Assessing Alternative R&D Investment Projects under Uncertainty*

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Abstract: In this paper, we consider two kinds of technologies distinguished by their level of productivity and associated costs, such that one technology increases the quantity of output more than the alternative technology. The firm must then decide in which technology to invest and when to invest in the chosen R&D project so as to maximize profit. To solve this problem, we formulate it as an optimal stopping problem for the firm. Using this analysis, we endogenously obtain the firm's expenditure on R&D investment and numerically locate the R&D investment thresholds for the firm's projects.

Keywords: Alternative technology; R&D; Real options; Optimal stopping

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1 Introduction

According to OECD [12], firm research and development (R&D) expenditures in 1993 amounted to USD 297.6 billion, increasing 1.7 times by 2007 to USD 517.4 billion (both in 2000 purchasing power parity terms). R&D investment thus appears to play an important role in increasing firm performance. Indeed, Griliches [7], Ito and Pucik [10], Ettlie [6], and Sher and Yang [15], among others, conclude there is positive relation between R&D investment and firm performance¹. However, when firms undertake investment in R&D, firms face uncertainty in both the potential success of the R&D project itself and the future cash flows following the initial R&D investment. Fortunately, because of some flexibility in investment decision making, firms are able to delay R&D investment. That is, firms have the option to invest in the R&D project (see, for example, Paxson [14] for details). It is then important to examine also the timing of investment in the R&D project.

Every year, new products utilizing new technologies, including fuel batteries and electric lights, come to market. The purpose of this paper is to focus on a product that draws upon a new technology developed by an R&D project. We assume that the firm that produces the product has two technology options. In turn, we can distinguish between these technologies by using their level of productivity and associated costs, such that one technology increases the quantity and/or quality of output more than the alternative technology. The firm then must choose one project from the two competing projects². The firm's problem is then to decide in which technology to invest and when to invest in the chosen R&D project to maximize the present value of the cash flows generated by the new product.

To solve the firm's problem, we take two steps. First, we assume that the firm has just one R&D project option available, but not both. In this case, we can formulate the firm's problem as an optimal stopping problem, as solved by the associated variational inequalities. Using this analysis, the firm invests in the R&D project when the values of the cash flows reach a certain threshold. Furthermore, we endogenously obtain the R&D investment expenditure as in Shih and Hung [16]. Second, we assume that the firm has two R&D project options and that the firm invests in either R&D project. We also formulate this problem as an optimal stopping problem, again solved by the associated variational inequalities. In terms of relevant work, Décamps, Mariotti and Villeneuve [4] investigate the investment decision problem of two alternative projects and show there are three thresholds involved when investing in these projects. We follow the framework in their analysis and show numerically the thresholds of the R&D investment projects. Representative findings indicate that the continuation region increases in the volatility of cash flow, that is, with uncertainty, while the R&D project investing regions decrease in volatility. Moreover, the amount of investment for both projects increases as uncertainty increases. As the mean arrival rate of R&D project i increases, it is then easy for project i to be chosen.

Our analysis differs from and relates to previous work in this area in several respects. For instance, while Shih and Hung [16] also investigate the R&D investment problem and endogenously obtain the level of investment expenditure when the firm has only a single technology option, we analyze the

¹Conversely, some other studies find a negative relation between R&D investment and firm performance. See, for example, Chan, Martin and Kensinger [2]. Yang, Chiao and Kuo [17] attempt to address this particular puzzle by using a three-stage sigmoid curve model.

²As another research perspective, it is also important to investigate the portfolio evaluation of R&D projects. See, for example, Chien [3].

outcomes when the firm has two alternative technology options. We thus refer to the former as a single R&D investment project and the latter, as developed in this paper, as alternative R&D projects. In other related work, Décamps, Mariotti and Villeneuve [4] examine the investment decision problem of two alternative projects without technological innovation, thereby indicating the value of flexibility when the agent can choose between alternative projects. More recently, Bobtcheff and Villeneuve [1] examine the existing technologies choice problem under uncertainty by considering that the input and output prices follow stochastic differential equations. Lastly, Nishihara and Ohyama [11] also investigate the technology choice problem, but by assuming the existence of a rival firm. However, none of this research endogenously obtains the investment cost.

