition of the most critical stage, for which condition the port area of the exit valve, \(A_{o, e}\), is determined in order for the mass flow to flow out under the pressure difference across the valve. It is the time to start the dynamic calculation phase in SRGTRAN, based on the initial steady-state information. For example, when the stalling point of the steady-state performance given by + mark in Fig. 4 is arrived at, the dynamic simulation process starts.

In order to secure the surging condition, the relative opening of the valve, \((A_o/A_{o, e})\), is throttled first to 0.9 of \((A_o/AR)/A_{o, e})\) in the time \(k\) from 0 to 10,000 \((=k_{ini})\). After the time, \((A_o/AR)\) is opened to the specified value of \((A_o/AR)/A_{o, e})\) in the time \(k_{ini}\) to \(k_{fin}\). For the time being, \(k_{ini}\) is set temporarily to be 20,000. After that, the valve area \(A_o\) is maintained constant at \(A_o/AR\). Here, \(A_o/AR\) and \(A_{o, e}\) are the initial valve area and the final one, respectively, and \(k_{ini}\) and \(k_{fin}\) are time numbers of the initial valve operation and the final one, respectively.

The valve treatment time is rather short. Typical times from initial stalling to \(k_{fin}\) in the present simulations are roughly 0.36-0.04 seconds for Comp11 and 0.30-0.13 seconds for Comp15. Resulting surge behaviors after a sufficient lapse of time are paid attention to.

4. Surge Behaviors in the Single-stage Compressor, Comp11 at 10000rpm

As an example of the situations related with low pressure-ratio compressors, surge behaviors in the single-stage compressor, Comp11, combined with various flowpath conditions are studied. While the identical suction flowpath is used, i.e., with values of \(L^*/L_{eo}\), and \(A_o\) kept constant, respectively, the delivery flowpath length \(L_o\) or the relative compressor location \((L^*/L_{eo})\) is specified. In the conditions, the sectional areas \(A_o\) of the delivery flowpath and therefore the \(f_{FINAULT}\) values are changed widely; thus the systematic variations in the surge behaviors could be observed numerically-experimentally. The relative opening of the exit valve \((A_o/AR)\) is kept 0.9\((=A_o/AR=AR/AR)\) throughout the initial survey phase.

Since the single-stage condition is unaffected by stage matching problems such as experienced in multi-stage conditions, it is convenient to watch the basic behaviors of surge.

With respect to Comp11 at 10000rpm, Fig. 6 shows behaviors of \(W_{c,2Max}\) and \(W_{c,2Min}\), i.e., the maximum values and the minimum ones of the surge mass flow ranges at the compressor exit (c2), against the AR values. The upper data groups and the lower ones are \(W_{c,2Max}\) and \(W_{c,2Min}\), respectively. The minor-loop data in subharmonic surges are shown by marks ‘×’. Data point groups are for several relative compressor locations \((L^*/L_{eo})\) ranging from near the flowpath inlet to roughly three-fourths of the total flowpath length.

Figure 6(a) shows behaviors of \(W_{c,2}\) for changes in the sectional area ratios, AR. Deep surges and degenerated surges are suggested by vertically wide gaps and narrow ones, respectively, between the upper data groups and the lower ones. The behaviors of \(W_{c,2}\) varying for changes in the AR values from larger ones to smaller ones suggest the manner of the surge degenerations affected by the AR values and the compressor locations. Particularly, for the compressor locations in the downstream side of the flowpath, abrupt and apparently discontinuous degenerations from deep surges to seriously degenerated situations are observed. On the other hand, for the compressor locations in the relatively upstream side of the flowpath, comparatively gradual changes to degeneration are observed. The AR values for the surge degeneration boundaries change much, depending on the relative compressor locations, particularly in the downstream locations of the compressor for \((L^*/L_{eo})\) greater than 0.6.

The situations are shown against the \(f_{FINAULT}\) values in Fig. 6(b). The changes in the gap widths are seen to be concentrated around \(f_{FINAULT}\) values of 1.5-1.6, which means the occurrence of surge degeneration boundaries there. The results agree with those in Zones ① and ② in Fig. 2, although Fig. 2 does not include those for \(f_{FINAULT}\) Values below degeneration boundaries.

The \(W_{c,2Max}\) behaviors against the \(f_{FINAULT}\) values are nearly the same for all LR values or relative compressor locations. However, the behaviors from near degeneration boundaries to degenerated surges are seen to be affected much by the relative compressor locations. For the relative compressor locations \((L^*/L_{eo})\) greater than 0.6, deep surges change discontinuously into seriously-degenerated surges. For the upstream locations, the changing manner is rather gradual. The downstream location of the compressor means that the delivery length ratio LR is smaller, which counteraction could appear in the tendency toward the larger AR values in Fig. 6(a). It might be related with the delivery volumes. The following parameter of volume-pressure ratio is paid attention to;

\[
VPR = LR \times AR \times PR
\]

(11)

Here, PR is the pressure ratio at the stall point.

Figure 6(c) shows the \(W_{c,2}\) behaviors against the VPR values. Discontinuous and abrupt changes around the degeneration are seen to gather together for the downstream side location of the compressor. The apparently discontinuous behaviors might be explained by the action of the relatively smaller volumes of the delivery flowpath.
By the way, if the flowpath in Fig. 1 is simplified to a configuration consisted of two ducts of suction and delivery having respective constant sectional areas, the particle passing time $\tau_{av}$ could be expressed approximately as follows:

$$\tau_{av} = \left[ \frac{L_{c}^{**}}{(u_{f} \phi_{f})} \right] + \left[ (u_{f} \phi_{f}) / AR / PR^{(k)} \right] = \left[ \frac{L_{c}^{**}}{(u_{f} \phi_{f})} \right] + LR \times AR \times PR^{(k)} \quad \text{(12)}$$

Here, $[L_{c}^{**}/(u_{f} \phi_{f})]$ is the particle passing time through the suction flowpath, and $[LR \times AR \times PR^{(k)}]$ is the ratio of the passing time through the delivery flowpath to the suction one. The latter term is akin to the $VPR$ parameter, eq. (11). It could explain a part of the tendency concerning the influences of the relative locations of the compressor, which is observed in Fig. 6(a) and (c).

Some examples of surge loops will be shown below.

