Original Paper

Analytical Surge Behaviors in Systems of a Single-stage Axial Flow Compressor and Flow-paths

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Abstract

Behaviors of surges appearing near the stall stagnation boundaries in various fashions in systems of a single-stage compressor and flow-path systems were studied analytically and were tried to put to order. Deep surges, which enclose the stall point in the pressure-mass flow plane, tend to have either near-resonant surge frequencies or subharmonic ones. The subharmonic surge is a multiple-loop one containing, for example, in a (1/2) subharmonic one, a deep surge loop and a mild surge loop, the latter of which does not enclose the stall point, staying only within the stalled zone. Both loops have nearly equal time periods, respectively, resulting in a (1/2) subharmonic surge frequency as a whole. The subharmonic surges are found to appear in a narrow zone neighboring the stall stagnation boundary. In other words, they tend to appear in the final stage of the stall stagnation process.

It should be emphasized further that the stall stagnation initiates fundamentally at the situation where a volume-modified reduced resonant-surge frequency becomes coincident with that for the stagnation boundary conditions, where the reduced frequency is defined by the acoustical resonance frequency in the flow-path system, the delivery flow-path length and the compressor tip speed, modified by the sectional area ratio and the effect of the stalling pressure ratio. The real surge frequency turns from the resonant frequency to either near-resonant one or subharmonic one, and finally to stagnation condition, for the large-amplitude conditions, caused by the non-linear self-excitation mechanism of the surge.

Keywords: Fluid Machine, Axial Flow Compressor, Surge, Analytical Simulation, Frequency, Fluid Dynamics

1. Introduction

On surges of pumps, the fundamental line of thinking and the basic equations, and description about the essential nature of the phenomena were given first by Professor Sumiji Fujii of Tokyo University in 1947, whose paper regrettably was written not in English but in Japanese. In the field of surges of axial flow compressors, several general features of the oscillatory flow behaviors were clarified by Greitzer [1] in 1976 along a similar way of thinking by use of a simplified system of a compressor duct, a delivery plenum, and a throttling duct. He proposed “B parameter” that has been shown to govern the characteristic feature of the surge behaviors, such as the stall stagnation phenomena and its boundary. Since then, researches on the compressor surges were concentrated on the development of more precise methods of surge analysis (for example, Corbett and Elder [2], Davis and O’Brien [3]), and on the clarification of details of specific incidents of surge and effects of its countermeasures (for example, Boyer and O’Brien [4]). However, very few studies since then clarified further general features of the surge phenomena comprehensively.

The real phenomena in site could be much more complicated to be understood pertinently. For example, ones of such phenomena are surge frequencies and their behaviors, which are of fundamental significance in surge as an oscillation system. Although the surge frequencies are considered to be related intimately with the acoustical resonance frequency in the flow-path system, the author’s analytical experiences (for example, Yamaguchi [5, 7, 8]) have shown that the surge frequencies are often far from the system resonance ones. The field of compressor surges is still staying as an unclarified area.

Some of the influential factors are considered as follows. The detailed flow-path geometry could affect the flow conditions having often been treated as a concentrated volume and a one-body fluid motion so far. Flow loads such as large flow resistances, existence of heat exchangers, combustors, and pipe frictions in long flow-paths could affect the phenomena. The exit valve could act also as an important boundary condition to the flow motions. The stalling pressure ratios of the compressors could affect the time required to fill and empty the gas into and out of the delivery volume, which is the essential feature of the compressor surge.

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For multi-stage compressors, in addition to the above effects by the stalling pressure ratios, stage-wise distributions of different working conditions of the respective stages, particularly at off-design speeds, could affect the phenomena in a different manner because of the closely located unstable stages and stable ones.

The author has provided information mainly on the geometrical stall-stagnation boundaries (Yamaguchi [5, 7, 8]). Some of the frequency aspects have been treated mainly in a recent study (Yamaguchi [8]), which has given some new findings about the surge frequency phenomena, though in a very qualitative manner. As a continuation of the study, the present paper describes more thoroughly about the nature of surge frequencies and the behaviors in the simplified condition of a single-stage compressor, i.e., in the absence of the multi-stage effects. The results have been obtained from surveys and examinations on many case studies by use of a surge transient analysis and simulation code developed by the author, SRGTRAN. The information could be useful for understanding some parts of the surge phenomena.

The present paper provides, in the first place, some examples of surges appearing in various environments, such as in near-stagnation conditions in Section 4, in changing flow-path geometries in Section 5, and in changing compressor-speeds conditions in Section 6. In the second place, in Section 7, it summarizes their positioning relative to the stagnation boundaries. And finally in Section 8, it makes clear the initiation condition of the stall stagnations on the basis of examinations on the related frequency parameters for the above results.