The rest of the paper is organized as follows. Section 2 describes the firm's single R&D investment problem. Section 3 examines the alternative R&D investment problem. We present the comparative static analysis in Section 4. Section 5 concludes the paper.

2 Single R&D Investment

Suppose that a risk-neutral firm is considering investing in an R&D project where the level of technology progresses by investing in the R&D project. We assume that there are two technology options available where one project entails higher-level technology than the other project. The former is labeled H and the latter is labeled L . In this section, we assume that the firm has either R&D project H or L available as R&D project options, but not both. If the firm chooses project i ($i = \{H, L\}$), it generates a net cash flow $a^i X_t$, where a^i represents the level of technology with $a^H > a^L$ and X_t represents the dynamics of the net cash flow. We assume that the process of X_t is governed by the following stochastic differential equation:

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \quad X_0 = x (> 0), \quad (1)$$

where $\mu \in \mathbb{R}$ and $\sigma (> 0)$ are constants and W_t is a standard Brownian motion.

We assume that the success of the R&D investment depends on a Poisson distribution with mean arrival rate λ . $1/\lambda$ is the expected time until the new technology is available during a time interval of infinitesimal length dt . Let θ_t be governed by the following Poisson process:

$$d\theta_t = \begin{cases} 1, & \text{w.p. } \lambda(I^i)dt, \\ 0, & \text{w.p. } 1 - \lambda(I^i)dt, \end{cases} \quad (2)$$

where I^i is the R&D investment with $I^H > I^L$. As in Shih and Hung [16], we assume that the mean arrival rate is given by:

$$\lambda(I) = b^i I^i, \quad (3)$$

where b^i is a constant with $b^i I^i \in (0, 1)$ and $0 < b^H < b^L$.

When the firm invests in R&D project i , using the strong Markov property and the tower property of conditional expectation, the firm's expected discounted profit $J^i(x; \tau_S^i)$ is given by:

$$J^i(x; \tau_S^i) = \mathbb{E}_x \left[e^{-r\tau_S^i} \left(\int_{\tau_S^i}^{\infty} e^{-\lambda(I^i)(t-\tau_S^i)} \lambda(I^i) e^{-r(t-\tau_S^i)} a^i X_t dt - I^i \right) \right], \quad (4)$$

where the expectation operator $\mathbb{E}_w$ is conditional on the current state w , r ($> \mu$) is the discount rate, $\tau_S^i \in \mathcal{T}$ is the investment time of the R&D project i , and $\mathcal{T}$ is the set of all admissible investment times. Therefore, the firm's problem is to choose the investment time that maximizes the expected discounted profit:

$$V^i(x) = \sup_{\tau_S^i \in \mathcal{T}} J(x; \tau_S^i) = J(x; \tau_S^{i*}), \quad (5)$$

where V^i is the value function and τ_S^{i*} is the optimal timing to invest in the R&D project i .

We formulate the firm's problem (5) as an optimal stopping problem. As is well known, optimal stopping problems are solved by variational inequalities. See, for example, Hu and Øksendal [8], Dupuis and Wang [5], and Øksendal [13].

Let $G^i(X_t)$ be given by:

$$\begin{aligned} G^i(X_t) &= \mathbb{E}_{X_t} \left[\int_t^\infty e^{-\lambda(I^i)(s-t)} \lambda(I^i) e^{-r(s-t)} a^i X_s ds - I^i \right], \\ &= \frac{b^i I^i a^i X_t}{r + b^i I^i - \mu} - I^i. \end{aligned} \quad (6)$$