(a) **Comp11, 10000rpm, ($L_{c}^{**}/L_{tot}$)=0.065 ($LR$=14.42), ($A_{v}/A_{v0}$)=0.9-0.9**

Figure 7(i) shows behaviors of mass flow ranges in terms of $W_{c2max}$ and $W_{c2min}$ vs. AR values of surge loops in Comp11 located very near the flowpath inlet, which is shown extracted from Fig. 6(a). Corresponding $f_{R1avMt}$ values (multiplied by 10) are included for reference in Fig. 7(i). When subharmonic surges occur, their data of $W_{c2max}$ and $W_{c2min}$ are additionally plotted as marks “×”. Surge degenerations tend to occur around the value of $f_{R1avMt}$ of 1.5-1.6. Above the value, deep surges occur, and below the value, the surge amplitudes tend to decrease, resulting in surge degeneration. Subharmonic loops appear around $f_{R1avMt}$ of 1.5 and the $W_{c2max}$ value decreases abruptly around $f_{R1avMt}$ of 1.45, showing the occurrence of surge degeneration. In the situation, the $W_{c2max}$ value immediately after the degeneration appears to be nearly equal to the $W_{c2max}$ value in a continuous manner. Therefore, the minor-loop in the subharmonic surge tends to predict the degenerated surge size.

Figure 8(i) (a)-(c) show surge loops $P_{c2}$ vs. $W_{c2}$ for the condition of Comp11, 10000rpm, and ($L_{c}^{**}/L_{tot}$) =0.065 for constant opening of the exit valve, ($A_{v}/A_{v0}$)=0.9-0.9, for three AR values, which reduces downward. Figure 8(i) (a), (b) and (c) show, respectively, a deep surge behavior near the core condition, a subharmonic deep surge showing a core deep loop and a minor-loop alternately, and a seriously-degenerated surge. The stage characteristics is shown by thin lines in each subfigure. The surge loop behaviors relative to the stage characteristics are understood. The surge behavior in Fig. 8(i)(b) is the same one as shown in Fig. 4.
(b) Comp11, 10000rpm, \( (\frac{L_{c}^{**}}{L_{t}}) = 0.51 \) (LR=0.9615), (Av/Aw)=0.9-0.9

Figure 7(ii) shows similar data as in Fig. 7(i) for Comp11 located near the central position of the flowpath, which is extracted from Fig. 6(a). Relatively continuous decrease in the \( W_{c2} \) gap width is seen in this situation. Figure 8(ii)(a)-(c) show surge loops at the typical points in Fig. 7(ii).
Figure 8(ii) shows the deep surge loop for AR of 2.4 and \( f_{R1avMt} \) of 1.565, which, however, includes deep loops having slightly different trajectories one another and could be regarded more precisely as (2/3) subharmonic deep surge without minor loops. It is just before the degeneration. In Fig. 8(ii)(b) the surge loop detaches from the stalling point ( mark “×”) of the steady-state performance, and settled in the post-stall zone. It is a minor-loop surge classified as a degenerated surge. Figure 8(ii)(c) shows a seriously degenerated surge behavior. As a whole, the surge loops tend to have shapes inflated upward and downward around the steady-state in-stall behaviors in comparison with those in Fig. 8(i).

(c) Comp11, 10000rpm, \( (Lc^{**}/L_{tot})=0.675 \ (LR=0.481), (A_v/A_v0)=0.9-0.9 \)

Figure 7(iii) shows similar data as in Fig. 7(i) for Comp11 located at about two-thirds of the flowpath, which is extracted from Fig. 6(a). At AR of 3.65 and \( f_{R1avMt} \) of 1.6, the \( W_{c2} \) range decreases discontinuously, and the surge degeneration occurs. When the compressor is located downstream as such, the surge degeneration tends to appear discontinuously, which is a situation drastically different from those for the relative locations \( (Lc^{**}/L_{tot}) \) of 0.05 and 0.51 in the above sections (a) and (b).

Figure 8(iii)(a)-(c) show varying surge loops for changes in the AR values, a deep surge, a degenerated surge, and, particularly, a seriously-degenerated one, respectively.

The surge behaviors in Comp11 at 10000rpm could be summarized as follows: For the compressor located in the upstream side in the flowpath, the surge mass flow amplitudes tend to decrease gradually for decreases in the values of AR or \( f_{R1avMt} \), and the degenerations occur continuously. On the other hand, the compressor located relatively downstream side, for example, at \( (Lc^{**}/L_{tot}) \) of 0.675, tends to fall abruptly into a seriously-degenerated condition for decreases in the values of \( f_{R1avMt} \). These surge behaviors depend on the selection of the values of LR and AR in the design stage of the flowpath configurations. If the deep surge behaviors are wanted, appropriate values of LR and AR should be selected with attentions to the \( f_{R1avMt} \) value.
5. Surge Behaviors in the Five-stage Compressor, Comp15 at 12000rpm

As an example of situations for comparatively high pressure-ratios, surge behaviors of the five-stage compressor, Comp15 at 12000rpm are shown in Fig. 9 in a similar manner to Fig. 6 for Comp11. The relative opening of the exit valve is kept at 0.9 ($=A_{Ao}/A_{Vo}=A_{Ao}/A_{Vo}$). Behaviors of $W_{c2}$ vs. $AR$ are shown in Fig. 9(a). For the compressor locations relatively upstream side in the flowpath, deep surges tend to degenerate abruptly and directly into seriously-degenerated surges. For some compressor locations in the downstream side, however, definite degenerations have not been calculated in the range of the present simulations, although some tendencies toward degenerations were observed. The causes for the difference have not yet been clear, but it might have some relation with the difficulty of simulations in the environment of the high pressure and the much limited delivery volume.

Figure 9(b) shows that the surge degenerations occur for the $f_{inj}$ values of 1.4 – 1.6. However, for the compressor location very near the flowpath inlet, ($L_{c}^*/L_{tot}$) of 0.091, the degeneration is seen to occur at very low $f_{inj}$ value. The phenomenon is seen in Fig. 2 also, where the degeneration points of Comp15 and Comp19 in zone ③ tend to come close to the resonance line ($f_{s0}=f_{r}$).

$$L_{c}^*/L_{tot}=0.091$$
$$L_{c}^*/L_{tot}=0.50$$
$$L_{c}^*/L_{tot}=0.667$$

Fig. 10 Surge flow ranges in Comp15, 12000rpm for changes in $AR$ values for reference of the working points in Fig. 11. 

(a) $AR=0.8$, $f_{inj}=0.8304$, deep surge
(b) $AR=0.725$, $f_{inj}=0.8576$, deep surge
(c) $AR=0.8$, $f_{inj}=1.0163$, serious degeneration

$+$ mark stands for the steady-state stalling point.
Figure 9(c) shows the $W_2$ behaviors against the VPR values. It is seen that for the compressor located upstream, $(L_2^*/L_{2ud})$ less than 0.25, degenerations tend to occur for VPR greater than 10, and for the compressor located downstream, $(L_2^*/L_{2od})$ greater than 0.5, degeneration occurs discontinuously, for VPR values less than 3. These tendencies are relatively similar to those for Comp11.