2. Situation of the Surge Analyses

2.1 Method of Analysis (SRGTRAN)

A detailed description about the code for the surge transient analysis and simulation, SRGTRAN, employed for the present study, can be found in Yamaguchi [7]. It is spatially a one-dimensional code in x-direction, in which the flow-path is divided into a suitable number of control volumes (CVs). Conservation laws of mass, momentum, and energy are applied to each CV, and solved with a time step \( \Delta t \). In the compressor, each stage is regarded as a CV, and the concerned equations are solved according to the two-stage Lax-Wendroff scheme. Stage characteristics of pressure rise and temperature rise versus flow are supposed and applied to each stage. In the ducts, the method of characteristics is applied to each CV. It does not consider swirl components of flow and neglects rotating stall phenomena.

The time step \( \Delta t \) is given as to satisfy the Courant-Friedrics-Lewy condition. Time is given by the following equation:

\[
t = k \Delta t
\]

Here, \( t \): time (s), \( k \): time number (integer), and \( \Delta t \): time step (s).

2.2 Compressor for Study

The compressor employed here for study is a single-stage one named Comp11. It has a constant-hub annulus configuration designed for a constant axial flow velocity for 11300 rpm and a blade tip peripheral speed \( u_t \) of 300 m/s.

The wide-range stage characteristics given in Fig. 1 is employed for the analyses where the coefficients of flow, pressure, and temperature are normalized with respect to the blade tip speed \( u_t \). The characteristics are supposed on the basis of literature survey results and the author’s experiences (Yamaguchi [7]). It has been applied in all the present cases for variable conditions of the Reynolds numbers and the Mach numbers, whose possible effects on the surge phenomena should be treated as a separate matter of interest.

Flow coefficient:

\[
\phi_t = \frac{u_t}{u_t}
\]

Pressure coefficient:

\[
\psi_{Pt} = \frac{C_p}{(1/2)u_t^2}\left(\frac{P_{T2}}{P_{T1}}\right)^{(k-1)/k} - 1
\]

Temperature coefficient:

\[
\psi_{Pt} = \frac{C_p}{(1/2)u_t^2}\left(T_{T2} - T_{T1}\right)
\]
Here, \( u_\text{an} \): annulus-averaged axial flow velocity, \( u_c \): compressor tip peripheral speed of the first stage (reference speed), \( P_{T1} \) and \( T_{T1} \): total pressure and temperature at the stage inlet, respectively, \( P_{T2} \) and \( T_{T2} \): total pressure and temperature at the stage exit, respectively, \( C_p \): specific heat at constant pressure, and \( \kappa \): ratio of specific heats.

2.3 Model of Flow-paths and Typical Parameters

Figure 2 shows schematically the configuration of the compressor-flow-path system for the present study. Here, the main part of the delivery flow-path is called often “plenum”, which includes not only short-and-fat plenum ducts but also long-and-narrow ducts. The distance between the compressor exit and the plenum inlet is very small.

The basic sizes and the typical parameters are as follows;
- \( A_p \) and \( L_p \): the sectional area (m\(^2\)) and the length (m) of the delivery.
- \( A_{c2} \) and \( L_{c} \): the sectional area of the compressor exit (m\(^2\)), and the length of the suction flow-path from the duct inlet to the compressor exit (m).

\( PR \): the stalling pressure ratio of the compressor.
- \( f_{s0} \): surge frequency (Hz)
- \( f_1 \): the first-mode frequency of the acoustic resonance in the compressor-flowpath system (Hz)
- \( \Lambda \): the wavelength of the acoustic resonance frequency \( f_1 \) (m)

The effects of the plenum configurations on the surge behaviors are studied analytically by giving arbitrarily selected plenum values of the sectional area \( A_p \) and the length \( L_p \), which are specified by applying adjusting factors \( A_{mod2} \) and \( X_{mod2} \), respectively, to a referential model configuration given for both factors of unity.

2.4 Flow-paths for Study

Some distributions of the flow-path sectional area \( A \) versus axial location \( x \) are shown in Fig. 3. These data are at stall-stagnation boundaries for 10000 rpm. The suction flow-path has a gradually reducing sectional area up to the compressor inlet. The situation is identical in all the present cases. The delivery flow-path expands immediately downstream of the compressor exit and forms a plenum downstream. To the final section of the delivery flow-path a short exit pipe is connected and terminated with an exit valve. The flow-paths are divided into forty-nine control volumes (CVs) in total in the present study; fourteen CVs for the suction duct, and thirty-four CVs for the delivery duct. The compressor has a CV corresponding to the single stage situation.

The explanatory notes in Fig. 3 gives the plenum configurations given by the adjusting factors \( A_{mod2} \) and \( X_{mod2} \). For example, a pair of values of large \( A_{mod2} \) and small \( X_{mod2} \) means a short-and-fat plenum configuration, and a pair of values of small \( A_{mod2} \) and large \( X_{mod2} \) means a long-and-narrow delivery flow-path configuration.

