Nonlinear flow regimes and fractal dimensions of rock fracture networks

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Received 24 06 2017; accepted 19 07 2017

ABSTRACT

This article is the summary for the thesis (Liu, 2016) receiving the best doctoral thesis award from the Japanese Society for Rock Mechanics (JSRM) in the fiscal year of 2016. First, a review study is presented to introduce previous studies on estimating permeability of discrete fracture networks. Mathematical expressions of permeability are summarized with the geometric properties, including: fracture length distribution, aperture distribution, fracture surface roughness, fracture dead-end, number of intersections, hydraulic gradient, boundary stress, anisotropy, and scale. Second, high-precision fluid flow tests and numerical simulations by solving the Navier-Stokes equations are conducted to investigate the nonlinear flow properties of fluid in both intersections and fracture networks. The results show that with the increment of the hydraulic gradient, the ratio of flow rate to hydraulic gradient decreases. When taking account of fracture surface roughness, the ratio of flow rate to hydraulic gradient would reduce by 0 ~ 26.55%. Finally, the fractal properties of rock fracture networks are characterized and a fractal model is proposed to link the fractal characteristics with the equivalent permeability of the fracture networks. The fractal dimension \( D_T \) that represents the tortuosity of the fluid flow and another fractal dimension \( D_f \) that represents the geometric distribution of fractures in the networks, are introduced into the model. The results indicate that compared with the parallel plate model, the maximum deviation of the calculated flow volume that considers the effect of tortuosity can be as high as 19.51%. A multiple fractal model for estimating the permeability of dual-porosity media is proposed. Analytical expressions for the fractal aperture distribution, the total flow rate, the total equivalent permeability, and the dimensionless permeability are derived. Thus the permeability of fractured rock masses could be easily assessed using the proposed method as a first order estimation.

Keywords: Fracture network, Permeability, Fractal dimension, Surface roughness, Tortuosity, Nonlinear flow

1. INTRODUCTION

Fluid flow in rock fractures plays an important role in applications of many projects in geosciences and geoengineering, such as CO\(_2\) sequestration, enhanced oil recovery, and geothermal energy development (Berkowitz, 2002). The permeability of a fracture network is governed by the inherent properties of fractures themselves (i.e., surface roughness, aperture, orientation, density, persistence, infilling, and intersection) and the applied mechanical and hydraulic pressures (i.e., boundary stress and hydraulic pressure) (Liu et al., 2016). Because the permeability of fracture networks is usually much greater than that of the rock matrix in the fractured rock masses (e.g., granite and basalt), the discrete fracture network (DFN) approach that neglects fluid flow in rock matrix has gained much attention in recent explorations (Jing, 2003; Latham et al., 2013). However, there are still many problems that are unsolved, due to the numerous uncertainties such as fracture geometries, locations, and stress environment in the un-visual underground and the limitation of computing power (Tsang et al., 2015). This thesis is focused on contributing to two aspects: nonlinear flow and fractal properties of fractured rock masses.

2. NONLINEAR FLOW IN CROSSED FRACTURES

Significant efforts are being devoted to the understanding of nonlinear flow behaviors through single fractures and single fracture intersections. However, there are few works focusing on the calculation of hydraulic aperture of each fracture segment, and the hydraulic properties of fluid flow at single intersections have not been systematically reported with mathematical expressions. In this section, fluid flow tests are carried out and numerical simulations based on the experimental models and large scale numerical models are performed to study the roles of hydraulic gradient, fracture surface roughness, intersecting angle, and scale effect. Finally, an empirical expression is proposed to calculate the hydraulic aperture mathematically.

Two square glass plates were cut to generate fracture models for flow tests through fracture intersections as shown in Figure 1. One of them, the smooth-walled model, was manufactured using a cutting machine to create smooth fracture surfaces. The other one, the rough-walled model, was cut based on a pre-designed fracture sketch to generate rough fracture surfaces.
The nonlinearity of fluid flow through crossed fracture models is primarily induced by fracture intersection, aperture variation, fracture surface roughness, and hydraulic gradient. The visualization techniques utilized in this study can accurately obtain the geometries of fracture surface. One of the conservative conditions to apply the cubic law at fracture intersections is suggested as: \( J < 10^3, E/R_h < 10^4 \), and JRC = 0.