Generally, in Comp15, the surge degenerations appear to occur discontinuously, falling directly into seriously-degenerated surges, in contrast to those in many situations of Comp11. Some examples of the surge behavior are shown below.

(a) **Comp15, 12000rpm, $(L_2^*/L_{2ud})=0.9090$ (LR=10), $(A/A_o)=0.9-0.9$**

Figure 10(i) shows the behaviors of $W_2$ values, and $f_{Riastb}$ values multiplied by 10 against AR values for Comp15 located near the flowpath inlet. It is a part of Fig. 9(a). Deep surges occur for AR values greater than 0.725, and seriously-degenerated surges for AR values less than 0.713. Surge degeneration boundary exists about AR of 0.713-0.725 and $f_{Riastb}$ of roughly 0.85. Typical surge loops corresponding to some values of AR in Fig. 10(i) are shown in Fig. 11(ii)(a) – (c). Figure 11(i)(a) shows a deep surge condition, (b) the one immediately before degeneration, and (c) the seriously-degenerated surge condition, in which the pressure has dropped to the bottom and the mass flow stagnates near zero flow. A discontinuous change occurs between (b) and (c) in Figs. 11(i)(b) and (c).

(b) **Comp15, 12000rpm, $(L_2^*/L_{2od})=0.95$ (LR=1), $(A/A_o)=0.9-0.9$**

Figure 10(ii) shows the behaviors of $W_2$ Values, and $f_{Riastb}$ values multiplied by 10 against AR values for Comp15 located at the central position of the flowpath. It is also a part of Fig. 9(a). Some examples of surge loops for the situations are shown in Fig. 11(ii)(a) – (c). Deep surges occur for the conditions of the values of AR and $f_{Riastb}$ greater than 1.15 and 1.64, respectively, as shown in Fig. 11(ii)(a). The surges become degenerated for the conditions of the values of AR and $f_{Riastb}$ less than 0.9-0.8 and 1.56, respectively, as shown in Fig. 11(ii)(c). For the conditions of the values of AR of 1.1-0.95 and $f_{Riastb}$ of 1.575, both the deep loops and the degenerated ones are seen to coexist in Fig. 11(ii)(b), forming subharmonic deep surges, which is suggested in Fig. 10(ii).

(c) **Comp15, 12000rpm, $(L_2^*/L_{2od})=0.667$ (LR=0.5), $(A/A_o)=0.9-0.9$**

Figure 10(iii) shows the behaviors of $W_2$ values, and $f_{Riastb}$ values multiplied by 10 against AR values for Comp15 located at the (2/3) position of the flowpath. Some typical surge loops are shown in Fig. 11(ii)(a) – (c). Deep surges occur for the conditions of the values of AR and $f_{Riastb}$ greater than 0.56 and 1.56, respectively, as shown in Fig. 11(iii)(a). When the surges come close to degeneration, some disturbances appear and superpose on the trajectories, as seen in Fig. 11(iii)(b) at values of AR and $f_{Riastb}$ of 0.563 and 1.556, respectively. Figure 11(iii)(c) shows the degenerated situation at values of AR and $f_{Riastb}$ of 0.547 and 1.553, where the surge once falls into the bottom zone, then some high-frequency phenomenon superposes. It has a possibility of some calculation problems in the code. In very narrow volumes, the flow appears to be hard to be stabilized in calculation.

The surge behaviors in Comp15 at 12000rpm could be summarized as follows; In the situations of the five-stage compressor, two basic surge behaviors of a deep surge and a surge degenerated to the bottom zone, or a seriously-degenerated surge, exist. In the intermediate situations around the degeneration boundary, surges tend to have both basic loops combined alternately in time, which reduces the time-averaged mass flow, and results in a subharmonic surge. It is somewhat different from the single-stage compressor situations where the surge mass flow ranges tend to show gradual variations in many cases.

### 6. Recoveries from Seriously-Degenerated Surges

Out of the combined systems of the compressors and the flowpaths studied above, several situations of seriously-degenerated surges are selected. In the selected situation as the initial condition, the exit valve is opened from the initial relative opening $(A_{rel}/A_o)$ of 0.9 to several specified values for the final opening $(A_{op}/A_o)$, then kept as such; and the finally resulting surge behaviors are watched. The following results are the flow ranges obtained finally for respective relative valve openings, which are not the time histories following the valve opening.

#### 6.1 Recoveries from Seriously-Degenerated Surges for Comp11 at 10000rpm

In some examples of seriously-degenerated surge conditions in Comp11 at 10000rpm shown in Fig. 8, the fully settled surge behaviors are watched after a sufficient lapse of time after setting the final relative opening of the exit valve. The behaviors of the $W_2$ values plotted against several values of $(A_{rel}/A_o)$ could suggest the recovery process as a quasi-static tendency.

(a) **Comp11, 10000rpm, $(L_2^*/L_{2od})=0.0655$ (LR=14.42)**

The flowpath condition of LR and AR values of 14.42 and 1.2, respectively, is selected. The compressor is located near the inlet of the flowpath. Variations in the surge loops are seen in Fig. 12(ii) (a)–(c). The thin lines show the stage characteristics for reference. The final $W_2$ data are plotted against $(A_{rel}/A_o)$ values in Fig. 13(i). In the example, it is seen that the $W_2$ amplitudes, or the gap $(W_{2max}-W_{2min})$ do not change much, but the average $W_2$ increases. Finally, it recovers the steady-state sound flow condition at about $(A_{rel}/A_o)$ of 2. The recovery requires doubling the exit valve area. The recovery is attained without experiencing deep surges, with the $W_2$ amplitudes kept very small in comparison with deep surge ones.

(b) **Comp11, 10000rpm, $(L_2^*/L_{2od})=0.51$ (LR=0.962)**

The compressor is located near the central position of the flowpath. The flowpath condition of LR and AR values of 0.962 and 1.2, respectively, and $f_{Riastb}$ of 1.387 is selected, which is sufficiently degenerated as seen in Fig. 8(ii). Surge loops for the exit valve being opened are shown in Fig. 12(ii)(a)-(c), and the $W_2$ behaviors are shown in Fig. 13(ii). In the situation, recovery to the steady-state performance has been attained by opening the exit valve to 2.5 times of the initial setting $(A_{rel}/A_o=2.5)$. In the process, the $W_2$ amplitudes in the surge loops are 70-40 % of that of the ordinary deep surges.