The opening area \( A_v \) of the exit valve is set to 90% of the stalling valve area \( A_{v0} \), which is determined at the stalling point. In the condition of the gas motion through the valve described by the model described in SRTRAN (Yamaguchi [6]), the following values of adjusting factors are assumed for the present study:

\[ \beta = 2, \quad \epsilon = 1 \]  
\[ \lambda = 0.02 \]  

The following pipe friction factor is assumed so as to make clearer the friction effects in a rather exaggerated manner;

3. Significant Non-dimensional Quantities

In order to express the results in a way easier to understand, the following non-dimensional quantities are paid attention to.

3.1 Parameters Concerning the Flow-path Geometries

The following geometrical parameters related with the flow-path configurations are paid attention to.

- Flow-path area ratio: \( AR = \frac{A_p}{A_{c2}} \)  
- Flow-path length ratio: \( LR = \frac{L_p}{L_c} \)  
- Area-pressure ratio: \( APR = \frac{A_p}{A_{c2}} PR \)

In order to express the relative size of the flow-paths against the reference acoustical wavelength in the system, the following relative lengths are defined;
Relative suction flow-path length: \( RLC = \frac{Lc}{\Lambda} \)
Relative delivery flow-path length: \( RLP = \frac{Lp}{\Lambda} \)

Here, \( \Lambda \) is the acoustical wavelength of \( f_1 \) and evaluated as follows:

\( \Lambda = \frac{a}{f_1} \)

The reference frequency \( f_1 \) is the first-mode resonance frequency. It is estimated by one-dimensional acoustic equations for the still air or gas having a uniform pressure and a temperature distribution given by the stalling condition throughout the system, coupled with the boundary conditions of the open inlet and the closed exit. \( a \) is the average speed of sound, for example, corresponding to the mass-averaged temperature in the whole flow-path.

3.2 Parameters Concerning Surge Frequencies

First of all, from the aspect of acoustical oscillation phenomena, the relative surge frequency is paid attention to;

\( f_{s0}/f_1 \)

Here, \( f_{s0} \) is the surge frequency.

Next, from the aspect of flow-related vibration phenomena, the reduced surge frequency is paid attention to. An ordinary reduced frequency has the following definition with respect to the plenum:

\( f_{RPs} = \frac{f_{s0}L_p}{u_t} \)

The above simple reduced surge frequency is seen to change much for changing compressor speeds (for example, Yamaguchi [7]).

In order to take into consideration the essential surge action of filling and emptying gas into and out of the delivery volume, the following volume-modified reduced surge frequency is proposed (Yamaguchi [7, 8]):

\( f_{RPVVs} = \frac{f_{s0}L_p(1 - PR^{1/k})}{u_t} \frac{A_p}{A_{C2}} = f_{RPs} \frac{A_p}{A_{C2}} (1 - PR^{1/k}) \)

Here, \( PR \): stalling pressure ratio, and \( k \): ratio of specific heats. It has a form of the ordinary reduced surge frequency \( f_{RPs} \) modified by the sectional area ratio \( AR \) and the density-affected term.

In addition to the above, the following volume-modified reduced resonant-surge frequency \( f_{RPVVs1} \) is paid attention to with the surge frequency \( f_{s0} \) replaced by the resonance frequency \( f_1 \):

\( f_{RPVVs1} = \frac{f_1 L_p(1 - PR^{1/k})}{u_t} \frac{A_p}{A_{C2}} = f_{RPVVs} \frac{f_1}{f_{s0}} \)

The above \( f_{RPVVs1} \) parameter considers a hypothetical situation where the surge process of emptying and filling gas out of and into the delivery flow-path is assumed to occur with the acoustic resonance frequency. It could have possibly some relation with the stall stagnation initiation. However, some calculation results have shown that, at the stall stagnation boundaries, it tends to vary slightly, influenced by compressor speeds. So, the following adjustment by the tip Mach numbers is tentatively proposed in order to fit the data for the speed effects, after some trials.

\( f_{RPVVs1} = f_{RPVVs} \sqrt{\frac{1}{M_t}} \)

or

\( f_{RPVVs1} = \frac{f_1 L_p(1 - PR^{1/k})}{u_t \sqrt{M_t}} \frac{A_p}{A_{C2}} \)

Here, \( M_t \) is the rotor tip Mach number of the compressor. The apparent differences among compressor speeds in the \( f_{RPVVs1} \) parameters have reduced as shown later in Fig. 16, Section 8.

The author would like to name the parameter \( f_{RPVVs1} \) as volume-modified reduced resonant-surge frequency or, more simply, reduced resonant-surge frequency.

4. Surges at Stall-Stagnation Boundary

First of all, the analytical results on the geometrical stall stagnation boundaries for Comp11 for 6000-15000 rpm are shown in Fig. 4. The results are similar to those given in related former papers (Yamaguchi [5, 8]). It is to be noticed that the variable for the abscissa is changed to \( RLC \) in place of \( RLP \) in the present paper for better correlation with the succeeding contents in this study. In Fig.4, both of the data points given by the coordinates \( (RLC, RLP) \) and \( (RLC, APR) \) being within the respective areas of deep surge indicate an occurrence of a deep surge, and both or either of them in the stall stagnation area indicate an occurrence of stall stagnation condition, i.e., a mild surge or a decaying surge.
With respect to the stagnation boundary, the zone where the value of RLC is about 0.15-0.2 for very small RLP values is named “B boundary zone”, and the zone where the value of RLP is about 0.2 for very small RLC values is named “A boundary zone”. The intermediate zone between the zones B and A is named “B-A transition zone”. The zones A and B correspond, respectively, to flow-path configurations of a short-and-fat plenum type delivery duct coupled with a long suction duct, and to those of a long-and-narrow delivery ducts coupled with a short suction duct.

