1. Introduction

Noise from distortion due to camera lenses and digitizing errors are generally found in motion analysis data. Several smoothing methods, such as the Butterworth digital filter, the cubic spline and Fourier series, have been used to remove such noise in previous biomechanical studies. In the motion analyses of tennis, however, the displacement data of the racket just after ball impact is changed suddenly by the collision with the ball. Consequently, the differentiation of displacement data reveals a decrease in the peak velocity and a shift in its appearance time when smoothing is applied through impact using these techniques (Elliott, et al., 1989; Sprigings, et al., 1994; Elliott and Christmass, 1995). Even if the data before and after impact are separated to avoid the influence of the collision, a distortion of the endpoint (endpoint error) is produced in the process of smoothing and is augmented by the differentiation. A common solution to the problem of endpoint errors is to extrapolate 5 - 20 points at the end of the raw data. A number of extrapolation methods have been proposed to correct endpoint error (Smith, 1989; Vint and Hinrichs, 1996; Giakas, et al., 1998; Knudson and Bahamonde, 2001).

Smith (1989), who tested the linear extrapolation, duplication and reflection methods using the angular displacement data reported by Pezzack, et al., (1977), suggested that the reflection method was somewhat superior to the linear extrapolation method. Vint and Hinrichs (1996) investigated the effects of linear extrapolation and reflection using the data of Pezzack, et al., (1977), and showed that linear extrapolation yielded more accurate results than reflection did. Giakas, et al., (1998), who examined
the effect of the least-squares and prediction methods using the same data, reported that the endpoint distortion obtained using the least-squares and prediction methods was smaller than that obtained using the linear extrapolation method. Thus, although the same data was used in each of these studies, there was no agreement on which would be the most suitable extrapolation method to correct the endpoint distortion. In these studies, these extrapolation methods were not applied to practical biomechanical data.

Knudson and Bahamonde (2001), who tested five-point linear extrapolation and polynomial extrapolation using wrist goniometric angle data in the tennis forehand stroke, concluded that both extrapolation methods were effective for removing noise in the goniometric angle data. Moreover, Knudson and Bahamonde (2001) applied these extrapolation methods to three-dimensional kinematic data from the tennis forehand stroke. However, they confirmed only that the resultant racket velocity attained a peak value at impact under both extrapolation conditions, and did not investigate whether these extrapolation methods were appropriate for kinematic data.

The purposes of the present study were to examine the distortions of three-dimensional high-speed kinematic data (1000 Hz) using existing extrapolation methods (reflection, linear extrapolation and least-squares regression) and to determine the most effective extrapolation method for removing the influence of the collision between the racket and the ball during the flat power serve in tennis (Elliott, et al., 1995). In the practical analyses of tennis movement, the motion is usually recorded at 200 Hz or less (Elliott, et al., 1989; Sprigings, et al., 1994; Elliott, et al., 1995) which means that a maximum of one image can be captured during ball contact. Consequently, researchers are unsure which frame should be selected as the start frame for extrapolation. Therefore, the present study also attempts to determine the proper start frame for extrapolation when performing analyses using 200 Hz kinematic data.

2. Methods

Data for motion that includes ball contact cannot be used to investigate the validity of the extrapolation method because of the influence of the collision between the racket and the ball. However, shadow swing data is not distorted by smoothing or differentiation because such data does not include the influence of the collision with the ball. In analyses of motion that include ball contact, the data before impact can be assumed to correlate to the data before pseudo-impact in the analysis of the shadow swing, because this portion of the movement is not affected by the collision with the ball. In the present study, shadow swing data was used to investigate the validity of the extrapolation method. That is, the differentiated data calculated using the non-extrapolated data (original data) of the shadow swing was defined as criterion data. The extrapolated data was created by padding the data after pseudo-impact during the shadow swing using three existent extrapolation methods. The differentiated data calculated using the extrapolated data was then compared with the criterion data to determine the extrapolation method that produced results that were most similar to the criterion data.