(c) **Comp11, 10000rpm, $(L_2^*/L_{2od})=0.6755$ (LR=0.481)**

The compressor is located at roughly two-thirds of the flowpath. The flowpath condition of LR and AR values of 0.481 and 1.0, respectively, and $f_{Riastb}$ of 1.481, is selected, which is sufficiently degenerated as seen in Fig. 8(iii). Surge loops for the exit valve being opened are shown in Fig. 12(iii)(a)-(c), and the $W_2$ behaviors are shown in Fig. 13(iii). Opening the exit valve area does not necessarily increase the $W_2$ amplitudes, and the average mass flow does not increase much until $(A_{rel}/A_o)$ value of about 2.3, at which abrupt recovery in the flow takes place. Steady-state sound performance is attained at about $(A_{rel}/A_o)$ value of about 2.5. The recovery does not go through deep surge loops.
The surge recoveries described above on the single-stage compressor, Comp11 at 10000rpm, is summarized as follows: 1. In the recoveries from the seriously-degenerated surge conditions in Comp11 at 10000rpm by opening the exit valve area, 2. 3 to 2.5 times of the initial valve area are required. In the recovery process, the surge flow amplitudes used to be much smaller than those in the ordinary deep surges. It could be explained by significantly low levels of the instability enhancement factor ($\sigma$-factor) (Yamaguchi[16]) in the degenerated situations, which suppresses the growth of the amplitude, although it is an intrinsically unstable phenomenon. Section 7(4) will explain briefly about the situation concerned.

![Surge loops](image)

**Fig. 12** Surge loops for conditions of Comp11, 10000rpm, for port-areas of the exit-valve being opened for recovery purpose from seriously-degenerated surge conditions. Top figures: degenerated-surge condition, and bottom figures: recovered condition by sufficiently-wide opened valves. The surge flow ranges are shown in Fig. 13 below. "**" mark stands for the stalling point.

![Surge ranges](image)

**Fig. 13** $W_2$ ranges in surge for conditions of Comp11, 10000rpm, for port-areas of the exit-valve being opened for recovery purpose from seriously-degenerated surge conditions.
Recoveries from Seriously Degenerated Surges for Comp15 at 12000rpm

In some examples of seriously-degenerated surge conditions in Comp15 at 12000rpm shown in Fig. 10, the surge behaviors are watched after a sufficient lapse of time after setting the final relative opening of the exit valve. The behaviors of the $W_{c2}$ values plotted against several values of $(A_{v_{fin}}/A_{v0})$ could suggest the recovery process as a quasi-static tendency.

Fig. 14 Surge loops for conditions of Comp15, 12000rpm, for port-areas of the exit-valve being opened for recovery purpose from seriously-degenerated surge conditions. Top figures: degenerated-surge condition, and bottom figures: recovered condition by sufficiently-wide opened valves. The surge flow ranges are shown in Fig. 15. “+” mark stands for the steady-state stalling point.

6.2 Recoveries from Seriously-Degenerated Surges for Comp15 at 12000rpm

In some examples of seriously-degenerated surge conditions in Comp15 at 12000rpm shown in Fig. 10, the surge behaviors are watched after a sufficient lapse of time after setting the final relative opening of the exit valve. The behaviors of the $W_{c2}$ values plotted against several values of $(A_{v_{ini}}/A_{v0})$ could suggest the recovery process as a quasi-static tendency.

(a) Comp15, 12000rpm, $(L_{c}^{**}/L_{tot})=0.0909$ ($LR=10$), $AR=0.6$

For the compressor located near the inlet of the flowpath, $(L_{c}^{**}/L_{tot})$ of 0.0909, the AR value of 0.6 is selected as a typical example in a seriously-degenerated surge condition as can be seen in Fig. 10(i). The recovery process by opening the exit valve from the initial value of the relative valve opening $(A_{ini}/A_{0})$ of 0.9 is shown in Fig. 14(i)(a)-(c) and Fig. 15(i). The surge behaviors change abruptly to
a deep one at $(A/A_0)$ of 0.9875, which continues up to about 1.5, and recovers onto the sound performance line at 1.6. The surges appear to have only the two modes of the deep surge and the seriously-degenerated surge. Changes between both modes occur in an apparently discontinuous manner.

In this particular case given in Fig. 14(i) and Fig. 15(i), the extent of the relative opening of the exit valve required to recover is rather small, compared with the single-stage cases and the following result for the five-stage case in the next paragraph. It suggests a chance to facilitate the surge recovery. This result is considered to have a strong relation with the degeneration boundary extended toward much lowered $f_{res,Mt}$ value, observed for the relative compressor location very near the flowpath inlet in Fig. 9(b).

The behaviors are different also from those in the single-stage compressor, Comp11. The surge behaviors in the process for the five-stage compressor change rather early to the deep surge mode including the stalling point (Fig. 14(ii)), although for the single-stage compressor, the average surge mass flows increase gradually with the amplitudes kept rather small (Fig. 13). The differences might be related to the differences in the pressure levels and the volume ratios of the delivery flowpaths, and also to the differences in the near-resonant surge character for the single-stage compressor and the convective surge character for the five-stage compressor.

(b) **Comp15, 12000rpm, $(Ls''/Ltot)=0.5, (LR=1), AR=0.8**

For the compressor located near the central position of the flowpath, $(Ls''/Ltot)=0.5$, the $AR$ value of 0.8 is selected as a typical example in a seriously-degenerated surge condition as can be seen in Fig. 10(ii). The recovery process by opening the exit valve from the initial value of the relative valve opening $(A_{rel}/A_0)$ of 0.9 is shown in Fig. 14(ii)(a)-(c) and Fig. 15(ii). The following processes are observed: degenerated surges for $(A/A_0)$ of $0.9 - 1.0$, and subharmonic deep surges for $(A/A_0)$ of $1.05 - 1.1$, deep surges for $(A/A_0)$ more than 1.1, and recovery and stabilizations at 2.3. Similarly to the above case, only the two basic modes of the deep loop and the degenerated loop exist. Subharmonic surges could appear as combinations of the two. The behavior for the $(A/A_0)$ of 1.1 in Fig. 14(ii)(b) corresponds to $(2/3)$ subharmonic one.

In this case of the compressor located at the central position of the flowpath, the surge recovery requires much wider opening of the exit valve up to $(A_{rel}/A_0)$ of 2.3, in comparison with that for the compressor located very near the flowpath inlet.

The surge recoveries in the five-stage compressor, Comp15 at 12000rpm, is summarized as follows: Two basic surge modes of the deep one and the seriously-degenerated one appear to exist during the recovery process by opening the exit valve. Either of the two appears basically during the recovery process. In some narrow zones of the recovery process, subharmonic surges including both of the two could appear.

From the stand point of the recoveries, it would be favorable to locate the compressor upstream as far as possible in the flowpath. As is observed in Fig. 15(i), the recovery could be relatively quick for the compressor located very near the flowpath inlet, where the relative opening $(A_{rel}/A_0)$ of nearly 1.5 can do, whereas for the compressor located near the central position is required the relative opening $(A_{rel}/A_0)$ of roughly as large as 2.3 in Fig. 15(ii). To return only to the deep surge conditions, the relative opening $(A_{rel}/A_0)$ can be relatively small, such as about 1.0.