Figure 5 shows the surge frequency behaviors along the stall stagnation boundaries for Comp11 against RLC. Explanatory notes mean the compressor rpm/the relative surge frequency or the surge type. Data-points having the same color stand for the same compressor speeds.

The lower data-points of marks “+” and “x” in Fig.5 show the behaviors of RLP vs. RLC that are essentially the same as those in the bottom of Fig. 4, but the situations are differentiated by marks “+” for the near-resonant surge condition (Type-I, as described below) and by marks “x” for the subharmonic one (Type II, as described below).

The upper data-points of small solid-circle marks in Fig. 5 show the behaviors of relative surge frequencies $f_{s0}/f_1$, which are seen to be affected much by the compressor speeds. For 6000rpm, the relative frequencies $f_{s0}/f_1$ are near unity, meaning near-resonant frequencies. On the otherhand, for example, for 15000rpm, the relative surge frequencies $f_{s0}/f_1$ below roughly 0.45 suggest an appearance of subharmonic surge frequencies, as described later. In the intermediate speeds, both types of relative frequencies appear in a combined fashion.

It is seen that, affected by the compressor speeds, the geometrical stagnation boundaries shift slightly and the extents of the subharmonic zone change. Points “a, b, c, and d” in Fig. 5 are cited for showing the pressure oscillograms and surge loops immediately before stall stagnations at 10000rpm in Fig. 6. Pressure oscillograms in the left-hand side of Fig. 6 show time-histories of the pressures at near the inlet of the suction duct (2), at the compressor exit (16), in the delivery plenum (34), and upstream of the exit valve (49). The abscissa of the oscillograms is physical time $t$ (s) give by Eq. (1) from the time numbers $k$ and the time step $\Delta t$ determined in relation with the Courant-Friedrics-Lewy condition. In all the oscillograms below in the present paper, time spans are arbitrarily selected to contain a sufficient number of surge cycles to watch macroscopically the temporarily changing behaviors.

Surge loops in the right-hand side of Fig. 6 show the compressor exit pressures $P_{comp_{delv}}$ against the mass flow at the compressor exit $W_{comp_{EXIT}}$. At points a and d, which are near the zone A and the zone B, respectively, deep surges are seen to occur, which enclose the stalling point by the particular loop.
Surge pressure oscillograms and surge loops along the stall stagnation boundary for Comp11, 10000rpm, corresponding to the situations indicated by tags “a”–“d” respectively in Fig. 5.
The author would like to name the surge “deep surge of Type I”. The surge loop configurations at points a and d show quite different behaviors in spite of the same Type I, reflecting the different nature of the respective zones.

On the other hand, in Fig. 6 at points b and c in the B-A transition zone shown in Fig. 5, repeated cycles of large and small pressure amplitudes are seen in the left-hand-side figures. The situation is made clearer by watching the surge loop behaviors in the right-hand-side figures of Fig. 6, where the stalling point is enclosed once by a complete deep loop accompanied by a mild loop completely within the stalled zone. It is observed that the deep loop and the mild loop have nearly the same loop period of time, which makes the complete surge period twice the near-resonance period, thus resulting in a (1/2) subharmonic frequency. The author would like to name this type of surge “deep surge of Type II”.

With respect to the compressor speed effects, the stagnation occurs from only deep surges of Type I for 6000rpm, and mostly from Type II for 150000rpm. It appears that the pressure ratios or the compressor speeds could have affected the surge behaviors in the neighborhood of the stagnation boundaries.

The whole flow-path length normalized by the reference wavelength is given by \((RLC+RLP)\). The values range approximately from 0.22 to 0.15 for the present stagnation boundaries. It suggests that the whole lengths are corresponding very roughly to a fourth of the wavelength for the simple pipe resonance condition with the inlet end open and the exit end closed. The parameters \(RLC\) and \(RLP\) are important in that they mean the relative location of the compressor and the relative length of the delivery plenum, respectively, within the field of the basic acoustic motions.

5. Surges Affected by Changes in the Flow-path Geometry

Surge frequency behaviors in the neighborhood of the stall stagnation boundaries are surveyed. For Comp11 at 10000 rpm, the flow-path sectional area ratio \(AR=Ap/\Delta c_2\) is changed with the length ratio \(LR=Lp/\Delta c\) kept constant at the stagnation boundary value. Figure 7 shows some flow-path geometries in terms of \(RLC\) and \(RLP\). The marks “+” “and “X” connected with dotted lines are the stagnation boundary lines shown with the same marks as those for the compressor speeds in Fig. 5. With these data as the background, isolated marks with explanatory notes “10000/varAR” and “varSpeed” mean, respectively, the conditions of and variable area ratios for a fixed length ratio for the compressor speed of 10000rpm, and the conditions for variable compressor speeds for a geometry determined for stagnation at 10000 rpm. Supplementary “near-res” and “(1/2)sub” stand for near-resonant conditions and (1/2) subharmonic conditions, respectively.