The region where the firm has not invested in R&D project i is defined by:

$$\mathcal{C}_S^i = \{x; V^i(x) > G^i(x)\}. \quad (7)$$

That is, $\mathcal{C}_S^i$ is the continuation region and this yields the timing of investing in the R&D project i , τ_S^i , given by:

$$\tau_S^i = \inf\{t > 0; X_t \notin \mathcal{C}_S^i\}. \quad (8)$$

We now define the variational inequalities of the firm's problem (5). The following relations comprise the variational inequalities of the firm's problem (5):

$$\mathcal{L}V^i(x) \leq 0, \quad (9)$$

$$V^i(x) \geq G^i(x), \quad (10)$$

$$\mathcal{L}V^i(x)[V^i(x) - G^i(x)] = 0, \quad (11)$$

where $\mathcal{L}$ is the differential operator defined by:

$$\mathcal{L} := \frac{1}{2} \sigma^2 x^2 \frac{d^2}{dx^2} + \mu x \frac{d}{dx} - r. \quad (12)$$

(11) is the complementary condition and can be rewritten as follows. If $x \in \mathcal{C}_S^i$, we then have:

$$\mathcal{L}V^i(x) = 0. \quad (13)$$

Alternatively, if $x \notin \mathcal{C}_S^i$, we have:

$$V^i(x) - G^i(x) = 0. \quad (14)$$

Next, we investigate whether the value function is a solution to the variational inequalities. From the formulation of the firm's problem (5), we conjecture the optimal R&D investment strategy as

follows. If the process $X = \{X_t\}_{t \geq 0}$ reaches some threshold x_S^i , the firm invests in R&D project i , otherwise it does not. Thus, the optimal timing of investing in R&D project i is given by:

$$\tau_S^i = \inf\{t > 0; X_t \geq x_S^i\}. \quad (15)$$

The variational inequalities imply that (13) holds for $x < x_S^i$. We conjecture that a solution to (13) is:

$$V^i(x) = B_{S1}^i x^{\beta_1} + B_{S2}^i x^{\beta_2}, \quad (16)$$

where B_{S1}^i and B_{S2}^i are unknowns to be determined. β_1 and β_2 are the solutions to the following characteristic equation:

$$\frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0, \quad (17)$$

and are calculated as:

$$\begin{aligned} \beta_1 &= \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \\ \beta_2 &= \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \end{aligned} \quad (18)$$

If x goes to 0, the firm does not make a profit. Then, we obtain the following boundary condition of the firm's problem:

$$\lim_{x \rightarrow 0} V^i(x) = 0. \quad (19)$$

It follows from (16) and (19) that we put $B_{S2}^i = 0$. Then, the value function is divided by the level of x as follows:

$$V^i(x) = \begin{cases} B_{S1}^i x^{\beta_1}, & x < x_S^i, \\ G^i(x), & x \geq x_S^i. \end{cases} \quad (20)$$

As in Shih and Hung [16], I^i is endogenously determined as a function of x by:

$$I^{i*}(x) = \arg \max \{G^i(x); I^i(x)\} = \arg \max \left\{ \frac{b^i I^i(x) a^i x}{r + b^i I^i(x) - \mu} - I^i(x); I^i(x) \right\}. \quad (21)$$

It follows from (21) that I^{i*} satisfies the following condition:

$$\frac{b^i a^i x (r - \mu)}{(r + b^i I^i(x) - \mu)^2} = 1. \quad (22)$$

The unknown B_{S1}^i and threshold x_S^i are calculated by the following simultaneous equations:

$$B_{S1}^i x^{\beta_1} = \frac{b^i I^i(x) a^i x}{r + b^i I^i(x) - \mu} - I^i(x), \quad (23)$$

$$\beta_1 B_{S1}^i x^{\beta_1-1} = \frac{d}{dx} \left[\frac{b^i I^i(x) a^i x}{r + b^i I^i(x) - \mu} - I^i(x) \right]. \quad (24)$$

These respective equations are well known as the value-matching and smooth-pasting conditions. Using the envelope theorem, (24) can be expressed as:

$$\beta_1 B_{S1}^i x^{\beta_1-1} = \frac{b^i I^i(x) a^i}{r + b^i I^i(x) - \mu}. \quad (25)$$