The surge behaviors in the recovery process are quite different between the five-stage compressor and the single-stage compressor, in that the latter tends to show gradual increases in the average surge mass flows with the amplitudes kept rather small (Fig. 13), although the former shows abrupt and discontinuous return to deep surge conditions from the seriously-degenerated conditions (Fig. 15).

7. Summary and Considerations

The above numerical-experimental study results will be summarized and investigated as follows;

1. **Changes in the surge behaviors affected by changes in the flowpath configurations**

When the values of the $M_t$-normalized flowpath-average reduced resonance frequencies $f_{res,Mt}$ are reduced by decreasing the area ratios $AR$ or the $f_{res,Mt}$ values, the surge behaviors tend to follow the metamorphosis process described below:

(a) Deep surge for sufficiently large values of $f_{res,Mt}$

- Core deep surge at the degeneration boundary,
- Subharmonic deep surge near the surge degeneration boundary,
- Minor-loop surge for further reduced level of $f_{res,Mt}$ value, and
- Seriously-degenerated surge (falling onto the bottom zone) for very low level of $f_{res,Mt}$ values.

The degenerated surges in the present study include above items (c)-(e). In the situations, the substantial surge mass flow amplitudes tend to decrease below the core deep surge one. In the subharmonic surges in (c), the time-averaged mass flow amplitude becomes smaller, which could be a sign of surge degenerations.

2. **Appearance of degenerated surges**

The following two types of behaviors of decreasing surge mass flow ranges are observed in the process of reductions in the $AR$ values or the $f_{res,Mt}$ values.

For Comp11, the surge mass flow amplitudes tend to decrease continuously(ya→(b)→(c)→(d)).

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Fig. 16 Values of $f_{res,Mt}$ at degeneration boundaries against the relative compressor locations. Marks x and + show existence of subharmonic deep surges.
For Comp15, the amplitudes tend to decrease discontinuously from the deep surge one (a) to the seriously-degenerated one (e), in many cases, by way of subharmonic surges (c).

In the latter situations, the deep surge mode (a) and the seriously-degenerated mode (e) are the basic surge modes. The two have definitely different magnitudes of surge amplitudes, respectively. At the complete degeneration boundary, the amplitudes decreases discontinuously. In some situations in the neighborhood of the degeneration boundaries, the surges could become subharmonic, in which both types of the modes or loops appear alternately or in series, thus lessening the time-average mass flow. For Comp11 located in the downstream side, a similar discontinuous degeneration is seen to occur.

The different manners of the surge appearances might have some relation with the near-resonant nature in Comp11 circumstances and the convective nature in Comp15 circumstances. In addition to that, it might be related with the pressure ratios and the volume ratios in the delivery flowpaths. Conclusive explanations about the distinctions have not yet been obtained.

(3) Surge degeneration boundary

The degeneration boundaries \( f_{\text{friction}}/(\text{degen}) \) read from the results are plotted in Fig. 16 against the compressor relative locations. Marks “+” and “x” show the occurrences of subharmonic surges between them. In the situations, the degeneration boundary is understood as the deep surge point immediately before (above) the subharmonic point.

The values of the degeneration boundaries are seen to be generally about 1.6 for Comp11, and 1.6-1.7 for Comp15. For the compressor locations from the inlet to about 10% of the flowpath, however, the values of \( f_{\text{friction}}/(\text{degen}) \) tend to be lower, particularly for the case of Comp15, suggesting that the compressors, particularly, Comp15, are reluctant to degenerate in the locations very near the inlet. The tendency has been observed also in Fig. 2 and in Yamaguchi [19 and 20].

For the compressor locations downstream, for example, from halfway, rather complicated behaviors of the degeneration boundaries are seen. In the circumstances, the compressors are located in the zone of the basic acoustic modes changeable locally. In addition to that, the values of the delivery flowpaths tend to be smaller in the situations. These two factors might be responsible for the appearance in that they could make the unsteady flow further complicated.

It is to be added here that the \( f_{\text{friction}}/(\text{degen}) \) value for the degeneration boundary is of qualitative nature, since it has been determined numerical-experimentally. It has not yet been confirmed experimentally or empirically in a strict sense. From comparisons with a few experiences, the author considers that the actual level for compressors having low pressure ratios might be as follows (Yamaguchi [20]):

\[
 f_{\text{friction}}/(\text{degen}) \to 3 \quad (\text{empirical})
\]

In other words, the values in the real field of compressors could be roughly twice the numerical-experimental ones.

(4) Cause of suppressed mass flow oscillations in degenerated situations

The surge mass flow ranges tend to be suppressed significantly in the degenerated environment. The situation could be explained as follows (Yamaguchi [16]). The circumstances could be caused by significantly low levels of instability enhancement factor (\( \sigma \)-factor), which tend to reduce the amplitudes of the unstable oscillation. The \( \sigma \)-factor depends on the flowpath geometry.

The \( \sigma \)-factor expresses the tendency of growth of oscillation amplitude in an extremely simplified system of a compressor element and a flowpath. The time behaviors of the flows and pressures in an infinitesimal surge in the system are assumed to be proportional to:

\[
 \exp(st)
\]

Here, \( t \) is time (s), and \( s \) is characteristic root of the system (s\(^{-1}\)), which is assumed as follows;

\[
 s = \sigma + j\omega
\]

Here, \( \omega \) is circular frequency (s\(^{-1}\)), and \( \sigma \) is growth rate of the amplitude(s\(^{-1}\)). Then, the \( \sigma \)-factor is defined as follows;

\[
 \sigma^* = (\sigma / \omega) / TT
\]

Here, \( TT \) is assumed as the non-dimensional characteristic slope of the compressor element;

\[
 TT = \frac{1}{\rho c} \left( \frac{dp}{du} \right)_{\text{stage}}
\]

Here, \( (dp/du)_{\text{stage}} \) is the slope of the compressor stage element, \( u \) and \( p \) are the flow velocity (m/s) and the pressure rise (Pa) through the element. \( \rho \) is the flow density (kg/m\(^3\)), and \( c \) is the sound speed (m/s). Yamaguchi [16] assumes the term \( (dp/du)_{\text{stage}} \) as a given constant value very close to zero. The \( \sigma \)-factor is shown to be analytically given uniquely by the flowpath geometry specified by a combination of values of \( LR \) and \( AR \).