Points A, B, C, and D enclosed by a red chain line for \(LR=1.12\) in Fig. 7 are cited for showing the pressure oscillograms and surge loops in Fig. 8. The subfigures are arranged in the same manner as in Fig. 6. When the \(AR\) values become larger from the boundary value, and the values of \(RLC\) and \(RLP\) become smaller, the surge situations transform from a mild surge with a near-resonant frequency for point A in Fig. 8(a), through a deep surge of Type II with a (1/8) subharmonic frequency for point B in Fig. 8(b) and then a deep surge of Type II with a (1/2) subharmonic frequency for point C in Fig. 8(c), finally to a deep surge of Type II with a near-resonant frequency for point D in Fig. 8(d). It is supposed that the order of the subharmonic surge frequency could change drastically in the narrow zone very near the stagnation boundary.

In the subharmonic situation, a set of surge loops having different sizes including mild ones tend finally to form a complete cycle of surge. Specifically, surge cycles of point B in Fig. 8(b) appear to form a complete surge cycle consisted by four groups of a basic set of two loops having a mild loop and a deep loop. It results finally in the (1/8) subharmonic frequency.

<table>
<thead>
<tr>
<th>(A_{mod2})</th>
<th>(X_{mod2})</th>
<th>(LR=Lp/\Delta c)</th>
<th>(AR=Ap/\Delta c_2)</th>
<th>Flow-path configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.15</td>
<td>0.12</td>
<td>0.33</td>
<td>5.8</td>
<td>Short-and-fat plenum type delivery flow-path</td>
</tr>
<tr>
<td>0.425</td>
<td>0.4</td>
<td>1.12</td>
<td>2.8</td>
<td>Comparable lengths of suction and delivery flow-paths</td>
</tr>
<tr>
<td>0.29</td>
<td>3.8</td>
<td>9.2</td>
<td>2.2</td>
<td>Long-and-narrow duct type  delivery flow-path</td>
</tr>
<tr>
<td>0.6</td>
<td>3.8</td>
<td>9.2</td>
<td>3.5</td>
<td>Different from stagnation-configuration</td>
</tr>
<tr>
<td>1.5</td>
<td>3.8</td>
<td>9.2</td>
<td>7.2</td>
<td>Different from stagnation-configuration</td>
</tr>
</tbody>
</table>

Fig. 7 Locations of flow-path configurations with variable values of \(AR\) (Sectional Area Ratio) and a fixed \(LR\) (Flow-path Length Ratio) of 1.12 on the \(RLP\) vs. \(RLC\) map. Letters A-D indicate corresponding surge oscillograms and loops in Fig. 8.
Fig. 8 Pressure variations and surge loops for Comp11 at 10000rpm for changing flow-path configurations for $LR$ of 1.12 and variable $AR$. Subfigures A-D correspond to the relative positions having the same tags in the $RLP$ vs. $RLC$ map in Fig. 7

(a) $LR=1.12\ AR=2.86$ /Mild surge ($A_{mod2}=0.45\ X_{mod2}=0.4$)

(b) $LR=1.12\ AR=3.27$/Deep surge of Type II ($A_{mod2}=0.55\ X_{mod2}=0.4$)

(c) $LR=1.12\ AR=3.89$/Deep surge of Type II ($A_{mod2}=0.7\ X_{mod2}=0.4$)

(d) $LR=1.12\ AR=4.30$/Deep surge of Type I ($A_{mod2}=0.8\ X_{mod2}=0.4$)

Comp11 10000rpm/variable AR configurations
6. Variation of Surge Frequencies in Compressor Speed Changes

In this section, effects of compressor speeds on the surge frequencies are surveyed for Comp11 coupled with several different flow-path configurations given in Table 1. The flow-path configurations are shown for reference in Fig. 9. The adjusting factors $A_{mod2}$ and $X_{mod2}$ for sectional areas and lengths, respectively, for the delivery duct are applied to a nominal configuration to specify the flow-path configurations. The present analyses use the same suction duct. The top three flow-paths are near the stagnation boundary one for 10000rpm. The bottom two flow-paths are far from the boundary ones. In Figs. 9-14, the data-points are shown with the length ratio $LR$ and the area ratio $AR$ as in the explanatory notes.

Figure 10 shows behaviors of surge frequencies for changes in the compressor speeds. Figure 11 shows the relative surge frequencies $f_s_0/f_1$ against compressor speeds. The $f_s_0/f_1$ values are 0.9 to 0.5 for near-resonant frequencies for deep surges of Type I and mild surges, and are 0.45 to 0.35 for subharmonic frequencies for Type II and IB, $(1/2)$ sub-harmonics in the situation. Representative surge conditions are shown in Figs. 12-14 corresponding to the point numbers “1A-3D” shown in Fig. 10. The subfigures below are arranged in the same manner as in Fig. 6.