3. NONLINEAR FLOW IN FRACTURE NETWORKS

Figure 4 shows a series of DFNs with realistic geometric characteristics of fractures. The fracture length follows a power-law distribution, and fracture orientation follows the Fisher distribution. The mechanical aperture of each fracture is presumed to follow the square root correlation with the fracture length. A total of 8 DFNs with side lengths ranging from 0.5 m to 2.0 m are generated, and Figure 4 exhibits 6 of them.

Figures 5(a) and (b) show the relationships of \( J \sim Q \) and \( Q/J \sim J \), respectively, with \( J = 10^8 \sim 10^9 \). The results show that the nonlinear \( J \sim Q \) relationship can be well described using the Forchheimer’s law. With the increment of \( J \), \( Q/J \) varies from constants to variable values, indicating that fluid flow experiences three stages: linear regime, weak nonlinear regime, and strong nonlinear regime.

A multi-variable regression algorithm is adopted to establish mathematical relationships among these parameters. \( h_x, N_i \), and JRC are three independent variables, and \( A, B, C \), and \( J \)c are three dependent variables. The best-fitted expressions are shown in Equations (1) to (3).

By assigning different values of \( \lambda \), and comparing the calculated results with the predicted results, the results show that a good agreement is obtained when \( \lambda = 0.8 \) mm$^{-1}$. 

\[ Q \sim f(J) \]

\[ Q/J \sim f(J) \]

\[ J \sim f(Q) \]
\[ A = \left( \lambda h \right)^{d_2} \cdot \exp \left[ 10.5 \left( \lambda h \right)^{d_3} - 0.01 \text{JRC} - 0.7 N_i^{d_4} + 303 \right] \] (1)

\[ B = \left( \lambda h \right)^{d_2} \cdot \exp \left[ 719 \left( \lambda h \right)^{d_3} - 0.01 \text{JRC} - 5 N_i^{d_4} - 7004 \right] \] (2)

\[ J_i = \left( \lambda h \right)^{d_2} \cdot \exp \left[ 300.8 \lambda h^{d_3} - 0.03 \text{JRC} - 3.3 N_i^{d_4} - 303.3 \right] \] (3)

where \( h_{is} \) is the mechanical aperture, \( \text{JRC} \) is the joint roughness coefficient, \( N_i \) is the number of intersections, \( A \) and \( B \) are two coefficients in Forchheimer’s law, \( J_i \) is the critical hydraulic gradient, and \( \lambda \) is a coefficient that quantifies the reduction of \( h_{is} \) due to the variations of fracture aperture in DFNs.

4. A FRACTAL PERMEABILITY MODEL

Mandelbrot (1982) verified that the cumulative size distribution of islands on the surface of the earth followed the fractal scaling law. Majumdar and Bhushan (1990) developed an equivalent equation to describe the distribution of islands. The distribution of fractures in 2-D rock masses is considered to be analogous to that of islands on the surface of the earth and that pores in porous media, which also follows the fractal scaling law, written as:

\[ N(L \geq l) = \left( \frac{l_{\text{max}}}{l} \right)^{D_f/2} \] (4)

where \( N \) is the total number of fractures with a length \( L \), \( D_f \) is the fractal dimension of fracture backbone in the range of \([1, 2]\) for 2-D fracture networks, and \( l_{\text{max}} \) is the maximum trace length of fractures in a rock mass.

Based on this theory, an expression for the fractal length distribution was derived, written as:

\[ l_i = \frac{l_{\text{min}}}{\left(1 - R_i\right)^{2/D_f}} = \left( \frac{l_{\text{min}}}{l_{\text{max}}} \right) \left(1 - R_i\right)^{-2/D_f} \] (5)

where \( l_i \) is the length of the \( i \)th fracture, \( i = 1, 2, 3, \ldots, N_l \) and \( N_l \) is the total number of fractures in the network.

Although real rock fractures have rough walls and variable apertures, fluid flow through rock fractures is usually described by the cubic law, which assumes that fractures consist of two smooth parallel walls. Under these conditions and considering Equation (5), the cubic law was corrected as:

\[ q(i) = \frac{e^{2 + D_f} \Delta P}{12 \mu l_i^{D_f/2}} \frac{l_{\text{min}}^{D_f/2}}{(1 - R_i)^{2/D_f}} \] (6)

where \( q \) is the flow rate, \( D_f \) is the fractal dimension of fracture backbone, \( D_t \) is the fractal dimension for the tortuous flow of single fractures.