2.1. Extrapolation Methods

Three extrapolation methods were examined in the present study (Figure 1):

1) Reflection (hereinafter referred to as REF), in which the impact frame value is used as a reference level, determining the Imp+i padding point value based on the Imp-i point value point-symmetrically, where Imp is the impact frame and i is the number of padding points.
2) Linear extrapolation (hereinafter referred to as LEX), in which extra points are calculated from the slope between the last two points.

3) Least-squares regression (hereinafter referred to as LES), in which extra points are calculated from a least-squares fit of a fourth-order polynomial to the final 10 data points.

2.2. Experimental Procedure

The subject was a skilled right-handed male tennis coach. Two high-speed video cameras (Photron Inc., Tokyo, Japan) were placed behind the baseline (Figure 2). These cameras were used to record a reference structure containing markers of known coordinates in the space that encompassed the field of movement during the serve. After an adequate warm-up period, the subject was requested to serve under the following two conditions from near the center mark of the baseline aiming for the center of the advantage service court until a successful trial was obtained:

1) Shadow swing as fast as possible while performing an imaginary flat power serve (hereinafter referred to as shadow serving). The obtained data was used to determine the validity of the extrapolation method.

2) Flat power serve as fast as possible, including collision with the ball (hereinafter referred to as actual serving). The obtained data was used to investigate the degree and direction of the influence of the impact. This data was also used to determine the proper start frame for extrapolation when performing analysis at 200 Hz.

The service motion was recorded using two cameras operating at 1000 Hz (exposure time 1/10000 s) focusing on the upper body.

2.3. Analysis and Treatment of the Data

Two-dimensional images of the reference structure (48 points) and subjects, including the racket head and tennis ball, were manually digitized. The images obtained from two cameras were mechanically synchronized. The transformation from digitized two-dimensional data to three-dimensional coordinate data was conducted by the direct linear transformation method. The static coordinate system used in the present study was a right-hand coordinate system in which the X-axis represents the horizontal direction toward the net (approximating the direction of the swing), the Y-axis represents the direction vertical to the ground, and the Z-axis represents the direction transverse to the sideline. The mean differences between the measured and estimated values in the calculation of the three-dimensional coordinates were 1.9±1.6 mm for the X-axis, 1.0±0.8 mm for the Y-axis and 1.3±1.1 mm for the Z-axis.

In the present study, we defined the frame just before (1 ms before) first contact between the racket...
and the ball, i.e., the frame that was closest to the collision but not influenced by it, as the impact frame in the actual serving data. The pseudo-impact frame in the shadow serving data was determined by comparison with the motion observed in the actual serving data. Twenty points were padded after impact using each of the three extrapolation methods in the shadow serving data. The displacement data of the non-extrapolated (original data) and extrapolated conditions were smoothed with a recursive Butterworth digital filter. The cut-off frequency was determined by residual analysis (Winter 1990), and the central difference method was used to differentiate the smoothed data. As mentioned above, this differentiated data calculated from the non-extrapolated original shadow serving data showed the criterion data. In the present study, the root mean square difference (RMSD) of 10 points before impact was computed from the following equation:

\[
\text{RMSD} = \sqrt{\frac{1}{10} \sum_{i=\text{imp}-9}^{\text{imp}} (D_{ci} - D_{exi})^2}
\]

where \(\text{imp}\) is the moment of impact, \(i\) is the \(i^{th}\) value of the differentiated data, and \(D_{ci}\) and \(D_{exi}\) are the criterion data and the differentiated data calculated from the extrapolated shadow serving data, respectively.

3. Results

3.1. Comparison of the Criterion Data and the Differentiated Data Calculated Using Smoothed Non-Extrapolated Data during Actual Serving.
The differentiated data calculated using the non-extrapolated data (original data) during shadow serving was defined as the criterion data in the present study. The differentiated data of the actual serving data were calculated using the same method as that employed for calculating the criterion data, and were compared with the criterion data to determine the degree of the influence of impact in each axis. Figure 3 shows the changes in the criterion data and the differentiated data during actual serving for the racket and the wrist. For the X-axis, the actual serving data showed that the changes in the racket-differentiated data differed markedly from the criterion data. That is, the criterion velocity data increased toward impact, and the criterion acceleration data peaked prior to impact and then decreased rapidly, approaching zero toward impact. On the other hand, the actual serving velocity data attained a peak value just before impact, decreasing thereafter until impact. The acceleration data decreased rapidly prior to impact, and a large negative acceleration appeared at impact. For the Y- and Z-axes, the actual serving data revealed that the changes in the racket-differentiated data were similar to the criterion data. The changes in the wrist-differentiated data during actual serving were also similar to the criterion data for all axes.