(a) Long-and-narrow delivery flow-path, $LR=9.2/AR=2.2$ for Points 1A-1D
The flow-path configuration corresponds to a long-and-narrow delivery duct coupled with a short suction duct. The situation is seen for points 1A-1D in Fig. 10. At 10000rpm, it shows a mild surge having a near-resonant surge frequency as shown in Fig. 12(a) for point 1A. For 10250-11750rpm, the surge frequencies reduce by half. The situation is a deep surge of Type II as shown in Fig. 12(b) for point 1B. Solid points in Figs. 10 and 11 stand for the basic near-resonant frequencies remaining and coexisting in the surge cycles as “(basic)” in the explanatory notes. Above 12000 rpm appear deep surges of Type I for points 1C and 1D as shown in Fig. 12(c) and (d).

(b) Comparable suction and delivery flow-paths, $LR=1.12/AR=2.8$ for Points 2A-2D
The flow-path configuration has comparable lengths of the suction duct and the delivery duct, corresponding to those in the B-A transition zone. The situation is seen for points 2A-2D in Fig. 10. For 10000-14750rpm appear mild surges of near-resonant frequencies as shown in Fig. 13(a) and (b) for points 2A and 2B. Above 15000rpm appear deep surges of Type II having double loops as shown in Fig. 13(c) and (d) for points 2C and 2D.

(c) Short-and-fat plenum delivery flow-path, $LR=0.33/AR=5.8$ for Points 3A-3D
The flow-path configuration shows a short-and-fat plenum delivery duct coupled with a long suction duct corresponding to the B stagnation boundary zone. The situation is seen for points 3A-3D in Fig. 10. Below 13250rpm and above 15500rpm appear deep surges of Type I having near-resonant
Fig. 12 Changing surge behaviors caused by speed changes of Comp11 in the flow-path system of $LR=9.2/AR=2.2$ shown in Fig. 9. Subfigure conditions are related with the corresponding points having the same tags in Fig. 10.
Fig. 13 Changing surge behaviors caused by speed changes of Comp11 in the flow-path system of $LR=1.12/AR=2.8$ shown in Fig. 9. Subfigure conditions are related with the corresponding points having the same tags in Fig. 10.
Fig. 14 Changing surge behaviors caused by speed changes of Comp11 in the flow-path system of $LR=0.33/AR=5.8$ shown in Fig. 9. Subfigure conditions are related with the corresponding points having the same tags in Fig. 10.
frequencies shown for points 3A and 3D in Fig. 14 (a) and (d), respectively.

For the intermediate speeds between 13500-15000rpm, however, appear deep surges of (1/2)subharmonic frequencies. In the situation, however, all the loops enclose the stall point as shown for points 3B and 3C in Fig. 14(b) and (c). Two deep loops change the top pressures alternately, resulting in a complete surge cycle. The author would like to name the situation a deep surge of “Type IB”. It has completely different surge loops from those of Type II. Topologically, Type IB could be classified into Type I having a near-resonant frequency.

(d) Far from the stagnation boundary, $LR=9.2/AR=3.5$ and 7.2 (marks “+” and “×”) These have flow-path configurations away from those for the stagnation boundary. They show deep surges of Type I (not shown here), having near-resonant frequencies with the relative surge frequencies changing continuously between approximately unity and 0.45, as shown by data points of marks “x” and “+” in Fig. 11.

Thus, the surge frequencies could show various unexpected ones, depending on the compressor speeds and the flow-paths.

7. Summary on the Surge Situations relative to the Stagnation Boundary

Figure 15 summarizes the information described above on the surge mode situations relative to the geometrical stall stagnation boundaries including the effects of the variable flow-path geometries, and variable compressor speeds. In the figure, stall stagnation boundaries are indicated by line marks “+” and “×” connected by dotted lines, and other conditions crossing the boundaries are indicated by isolated solid marks and outline marks. In the explanatory note, for example, “15000/stg” means the situation for the stagnation condition of Comp11 at 15000 rpm, and supplementary “near-res” and “/sub” mean near-resonant surge frequency and subharmonic one, respectively. With respect to solid marks and outline marks, cold-color marks mean variable area conditions (“variAR” in the explanatory note) and warm-color marks mean variable speed conditions (“variSpeed” in the explanatory note). For example, “10000/variAR/near-res” means the situation of near-resonant surge for variable AR at a constant LR for Comp11 at 10000rpm, and “/(1/2)sub” means a surge with (1/2) subharmonic frequency.

In general, deep surge conditions of Type I of near-resonant frequency are given by data points of marks “+” and solid circles, and subharmonic deep surge conditions of Type II by data points of “×” mark and outline marks.

As seen in Fig. 15, the subharmonic surges tend to gather together within the green-color shaded narrow zone neighboring the stagnation boundary.