When the surge degeneration boundaries obtained numerical-experimentally for single-stage compressors are compared with the calculated \( \sigma \)-factor data, they are seen to be corresponding to a zone of relatively small values of the \( \sigma \)-factor. The \( TT \) has a small value very close to zero, since it normalizes the stage characteristics slope near the stalling point. Therefore, the magnitude of \( \sigma \) is very small around the degeneration boundaries. Namely, the growth rate of the amplitudes of the unstable oscillation is very small in the surge degeneration zone, and the oscillations tend to be suppressed. Namely, in the surge degeneration zone, the oscillations are unstable but could hardly grow in the amplitudes.

The \( \sigma \)-factor has been derived only analytically in hypothetical infinitesimal surges, and it is seen to have a unique relation with the flowpath geometry. And the degeneration boundaries obtained for single-stage compressors numerical-experimentally tend to show relatively reasonable correlations with the \( \sigma \)-factor values. From these aspects, the \( \sigma \)-factor could be supposed to be in some relation with the time \( \tau_{es} \) for the fluid particle passing through the whole flowpath, which is determined mainly by the flowpath geometry. As
can be understood by the parameter $f_{k_{\text{Klo}}}$ defined by eq. (5), only when the fluid particle is excited by the resonance frequency more times than some critical value, the surge oscillation can grow sufficiently.

(5) Recoverability from the seriously-degenerated surges

It is difficult to investigate the in-situ situations of stall stagnations and recoveries therefrom in jet engines by the simple simulation procedures of SRGTRAN code. So, the present study has tried to see the changes in surge conditions in the compressors having once fallen into a seriously-degenerated surge condition, which situation is tried to get out from by opening the exit valve.

Recoveries from sufficiently degenerated surge conditions are observed to be possible by opening the exit valve to the relative opening of roughly 2.5 times of the initial setting within the studied cases. In the recovery process in Comp11 located rather upstream side in the flowpath, the average mass flows tend to increase gradually up to the steady-state performance curve, maintaining the small mass flow amplitudes, i.e., with no appearances of deep surges. For Comp11 located far downstream, the recovery process appear similar to the Comp15 situation described in the next paragraph.

In the recovery process in Comp15, the surge behaviors in the seriously-degenerated conditions are maintained up to some extent of opening of the exit valve, and the surge amplitudes enlarge abruptly and discontinuously, forming deep surges. After the appearance of the deep surges, furthermore opening of the exit valve is required in order to recover onto the steady-state performance curve. For the recovery, it appears necessary to open the exit valve to roughly 2 – 2.5 times of the initial setting. For Comp15 located very near the flowpath inlet, however, the recovery can be attained by opening only about 1.6 times of the initial setting.

To open the exit valve is equivalent in a sense to alleviate the pressure loads in the flowpaths in ordinary plants. However, it could be difficult in many occasions to reduce the loads which have been occupied for the purpose of satisfying the system specifications. Thus, the stagnation stalls mean the situations where the compressors having fallen into seriously-degenerated surge conditions are hard to recover to the steady-state performances because of insufficiency of operational surge margins and of control flexibilities. The term of stall stagnation should be used appropriately in the sense, in distinction from the term surge degeneration. The term “stagnation stall” appears to have been used exclusively in the field site of jet engines.

(6) Effects of the exit valve manipulations on the surge modes

It would be useful to watch the surge behaviors in relation with the valve effects. They are examined in the APPENDIX. In the ordinary deep surge conditions in Fig. A1, the behaviors of pressure vs. mass flow in the upstream locations show sufficiently wide mass flow amplitudes. In the immediately upstream of the valve, however, the behaviors tend to lie near the center line of the mass flow oscillations in the upstream zones. It shows the oscillation behaviors in the conditions of the open inlet and the half-closed exit.

In the degenerated surge conditions of Comp11, the initial surge loops shown in Fig. A2(a) are completely suppressed in comparison with those in the deep surges (Fig. A1(a)). By opening the exit valve (Fig. A2), the situation tends to be little improved, in which the average mass flows tend to increase, but the oscillation amplitudes kept suppressed as small as the one upstream of the valve.

In the situation, clear oscillation modes do not exist in the axial direction over the flowpath.

In the case of Comp15, the seriously degenerated surge condition in Fig. A3(a) is settled onto the bottom zone of flow and pressure, possibly determined by the compressor steady-state performance. It is quite different from the sufficiently deep surge conditions in Fig. A1(b). By opening the valve, it gets out of the circumstance, changing into subharmonic surges consisted of the seriously-degenerated surge loop and a deep loop as shown in Fig. A3 (b), and, finally, deep surges shown in Fig. A3 (c). In the process in Fig. A3 (b) and (c), the surge mode is seen to appear in the axial direction.

As is observed above, definite differences exist in the effects of opening the exit valve between the single-stage compressor and the five-stage compressor. It could have some relation with the level of the system pressure ratios and the compressor working conditions. The former could be related with either the near-resonant type surge or the convective surge of filling and emptying type, and the latter possibly with the stage matching conditions in multi-stage environments.

(7) Stagnation stalls in jet engines

The flowpaths of jet engines are considered in comparison with the present results, though very qualitatively. The turbine blade rows could be regarded very roughly as an equivalent to the exit valve in the present study, in the role of the flow throttling actions. The combustion chamber and the combustors could be corresponding to the delivery flowpath (plenum) in the present study, which could be rather short in the length and small in the sectional area. For the engines which tended to fall frequently into stall-stagnations, a relatively long passage of the air intake appear to be equipped. The environment could be corresponding to a flowpath having smaller values of $LR$ and $AR$, in which environment the tendency toward serious degenerations appear to be significant.

From the results of the present study, high pressure-ratio compressors tend to seriously degenerate in surge. For the recovery, the exit valve should be opened very widely, possibly much more in comparison with the 2-2.5 times of the initially-set value in the very low pressure ratio for Comp15. The equivalent load alleviations could hardly be possible in the real engine flowpaths having small surge margins due to large pressure losses as fixed loads.

Jet engines could stall by many causes, such as, ingestions of distorted inlet flows caused by hard maneuvers of the aircraft etc. and shocks given to the flow e.g. in lighting the afterburners, etc. In addition to the surge degeneration environments, the recoveries of the engines could be prevented, for example, by the engine power losses resulting from the engine speed reductions in the situations. Although the surge degenerations could be a compressor phenomenon in the post-stall zone, the stall stagnations are the phenomena as the whole system of the engine and the airframe caused by the many factors described above, coupled with the surge degenerations.

From the aspect of the present study, the following points might help the improvements of the surge degeneration tendency. One is to increase the $f_{k_{\text{Klo}}}$ value, mainly the value of $(f_t$ $\tau_w)$, by modification of the flowpath configuration. At the same time, locating the compressor near the inlet of the system could also help the improvement.