Then, from (22)–(25) we obtain that:

$$I^{i*} = \frac{(r - \mu)}{(\beta_1 - 1)b^i}, \quad (26)$$

$$x_S^i = \left(\frac{\beta_1}{\beta_1 - 1} \right)^2 \frac{(r - \mu)}{a^i b^i}, \quad (27)$$

$$B_{S1}^i = \beta_1^{-2} \left(\frac{\beta_1}{\beta_1 - 1} \right)^{2(1-\beta_1)} (a^i)^{\beta_1} \left(\frac{r - \mu}{b^i} \right)^{1-\beta_1}. \quad (28)$$

3 Alternative R&D Investments

In this section, we consider that the firm has two R&D project options and assume that the firm invests in either R&D project H or L . We follow the framework in Décamps, Mariotti and Villeneuve [4] who investigate the choice problem between two alternative investment projects. Let τ_A be the timing of investing project H or L given by:

$$\tau_A = \min [\tau_A^H, \tau_A^L], \quad (29)$$

where τ_A^i ($i = \{H, L\}$) is the timing of investing the R&D project i where the firm has two R&D project options. Then, the firm's expected discounted profit J is:

$$\begin{aligned} J(x; \tau_A) &= \mathbb{E}_x \left[\mathbf{1}_{\{\tau_A^H \leq \tau_A^L\}} e^{-r\tau_A^H} \left(\int_{\tau_A^H}^{\infty} e^{-\lambda(I^H)(t-\tau_A^H)} \lambda(I^H) e^{-r(t-\tau_A^H)} a^H X_t dt - I^H \right) \right. \\ &\quad \left. + \mathbf{1}_{\{\tau_A^H > \tau_A^L\}} e^{-r\tau_A^L} \left(\int_{\tau_A^L}^{\infty} e^{-\lambda(I^L)(t-\tau_A^L)} \lambda(I^L) e^{-r(t-\tau_A^L)} a^L X_t dt - I^L \right) \right] \\ &= \mathbb{E}_x \left[\mathbf{1}_{\{\tau_A^H \leq \tau_A^L\}} e^{-r\tau_A^H} G^H(X_{\tau_A^H}) + \mathbf{1}_{\{\tau_A^H > \tau_A^L\}} e^{-r\tau_A^L} G^L(X_{\tau_A^L}) \right]. \end{aligned} \quad (30)$$

Therefore, the firm's problem is to choose the timing of investing in either R&D project H or L to maximize their expected total discounted profit J :

$$V(x) = \sup_{\tau_A \in \mathcal{T}} J(x; \tau_A) = J(x; \tau_A^*). \quad (31)$$

From (7), the region where the firm invests in neither R&D project H nor L is defined by:

$$\mathcal{C}_A = \{x; V(x) > \max[G^H(x), G^L(x)]\}. \quad (32)$$

That is, $\mathcal{C}_A$ is the continuation region. Then, τ_A is given by:

$$\tau_A = \inf\{t > 0; X_t \notin \mathcal{C}_A\}. \quad (33)$$

As in Section 2, the firm's problem is formulated as an optimal stopping problem, solved using the variational inequalities. The variational inequalities of the firm's problem (31) are as follows:

$$\mathcal{L}V(x) \leq 0, \quad (34)$$

$$V(x) \geq \max[G^H(x), G^L(x)], \quad (35)$$

$$\mathcal{L}V(x) [V(x) - \max[G^H(x), G^L(x)]] = 0. \quad (36)$$

Let $\tilde{x}$ be the value of X such that $G^H(x) = G^L(x)$. Then, $\tilde{x}$ is calculated as:

$$\tilde{x} = (I^H - I^L) \left[\frac{b^H I^H h^H}{r + b^H I^H - \mu} - \frac{b^L I^L h^L}{r + b^L I^L - \mu} \right]^{-1}. \quad (37)$$

Let $G(x) = \max[G^H(x), G^L(x)]$. Then, for any $\varepsilon > 0$, we have:

$$\lim_{\varepsilon \rightarrow 0} \frac{G(\tilde{x} + \varepsilon) - G(\tilde{x})}{\varepsilon} > \lim_{\varepsilon \rightarrow 0} \frac{G(\tilde{x}) - G(\tilde{x} - \varepsilon)}{\varepsilon}. \quad (38)$$

This means that $G(x)$ is not differentiable at $\tilde{x}$ and $V'(\tilde{x}) \neq G'(\tilde{x})$. Thus, the smooth-pasting condition does not hold at $\tilde{x}$. We then have the following result.