Thus, the following general tendency could be summarized.

(1) Occurrence of subharmonic deep surges is considered to be a near-final stage of the process leading to stall stagnation.

(2) The process of stall stagnation is considered to be in principle as follows;

Deep surge of Type I → deep surge of Type II → mild surge → decaying surge

(3) Near the zone A and the zone B of stagnation boundary, stall stagnation tends to occur directly from Type I.

(4) In the B-A transition zone, stagnation tends to occur by way of Type II having subharmonic frequencies.

(5) For increases in the compressor speed, the extent of the subharmonic surges increases near the stagnation boundaries.

Here, the main types of the surge behaviors are looked over as follows;

(a) Surge of Type I: Every surge loop encloses once the stalling point. The relative surge frequency $f_{surge}/f_1$ has a value ranging continuously from approximately unity to above roughly 0.45. It occurs for ordinary situations of flow-path configuration and compressor working condition away from those for the stall stagnation boundaries. It tends to occur also in the close neighborhood of the zone B and the zone A of the stagnation boundary.

(b) Surge of Type II: It shows a subharmonic surge frequency with the relative frequency $f_{surge}/f_1$ below approximately 0.45. It accompanies the near-resonant frequency as the basic frequency. Namely, the basic deep loop and mild loops are coexisting, each having nearly the same time periods of near-resonance. The transition from Type I to Type II
occurs suddenly and discontinuously. It tends to appear in the neighborhood of the stall stagnation boundary, particularly in the B-A transition zone.

(c) Surge of Type IB: It shows a set of deep surge loops whose respective shapes are slightly different, caused possibly by a very close proximity to the boundary as described in Section 8. It could be classified topologically into Type I, although it appears to have a subharmonic frequency.

(c) Mild surge: It shows surge loops staying only in the stalled zone of the pressure vs. mass flow plane, not enclosing the stalling point. It has a near-resonant surge frequency. It could be a situation of stall-stagnated condition.

(d) Decaying surge: The surge decays from the mild surge condition or from the deep surge condition and converges to a point within the stalled zone. It is the final situation of the stall-stagnated condition.

8. Initiation of Stall Stagnations from the Aspects of the Frequency Parameters

In order to find the key to the conditions for initiation of stall stagnations, with particular attention to the frequency parameters, Fig. 16 collects and compares all the present frequency-related results on the relative surge frequencies $f_{s0}/f_1$ (top figure), the reduced resonant-surge frequencies $f_{RPV1}$ (middle figure), and the volume-modified reduced surge frequencies $f_{RPVV1}$ (bottom figure) against the relative compressor location, RLC, for the abscissa. Stall stagnation boundary data are given by small solid circle marks connected by dotted lines in all the three figures for 15000 - 6000 rpm, as shown “stg-boundary/rpm” in the explanatory notes. With these stagnation boundary data as the background, the data crossing the boundaries are indicated by outline marks and line marks “+” and “×”. Outline marks stand for surge data for the variable AR conditions (“variAR”) for some given LR at 10000 rpm. Line marks stand for surge data for variable speed conditions (“variSpeed”) for some fixed geometries of given LR and AR.

Fig. 16 Behaviors of non-dimensional frequencies concerned with those of resonant-surges and surges against the relative compressor locations. Attention should be paid to the ordinate for $f_{RPVV1}$ in the bottom figure which is scaled by ten in order to avoid overlapping of data points.

Fig. 17 Comparison of the reduced resonant-surge frequencies $f_{RPV1}$ with those for the stagnation boundaries and relative surge frequencies $f_{s0}/f_1$ to see the occurrence of subharmonic surges and stall stagnations during changing compressor speeds. Inserted numbers 1A-3D are corresponding respectively to those in Figs. 10 and 12-14.
The reduced resonant-surge frequency $f_{RPSV1}$ given by Eq. (17a) is a reduced frequency assuming an acoustic resonance frequency in a hypothetical emptying and filling process of surge. In the middle figure of Fig. 16, they show a fairly good consistency of the behaviors against the changes in $RLC$, though slight changes in the levels are seen against the compressor speed. It is observed that stagnations occur for the conditions where the $f_{RPSV1}$ lines for the stagnation boundaries happen to be crossed by the $f_{RPSV1}$ behaviors for both of the variable $AR$ (area ratio) conditions given by outline marks and the variable speed conditions by line marks "+" and "×". It could be justified by the observation that the $RLC$ values of the crossing points in the middle figure are coincident with those for sudden drops in the relative surge frequencies $(f_{SO}/f_1)$, i.e., for the occurrence of subharmonic surges in the top figure of Fig. 16.