In addition to the above, it is recommended to increase the stall margins of the compressors both in designs and operations. Langston [23] has recommended a variety of concepts for increasing surge margins, such as wide-chord blades, anti-stall devices,
variable geometries, less shocks to the flow, prevention of excursions of the operating conditions, etc., possibly aiming to increase the tolerability to inlet distortions and to improve the stall stagnation environments.

8. Conclusion

Phenomena of surge degenerations related with stall stagnations and recoveries therefrom are studied on the basis of numerical-experimental results by one-dimensional surge simulations on a single-stage axial flow compressor and a five-stage one. The phenomena are observed to show some different tendencies depending on the number of stages and the relative location of the compressor in the flowpath. The mass flow amplitudes, as the measure of the surge sizes, show behaviors of either continuous decrease or discontinuous ones in the degeneration process. The latter is seen in the five-stage compressor and the former in the single-stage one. The tendency appears to be influenced by the levels of the compressor pressure-ratios and by the relative compressor locations also. Recoveries from the seriously-degenerated surge situations are observed to be achieved by opening the exit valve very widely. It turned out that opening the valve very widely is indispensable for the recovery. It suggests that the margins for alleviating significantly the pressure loads are indispensable in the situation. In the sense, it could be said that the stall stagnation or non-recoverable stall is the surge degeneration phenomenon deteriorated further by operational factors on-site, such as insufficient stall margins and limited flexibilities of operational procedures.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>Annulus area of the compressor inlet (m²)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>Annulus area of the compressor exit (m²)</td>
</tr>
<tr>
<td>( A_e )</td>
<td>Area of exit valve port (m²)</td>
</tr>
<tr>
<td>( A_R )</td>
<td>Sectional area of delivery flowpath (m²)</td>
</tr>
<tr>
<td>( AR )</td>
<td>Ratio of the delivery sectional area to the compressor exit annulus area</td>
</tr>
<tr>
<td>( c )</td>
<td>Speed of sound (m/s)</td>
</tr>
<tr>
<td>( dp/du )</td>
<td>Slope of the characteristics of pressure-rise to flow velocity of a compressor element (Pa-st/m)</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>Resonance frequency in the flowpath (Hz)</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>Surge frequency of the compressor-flowpath system (Hz)</td>
</tr>
<tr>
<td>( f_{reduced} )</td>
<td>( M_t )-normalized flowpath-average reduced resonance frequency, or ( M_t )-normalized resonance excitation number</td>
</tr>
<tr>
<td>( \dot{f}_{reduced} )</td>
<td>( M_t )-normalized flowpath-average reduced surge frequency</td>
</tr>
<tr>
<td>( j )</td>
<td>Imaginary unit</td>
</tr>
<tr>
<td>( k )</td>
<td>Time number</td>
</tr>
<tr>
<td>( L_{suc} )</td>
<td>Length of the suction flowpath including the compressor (m)</td>
</tr>
<tr>
<td>( L_{suc}/L_{flow} )</td>
<td>Relative location of the compressor in the flowpath</td>
</tr>
<tr>
<td>( L_e )</td>
<td>Length of the delivery flowpath (m)</td>
</tr>
<tr>
<td>( LR )</td>
<td>Ratio of the length of the delivery flowpath length to the suction one</td>
</tr>
<tr>
<td>( L_{tot} )</td>
<td>Total length of the whole flowpath (m)</td>
</tr>
<tr>
<td>( M_t )</td>
<td>Mach number of the tip peripheral speed of the first rotor</td>
</tr>
<tr>
<td>( PR )</td>
<td>Compressor pressure ratio at stalling</td>
</tr>
<tr>
<td>( r_{21} )</td>
<td>Radius of the first-stage rotor blades (m)</td>
</tr>
<tr>
<td>( s )</td>
<td>Characteristic root (s²)</td>
</tr>
<tr>
<td>( t )</td>
<td>Time (s)</td>
</tr>
<tr>
<td>( \tau_{ave} )</td>
<td>( \tau ) (s)</td>
</tr>
<tr>
<td>( TT )</td>
<td>Normalized slope of pressure-rise to flow velocity of a compressor element</td>
</tr>
<tr>
<td>( u_{11} )</td>
<td>Tip peripheral speed of the first-stage rotor blades (m/s)</td>
</tr>
<tr>
<td>( P_e )</td>
<td>Pressure at the compressor exit (Pa)</td>
</tr>
<tr>
<td>( V_{ave} )</td>
<td>Velocity averaged over the whole flowpath</td>
</tr>
<tr>
<td>( VR )</td>
<td>Volume ratio</td>
</tr>
<tr>
<td>( VPR )</td>
<td>Volume-pressure ratio</td>
</tr>
<tr>
<td>( W )</td>
<td>Mass flow (kg/s)</td>
</tr>
<tr>
<td>( W_{c1} )</td>
<td>Mass flow at the compressor exit (kg/s)</td>
</tr>
<tr>
<td>( W_{c2} )</td>
<td>Mass flow at the compressor exit (kg/s)</td>
</tr>
<tr>
<td>( W_{cmax} )</td>
<td>Maximum mass flow in the surge cycle at the compressor exit (kg/s)</td>
</tr>
<tr>
<td>( W_{cmaxsub} )</td>
<td>Maximum mass flow in the subharmonic surge loop at the compressor exit (kg/s)</td>
</tr>
<tr>
<td>( W_{cmin} )</td>
<td>Minimum mass flow in the surge cycle at the compressor exit (kg/s)</td>
</tr>
<tr>
<td>( W_{cminsub} )</td>
<td>Minimum mass flow in the subharmonic surge loop at the compressor exit (kg/s)</td>
</tr>
<tr>
<td>( \Delta T )</td>
<td>Time unit for the simulations (s)</td>
</tr>
<tr>
<td>( \tau_{ave} )</td>
<td>Time for the fluid particle to pass through the whole flowpath (s)</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Flow coefficient of the compressor stage</td>
</tr>
<tr>
<td>( \varphi_{pr} )</td>
<td>Pressure coefficient of the compressor stage</td>
</tr>
<tr>
<td>( \psi_{pr} )</td>
<td>Temperature coefficient of the compressor stage</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Amplitude growth factor of the instability (1/s)</td>
</tr>
<tr>
<td>( \sigma^* )</td>
<td>( \sigma )-factor. Instability enhancement factor</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Flow density (kg/m³)</td>
</tr>
<tr>
<td>( \pi_e )</td>
<td>Stage pressure ratio</td>
</tr>
<tr>
<td>( \omega_{deg} )</td>
<td>Circular frequency of the instability (1/s) at surge-degeneration boundary</td>
</tr>
</tbody>
</table>

Suffixes

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>C2 ( )</td>
<td>Compressor exit</td>
</tr>
<tr>
<td>fin ( )</td>
<td>Final condition with respect to the exit valve</td>
</tr>
<tr>
<td>ini ( )</td>
<td>Initial condition with respect to the exit valve</td>
</tr>
<tr>
<td>plm ( )</td>
<td>Delivery plenum</td>
</tr>
<tr>
<td>t ( )</td>
<td>Rotor tip condition</td>
</tr>
<tr>
<td>V ( )</td>
<td>Exit valve</td>
</tr>
</tbody>
</table>
APPENDIX

Effects of Manipulations of the Exit Valve on the Surge Modes

In the recovery process modelled by opening the exit valve, observations on the effects of the valve conditions on the surge modes will help understanding the surge modes and the phenomena in the axial direction. At the same time, it is useful and interesting to watch the system oscillations with attentions to the valve effects.