Proposition 1 *When the cash flow is $\tilde{x}$, the firm invests in neither R&D project.*

Then, we obtain the timing of investing R&D project i .

Theorem 1 *Assume that the following inequality holds³:*

$$(a^H)^{\beta_1} (b^H)^{\beta_1-1} < (a^L)^{\beta_1} (b^L)^{\beta_1-1}. \quad (39)$$

Let x_A^i ($i = \{H, L\}$) be the threshold of investing in R&D project i when the firm has two R&D project options. The timing of investing R&D project L , τ_A^L , is given by⁴:

$$\tau_A^L = \inf\{t > 0; x_S^L \leq X_t \leq x_A^L\}. \quad (40)$$

Conversely, the timing of investing R&D project H , τ_A^H , is given by:

$$\tau_A^H = \inf\{t > 0; X_t \geq x_A^H\}. \quad (41)$$

The continuation region $\mathcal{C}_A$ is redefined by:

$$\mathcal{C}_A = \{x; x < x_S^L, x_A^L < x < x_A^H\}. \quad (42)$$

A proof can be found in Theorem 2.1 of Décamps, Mariotti and Villeneuve [4]. From (40), the region where the R&D project L is invested is defined by:

$$\mathcal{I}_L = \{x; x_S^L \leq x \leq x_A^L\}. \quad (43)$$

³This inequality is equivalent to $B_{S1}^H < B_{S1}^L$ and rules out that it is never optimal to invest in project L .

⁴The timing of investing in R&D project L is composed of a combination of the thresholds of project L in the single and alternative investment cases.

Similarly, from (41), the region where the R&D project H is invested is defined by:

$$\mathcal{I}_H = \{x; x \geq x_A^H\}. \quad (44)$$

The continuation region $\mathcal{C}_A$ is divided into two regions. The first region is defined by:

$$\mathcal{C}_{AL} = \{x; x < x_S^L\}, \quad (45)$$

where $\mathcal{C}_{AL}$ is the continuation region when the firm has only the R&D project L , so that $\mathcal{C}_{AL} = \mathcal{C}_S^L$. The second region is defined by:

$$\mathcal{C}_{ALH} = \{x; x_A^L < x < x_A^H\}. \quad (46)$$

This region arises from the flexibility whereby the firm can choose between R&D projects L and H .

From the variational inequalities (34)–(36), for $x \in \mathcal{C}_A$, we have:

$$\mathcal{L}V(x) = 0. \quad (47)$$

For $x < x_S^L$, when x reaches x_S^L , the firm invests in R&D project L . Then, we have V given by (20). For $x_A^L < x < x_A^H$, when x reaches x_A^L before x_A^H , the firm invests in R&D project L . Alternatively, when x reaches x_A^H before x_A^L , the firm invests in R&D project H . Thus, the firm has two types of flexibility in this region. The value function is then:

$$V(x) = B_{AH}x^{\beta_1} + B_{AL}x^{\beta_2}, \quad (48)$$

where B_{AH} and B_{AL} are unknowns to be determined. The first term on the right-hand side is the value of the flexibility from where the firm chooses the timing of investing in R&D project H . The second term is the value of the flexibility from where the firm chooses R&D project L . Then, the value function V is divided by the level of x as follows:

$$V(x) = \begin{cases} B_{S1}^L x^{\beta_1}, & x < x_S^L, \\ G^L(x), & x_S^L \leq x \leq x_A^L, \\ B_{AH}x^{\beta_1} + B_{AL}x^{\beta_2}, & x_A^L < x < x_A^H, \\ G^H(x), & x \geq x_A^H. \end{cases} \quad (49)$$