Equation (6) is the new proposed governing equation for fluid flow in each fracture. The flow rate of a fracture network was calculated by calculating the hydraulic head at each intersection after solving of a series of linear equations. The equivalent permeability was calculated according to the linear Darcy’s law.

Ten random numbers corresponding to each fractal dimension \( D_f \) were generated and applied to the DFN modeling. The relation between the equivalent permeability \( K \) and the value of \( D_f \) is shown in Figure 6. When \( D_f \) is small (e.g., 1.3), \( K \) varies by approximately 5 orders of magnitude. The range of variation decreases to less than 2 orders of magnitude when \( D_f \) becomes large (e.g., 1.7-1.8). With increasing \( D_f \), the influence of the random number decreases, which is reasonable because the permeability of a model with only a few fractures can be affected more by the randomness of the trace length, location and orientation of the fractures. By taking the mean value of the results for each \( D_f \), we obtained an approximate curve that indicates that the changes in equivalent permeability with \( D_f \) follows an exponential law.

![Figure 6. Relationship between \( K \) and \( D_f \).](image)

Figure 7 shows the relation between \( K \) and \( D_f \) with trace length ratios \([l_{\text{min}}/l_{\text{max}}]\) of \( 2.0 \times 10^{-4}, 1.0 \times 10^{-4}, \) and \( 0.5 \times 10^{-4} \), respectively. With an increase of \( D_f \), the equivalent permeability decreases because higher values of \( D_f \) represent greater tortuosity and thus greater resistance to fluid flow in the fractures, which results in a lower equivalent permeability. As the trace length ratio \([l_{\text{min}}/l_{\text{max}}]\) increases, the equivalent permeability decreases. The three regression functions show that the changes in equivalent permeability with \( D_f \) follow exponential laws, which are affected by the trace length ratio.

![Figure 7. Relationship between \( K \) and \( D_f \).](image)

Figure 8 shows the relation between relative error and \( D_f \) at different values of \( D_f \). The values of relative error vary little with different random numbers at a given \( D_f \), which indicates that the random number mainly represents the macro-scale properties of the fracture network. The values of relative error vary little among cases with different values of \( D_f \), which shows that \( D_f \) is another macro-scale parameter. When \( D_f = 1.018 \), which corresponds to a JRC value of 20, the maximum values of relative error range from

\[ y = \exp(-7.5x+4.6) \] (R=1)

\[ y = \exp(-7.5x+5.2) \] (R=1)

\[ y = \exp(-7.5x+6.0) \] (R=1)
17.64% to 19.51%, which indicates that the effect of tortuosity is not negligible and should be included in fractal models to accurately estimate the hydraulic behavior of fracture networks.

5. CONCLUSIONS

The mechanical apertures of experimental models can be measured using the visualization techniques with a CCD camera. The numerical simulations examined the accuracy of the measured mechanical apertures, and the flow tests verified the reliability of the FVM (finite volume method) code by solving the Navier-Stokes equations to quantify the hydraulic properties of fluid flow at fracture intersections. The nonlinearity of fluid flow through crossed fracture models is potentially caused by fracture intersection, aperture variation, hydraulic gradient, inflow/outflow tanks, and fracture surface roughness. With the increment of the hydraulic gradient, the ratio of the flow rate to the hydraulic gradient decreases and the relative deviation increases, due to the gradually increasing inertial effects.

A fractal model is established to assess the equivalent permeability of 2-D rock fracture networks. Fluid flow is simulated, and the relation between the fractal dimension and the equivalent permeability is analyzed. Compared with the parallel plate model, the maximum deviation of the calculated flow volume that considers the effect of tortuosity can be as high as 19.51%. Based on the fractal length distribution model, a governing equation for fluid flow in fractures that considers the effects of tortuosity and takes into account the out-of-plane geometry of fractures is proposed. The random number utilized to generate the fracture length has larger impacts on the calculated equivalent permeability than those for generating the orientation and center point of fractures.

Although this thesis established mathematical expressions for predicting permeability of fractured rock masses both experimentally and numerically, the predicted results are not compared with those of a real construction site or an in situ hydraulic test. In the future, we hope that our established mathematical expressions can be utilized in the construction and maintenance of the underground projects such as underground nuclear waste repositories, CO2 sequestration and enhanced geothermal systems. The author expects that the researches during the doctoral course could contribute to the evolving discipline.

REFERENCES