First of all, behaviors of pressure \( p \) vs. mass flow \( W \) in the situations of ordinary deep surges are paid attention to, for reference. The typical behaviors for the constant opening of the exit valve \( (A_{in}/A_{o}) = (A_{i0}/A_{o})=0.9 \) are shown in Fig. A1(a) and (b) for Comp11 and Comp15, respectively. The \( p \) vs. \( W \) loops indicated by data points of \( X \) marks (red-colored), \( \blacksquare \) marks (black-colored), and \( \circ \) marks (blue-colored) are those at the immediately upstream of the exit valve (V), at the compressor exit (C2), and at the in-plenum location (plnm), respectively. In the main body of the present paper, conditions for the compressor exit (C2) have been paid attention to.

The oscillations immediately upstream of the exit valve (red-colored) show small amplitudes of mass flows and wide ranges of pressures, compared with those at other locations. At the same time, the loops for the compressor exit and the in-plenum location tend to swing widely around the mass flows at the end of the flowpath, i.e. immediately upstream of the valve as the center. It suggests the surge modes in the flowpath as the ones having the boundary condition of the open-end inlet and the half-closed-end exit, analogous to some extent to the acoustic resonance mode. These figures show clearly the role played by the valve in the mass flow oscillations in the surges.

It is to be noted in Fig. A1(a) that the transition of the working points from the steady-state stalling point indicated by + mark to a point indicated by \( \text{O} \) mark is achieved by closing the exit valve \( (A_{in}/A_{o}) \) to the relative opening of 0.9, which is kept the same after the time in this situation. The red trajectory transits further to nearly the middle position of the mass flows between the stalling point and the zero flow, which could be the center of the mass-flow oscillations. Namely, in the red trajectory, two kinds of transitions appear, in which the first is the artificial one for securing the surge situation by closing somewhat the exit valve, and the second one by a spontaneous sequence of transition toward the final oscillations. Quite the same tendency can be observed for Comp15 in a higher pressure-ratio condition in Fig. A1(b).

![Fig. A1 Behaviors of pressure \( p \) vs. mass flow \( W \) in sufficiently deep surge conditions for Comp11 (a) and Comp15 (b). Both compressors are located at the central position of the flowpath. Mark \( \text{X} \) (red-colored): immediately upstream of the exit valve, mark \( \blacksquare \) (black-colored): at the compressor exit, and mark \( \circ \) (blue-colored): in the delivery plenum.](image)

Figure A2 shows the behaviors of pressure \( p \) vs. mass flow \( W \) in a similar manner to Fig. A1 for Comp11 at 10000rpm, located at the central position of the flowpath, having the AR value of 1.2. The situation is the recovery process from a seriously degenerated-surge condition by opening the exit valve, corresponding to those in Fig. 12(ii). From the steady-state stalling point (mark +), the valve is first closed by 10%, i.e., to \( (A_{in}/A_{o}) \) of 0.9, which is seen as an abrupt change upward to left to the point enclosed by \( \text{O} \) mark; then the valve is opened to a specified final value of \( (A_{in}/A_{o}) \).

The condition given in Fig. A2(a) is different from the one in Fig. A1(a) only in that it has an AR value of 1.2 and a constant value of 1.387, much smaller than the values of 2.7 and 1.607 in Fig. A1(a), respectively. The exit valve is opened from Fig. A2(a) through (b) to (c). The average mass flows through the compressor, the delivery plenum, and the valve are seen to increase together gradually by opening the valve. The final condition in Fig. A2(c) shows the situation recovered onto the normal performance, though the small loops remaining. No deep surges occur throughout the manipulation. The mass flow ranges through the compressor, the plenum, and the valve are limited, although the pressure changes widely in the vertical direction, which are also smaller in comparison with the ones in Fig. A1(a). It suggests that the suppressed condition of the mass flow amplitudes is maintained throughout the procedure of opening the exit valve, which could be governed possibly by the main flowpath conditions.

Figure A3 shows the results in the recovering process in Comp15 at 12000rpm, located at the central position of the flowpath having values of LR and AR of 1 and 0.8, respectively. The conditions shown in Fig. A3(a) having in the AR value of 0.8 and the \( f_{\text{Reson}} \) value of 1.525 are different only from those of 1.15 and 1.643 for the one in Fig. A1(b), respectively. The present case is the seriously-
degenerated surge whereas the former case in Fig. A1(b) is in a completely deep surge condition. The seriously-degenerated surge is considered to be forced by a combination of the extremely low performance yielded by the compressor and the suppressed character of the oscillations of the system.

By opening the exit valve, the behavior could become rather more active, to a subharmonic surge, as seen in Fig. A3(b), forming partially a deep loop, together with the degenerated condition maintained. Opening further the exit valve brings about complete deep surge behaviors as shown in Fig. A3(c). By further opening, the condition can get out to the normal performance conditions, as seen in Fig. A3(d).

![Fig. A2 Behaviors of pressure vs. mass flow for Comp11 at 10000rpm, LR of 0.962, AR of 1.2, located at the central position of the flow-path, in the process of recovery by opening the exit valve from (a) the seriously degenerated surge condition, (b) halfway in recovery, and finally to (c) near the ordinary performance. Mark X (red-colored): immediately upstream of the exit valve, mark ■ (black-colored): at the compressor exit, and mark ○ (blue-colored): in the delivery plenum. The situations are corresponding to those in Fig. 12(ii).](image)

![Fig. A3 Behaviors of pressure vs. mass flow for Comp15 at 12000rpm, LR of 1, AR of 0.8, located at the central position in the flow-path, in the process of recovery by opening the exit valve from (a) the seriously degenerated surge condition, through (b) and (c), and finally to (d) near the ordinary performance. Mark X (red-colored): immediately upstream of the exit valve, mark ■ (black-colored): at the compressor exit, and mark ○ (blue-colored): in the delivery plenum. The situations are corresponding to those in Fig. 14(ii).](image)