As in Section 2, we need to determine the unknowns: B_{AH} , B_{AL} , and the thresholds: x_A^L , x_A^H . These are calculated using simultaneous equations:

$$B_{AH}(x_A^L)^{\beta_1} + B_{AL}(x_A^L)^{\beta_2} = \frac{b^L I^{L*} a^L x_A^L}{r + b^L I^{L*} - \mu} - I^{L*}, \quad (50)$$

$$B_{AH}(x_A^H)^{\beta_1} + B_{AL}(x_A^H)^{\beta_2} = \frac{b^H I^{H*} a^H x_A^H}{r + b^H I^{H*} - \mu} - I^{H*}, \quad (51)$$

$$\beta_1 B_{AH}(x_A^L)^{\beta_1-1} + \beta_2 B_{AL}(x_A^L)^{\beta_2-1} = \frac{b^L I^{L*} a^L}{r + b^L I^{L*} - \mu}, \quad (52)$$

$$\beta_1 B_{AH}(x_A^H)^{\beta_1-1} + \beta_2 B_{AL}(x_A^H)^{\beta_2} = \frac{b^H I^{H*} a^H}{r + b^H I^{H*} - \mu}. \quad (53)$$

Unfortunately, as we are unable to derive these thresholds analytically, we numerically calculate their values in the following section.

Figure 1: Value functions of alternative R&D projects.

4 Comparative Static Analysis

In this section, we first numerically calculate the thresholds: x_S^L , x_S^H , x_A^L , and x_A^H , and the R&D investments: I^L and I^H . Further, we investigate the effects of changes in the parameters on the thresholds and the R&D investments. The basic parameter values are set out as follows: $r = 0.06$, $\mu = 0.01$, $\sigma = 0.2$, $a^H = 2$, $a^L = 1$, $b^H = 0.02$, and $b^L = 0.1$.

The value function V where the firm has two R&D project options is illustrated in Figure 1. The threshold values are calculated as $x_S^L = 2.000$, $x_S^H = 5.000$, $x_A^L = 2.891$, and $x_A^H = 5.185$ in the base case. The R&D investments are also calculated as $I^L = 0.5$ and $I^H = 2.5$. The indifference value of the shift variable is $\tilde{x} = 4.000$.

Next, we conduct the comparative static analysis of the thresholds and the R&D investments.

Proposition 2 *The investment region for the project L, $\mathcal{I}_L$, is decreasing in σ , a^H , b^H , while $\mathcal{I}_L$ is increasing in a^L and b^L .*

The proof of Proposition 2 is in the Appendix.

We provide the results of the comparative static analysis of the thresholds and the R&D investments in Figures 2–9. Figure 2 shows that the continuation region $\mathcal{C}_A (= \mathcal{C}_{ALH} \cup \mathcal{C}_{AL})$ is increasing in the volatility of the cash flow, σ . The investing region of both R&D projects H and L , $\mathcal{I}_H$ and $\mathcal{I}_L$, are decreasing in σ . Note that (39) doesn't hold for $\sigma \geq 0.253$ and the investing region $\mathcal{I}_L$ is disappeared. That is, it is never optimal to invest in the R&D project L for $\sigma \geq 0.253$. Then, for $\sigma \geq 0.253$ the continuation region is $\mathcal{C}_S^H$ where the firm has just one project option for higher-level technology. Figure 3 indicates the amounts of R&D investment of both projects, I^L and I^H , are increasing in σ . These results imply that the incentive to wait for new information about the business environment becomes stronger as the uncertainty in future cash flow increases. The amounts of both investments then increase as the uncertainty of future cash flow increases.