It is difficult to see the precise situation in the variable speed conditions in Fig. 16 because of the small changes in the $RLC$ values. To close up the situations, Fig. 17 shows behaviors of $f_{RPSV1}$ and $(f_{SO}/f_1)$ against compressor speeds. Small solid-circle marks connected with dotted lines show values of reduced resonant-surge frequency $f_{RPSV1}$ at the stagnation boundary estimated for the respective $LR$ values and the changing compressor speeds. Data point tags “1A-3D” correspond to the cases with $LR$ of 0.33 and $AR$ of 5.8, deep surges of Type IB occur at the situations 3B-3C above and very close to the stagnation boundary line. The observation supports the judgment that the deep surge of Type IB may be included in Type I. The situation could have been forced to vibrate in a rather unsettled way in close proximity to stagnating conditions.

Particularly, for the case with $LR$ of 0.33 and $AR$ of 5.8, deep surges of Type IB occur at the situations 3B-3C above and very close to the stagnation boundary line. The observation supports the judgment that the deep surge of Type IB may be included in Type I. The situation could have been forced to vibrate in a rather unsettled way in close proximity to stagnating conditions.

The values of the parameter $f_{RPSV1}$ for the deep surges of Type I are seen to be confined within the relatively narrow zone shaded by green-color, which is considered to be the range of ordinary deep surges of Type I. In the particular figure, it requires attention to that they are given by a reduced scale of magnitude, i.e., $f_{RPSV1}/10$ for convenience of representation.

9. Conclusion

The results of the present study on the surge behaviors in systems of a single-stage compressor and flow-paths are summarized as follows.

1. Stall stagnation initiates fundamentally when the volume-modified reduced resonant-surge frequency $f_{RPSV1}$ falls into coincidence with the conditions for the stagnation boundaries. The resonant surge frequency is the first-mode resonance frequency in the flow-path system. After the initiation incident, the situation proceeds, mostly by way of occurrence of subharmonic surges, to stagnation to a mild surge or a decaying surge. Thus the subharmonic surges are considered to be in a near-final stage of the stagnation process.

2. Real surge frequencies are analytically either near-resonant ones or their subharmonic ones, whose appearances are closely related with the geometrical locations relative to the stagnation boundaries. The respective surge behaviors are classified broadly as follows; (1) Deep surge of Type I, in which every loop encloses once the stalling point. The relative surge frequency $f_{SO}/f_1$ has a value from approximately unity to above roughly 0.45. (2) Deep surge of Type II which has a subharmonic surge frequency with the relative frequency below nearly 0.45, showing a coexistence of the basic deep loop and mild loops, (3) Deep surge of Type IB, in which a set of unsettled deep surge cycles form a complete cycle. It appears to have a subharmonic surge frequency but could be classified topologically as Type I having the basic near-resonant surge frequency. (4) Mild surge whose loops stay only in the stalled zone over the surge period. And finally, (5) Decaying surge which decays and converges to a point within the stalled zone. The mild surge and the decaying surge are situations of stall-stagnated condition.

10. Postword

The present study has found out some of the essential nature of the compressor surge phenomena, particularly in relation with the basic acoustic resonance frequencies and the wavelengths. It is to be noted, however, that the results would depend on the particular conditions employed in the analyses, such as, for examples, the stage characteristics, the exit valve conditions, the duct friction factor, etc. In the sense, the present results might be of qualitative nature. However, the qualitative information thus obtained could be useful in considering surge problems in site. At the same time, the analytical observations have not yet been supported sufficiently at present by experimental data or actual experiences. The author expects that further evidences concerned will be obtained in future.
Nomenclature

\( a \) Speed of sound (m/s)  
\( A_{c2} \) Sectional area of the compressor exit (m²)  
\( \text{Amod1} \) Adjusting factor of the suction duct sectional area  
\( \text{Amod2} \) Adjusting factor of the delivery duct sectional area  
\( A_p \) Sectional area of the delivery duct or plenum (m²)  
\( AR \) Sectional area ratio  
\( APR \) Sectional area \( \cdot \) pressure ratio  
\( f_1 \) First resonance frequency in the flow-path system (Hz)  
\( f_{SP} \) Reduced surge frequency  
\( f_{SPVVs} \) Volume-modified reduced surge frequency  
\( f_{SPVVs1} \) Volume-modified reduced resonant-surge frequency (reduced resonant-surge frequency)  
\( f_s0 \) Surge frequency (Hz)  
\( f_{w1} \) Relative surge frequency  
\( L_s^{**} \) Suction flow-path length (m)  
\( L_P \) Delivery flow-path length (m)  
\( LR \) Length ratio of the upstream and downstream flow-path  
\( LR \) Length ratio of the upstream and downstream flow-path  
\( M_t \) Rotor tip Mach number  
\( PR \) Stalling pressure ratio  
\( u_m \) Annulus-averaged axial velocity (m/s)  
\( u_t \) Tip speed of the first-stage rotor (m/s)  
\( X_{mod1} \) Adjusting factor of the suction duct length  
\( X_{mod2} \) Adjusting factor of the delivery duct length  
\( \phi_t \) Flow coefficient normalized by the tip speed \( u_t \)  
\( \psi_p \) Pressure coefficient  
\( \psi_T \) Temperature coefficient  
\( \kappa \) Ratio of specific speeds  

References