Figure 4 shows that the continuation region for both R&D projects L and H , $\mathcal{C}_{ALH}$, is increasing in the technology parameter for the R&D project H , a^H . Recall that the continuation region for the R&D project L , $\mathcal{C}_{AL}$, is independent of a^H . Then, the continuation region $\mathcal{C}_A$ is increasing in a^H . The investing region of the R&D project H , $\mathcal{I}_H$, is increasing in a^H , while the investing region of

Figure 2: Comparative statics of thresholds with respect to σ .

Figure 3: Comparative statics of R&D investments with respect to σ .

Figure 4: Comparative statics of thresholds with respect to a^H .

 Figure 5: Comparative statics of thresholds with respect to a^L .

the R&D project L , $\mathcal{I}_L$, is decreasing in a^H . These results mean that the higher the cash flow from project H , the easier it is for project H to be chosen when the technology parameter of project L does not change. Note that (39) doesn't hold for $a^H > 2.236$ and the investing region $\mathcal{I}_L$ is disappeared. Then, for $a^H > 2.236$ the continuation region is $\mathcal{C}_S^H$ where the firm has just one project option for higher-level technology. Note also that, from (26), I^H and I^L are independent of a^H .

Figure 5 shows that the continuation region $\mathcal{C}_A$ is decreasing in the technology parameter for the R&D project L , a^L . The investing region of the R&D project L , $\mathcal{I}_L$, is increasing in a^L , while the investing region of the R&D project H , $\mathcal{I}_H$, is decreasing in a^L . These results mean that the higher the cash flow from project L , the easier it is for project L to be chosen when the technology parameter of project H does not change. Note that (39) doesn't hold for $a^L \leq 0.894$ and the investing region $\mathcal{I}_L$ is disappeared. Then, for $a^L \leq 0.894$ the continuation region is $\mathcal{C}_S^H$ where the firm has just one project option for higher-level technology. Note also that, from (26), I^L and I^H are independent of a^L .

Figure 6 shows that the continuation region for both R&D projects L and H , $\mathcal{C}_{ALH}$, is decreasing in the parameter that determines the mean arrival rate of the R&D project H , b^H . Recall that the continuation region for the R&D project L , $\mathcal{C}_{AL}$, is independent of b^H . Then, the continuation region $\mathcal{C}_A$ is decreasing in b^H . The investing region of the R&D project H , $\mathcal{I}_H$, is increasing in b^H , while the investing region of the R&D project L , $\mathcal{I}_L$, is decreasing in b^H . Note that (39) doesn't hold for $b^H \geq 0.025$ and the investing region $\mathcal{I}_L$ is disappeared. Then, for $b^H \geq 0.025$ the continuation region is $\mathcal{C}_S^H$ where the firm has just one project option for higher-level technology. Figure 7 indicates the amount of R&D investment for project H , I^H , is decreasing in b^H . Note that, from (26), I^L is independent of b^H . These results mean that the higher the mean arrival rate of R&D project H , the easier it is for project H to be chosen when the mean arrival rate of R&D project L does not change. Decreasing the threshold x_A^H lowers the cash flow of the project, such that the amount of investment needed for project H decreases as b^H increases.

Figure 8 shows that the continuation region $\mathcal{C}_A$ is decreasing in b^L . The investing region of the R&D project L , $\mathcal{I}_L$, is increasing in b^L , while the investing region of the R&D project H , $\mathcal{I}_H$, is decreasing in b^L . Note that (39) doesn't hold for $b^L \leq 0.08$ and the investing region $\mathcal{I}_L$ is disappeared.

Figure 6: Comparative statics of thresholds with respect to b^H .

Figure 7: Comparative statics of R&D investments with respect to b^H .

Figure 8: Comparative statics of thresholds with respect to b^L .

Figure 9: Comparative statics of R&D investments with respect to b^L .

Then, for $b^L \leq 0.08$ the continuation region is $\mathcal{C}_S^H$ where the firm has just one project option for higher-level technology. Figure 9 indicates the amount of R&D investment of the project L , I^L , is decreasing in b^L . Note that from (26), I^H is independent of b^L . These results mean that the higher the mean arrival rate of R&D project L , the easier it is for project L to be chosen when the mean arrival rate of R&D project H does not change. Decreasing the threshold x_S^L lowers the cash flow of the project, such that the amount of investment needed for project L decreases as b^L increases.

5 Conclusion

In this paper, we examined the firm's R&D investment problem with alternative R&D projects. To solve this particular problem, we formulated it as an optimal stopping problem. We first investigated the single R&D project and obtain the closed form of the threshold, and then investigated the alternative R&D projects. Unfortunately, we were unable to derive explicitly the thresholds of the projects. Therefore, we conducted numerical and comparative static analysis. The representative findings indicate that the continuation region increases in the volatility of cash flow, that is, with uncertainty, while the R&D project investing regions decrease in volatility. In addition, the amounts of investment needed for both projects increase as uncertainty increases. That is, as the mean arrival rate of R&D project i increases, it is easier for project i to be chosen.

In concluding, we suggest a number of possible extensions for our model. First, we defer to future work to examine a finite time horizon. Second, in this paper we assume that a single stochastic differential equation provides the cash flow dynamics. It would then be an interesting extension to assume different cash flow dynamics for each project. Third, in actual business environments, firms compete. We could then consider a two-competing firms problem formulated as a game theoretic real options model, as in Huisman [9] and Nishihara and Ohyama [11]. We leave these topics for future research.

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A Proof of Proposition 2

Proof: Let a function f be defined by:

$$f(\sigma, a^i, b^i) := (a^L)^{\beta_1} (b^L)^{\beta_1-1} - (a^H)^{\beta_1} (b^H)^{\beta_1-1}, \quad i = \{H, L\}. \quad (54)$$

From (2.18), β_1 is a decreasing function of σ . That is, we have:

$$\beta_{1,\sigma} := \frac{\partial \beta_1(r, \mu, \sigma)}{\partial \sigma} < 0. \quad (55)$$

We then obtain that the following results.

$$\begin{aligned} \frac{\partial f(\sigma, a^i, b^i)}{\partial \sigma} = & \left[(a^L)^{\beta_1} (b^L)^{\beta_1-1} (\log a^L + \log b^L) \right. \\ & \left. - (a^H)^{\beta_1} (b^H)^{\beta_1-1} (\log a^H + \log b^H) \right] \beta_{1,\sigma} < 0, \end{aligned} \quad (56)$$

$$\frac{\partial f(\sigma, a^i, b^i)}{\partial a^H} = -\beta_1 (a^H)^{\beta_1-1} (b^H)^{\beta_1-1} < 0, \quad (57)$$

$$\frac{\partial f(\sigma, a^i, b^i)}{\partial a^L} = \beta_1 (a^L)^{\beta_1-1} (b^L)^{\beta_1-1} > 0, \quad (58)$$

$$\frac{\partial f(\sigma, a^i, b^i)}{\partial b^H} = -(\beta_1 - 1) (a^H)^{\beta_1} (b^H)^{\beta_1-2} < 0, \quad (59)$$

$$\frac{\partial f(\sigma, a^i, b^i)}{\partial a^L} = (\beta_1 - 1) (a^L)^{\beta_1} (b^L)^{\beta_1-2} > 0. \quad (60)$$

Given that it is obvious that f is a smooth function, there exist $\hat{\sigma}$, $\hat{a}^i$, and $\hat{b}^i$ for $i = \{H, L\}$ such that:

$$f(\hat{\sigma}, a^i, b^i) = 0, \quad (61)$$

$$f(\sigma, \hat{a}^i, b^i) = 0, \quad (62)$$

$$f(\sigma, a^i, \hat{b}^i) = 0. \quad (63)$$

Then, for $\sigma \geq \hat{\sigma}$, $a^H \geq \hat{a}^H$, $a^L \leq \hat{a}^L$, $b^H \geq \hat{b}^H$, and $b^L \leq \hat{b}^L$, the condition (3.11) does not hold and $\mathcal{I}_L = 0$.

The proof is completed. $\square$

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