### 3.2. Application of Extrapolation Methods

In the present study, the racket-differentiated data calculated using three extrapolation methods during shadow serving was compared with the criterion data to investigate the validity of the extrapolation method. Figure 4 shows the changes in the racket velocity and the acceleration calculated using the extrapolated data. The criterion data are also shown in the same figure for comparison. The RMSDs of racket velocity and acceleration using each extrapolation method are summarized in Table 1. For the X-axis, the velocities in REF and LEX were close to the criterion data. The acceleration obtained using REF was similar to the criterion data, whereas that using LEX and LES at impact was larger than the criterion data. For the Y-axis, the racket velocity and acceleration in LES were similar to the criterion data.
data. The accelerations before impact in REF and LEX were deviant from the criterion data. For the Z-axis, all extrapolation conditions resulted in a small negative velocity from before impact as the criterion data. Although the criterion data showed positive acceleration just before impact, all of the extrapolation methods yielded negative accelerations for the same time point. On all axes, the acceleration obtained using REF was closest to zero at impact, and that using LEX was high compared to that using REF. The acceleration/deceleration calculated using LES was the highest among all extrapolations.

4. Discussion

4.1. Suitable Extrapolation Method

The actual serving data revealed that the changes in the racket-differentiated data for the Y- and Z-axes were similar to the criterion data (Figure 3). For the X-axis, however, a rapid decrease in the racket-differentiated data was observed just before impact in the actual serving data. In the present study, since the X-axis and the direction of the swing corresponded approximately, the impulse of the ball impact primarily affected the displacement data of the X-axis and not the data of the Y- or Z-axes. The goal of extrapolation was to remove the influence of the collision with the ball. Thus, applying an extrapolation method to the Y- and Z-axes data, which were only slightly affected by the collision with the ball, is not necessary. The criterion velocity data for the X-, Y- and Z-axes at impact were found to be 46.3, 3.0 and -4.2 m/s, respectively. Most of the resultant racket velocity was generated by the velocity toward the X-axis and not the Y- or Z-axes. The effect on the resultant velocity of the Y- or Z-axes was very small, even when there was some influence due to the collision. These results indicate that extrapolation should not be applied to the Y- or Z-axes, and that the data before and after impact should be smoothed together without extrapolation.

In the case of the wrist, the actual serving data yielded changes in the differentiated data similar to the criterion data (Figure 3). That is, there was hardly any influence from the collision with the ball for the wrist. Therefore, the extrapolation method should be applied to the data only for the racket on the X-axis.

To verify that a similar extrapolation effect is obtained when using other data, the same subject was again requested to shadow serve under the same conditions as the first shadow serving was recorded. The differentiated data obtained using extrapolated and non-extrapolated data in the second shadow serving were calculated by the same procedure as that employed in calculating the first shadow serving data. The differentiated data calculated using non-extrapolated data were defined as the criterion data in the second shadow serving. The differentiated data under all conditions in the second shadow serving were similar to those in the first shadow serving on all axes (Figure 5). That is, the racket acceleration obtained using REF was consistent with the criterion data for the X-axis, and the acceleration in LES was similar to the criterion data for Y-axis. On all axes, the acceleration in REF was closest to zero at impact, and the accelerations in LEX and LES were high. Smith (1989) tested extrapolation using the data reported by Pezzack, et al., (1977), and found that the error of angular acceleration data at the endpoint was smallest when using REF. Vint and Hinrichs (1996) compared the endpoint error by extrapolation using the data of Pezzack, et al., (1977), and recommended LEX rather than REF for correcting the endpoint error. Giakas, et al., (1998) examined the effect of extrapolation using the same data, and reported that the distortion of the endpoint obtained using LES was less than that obtained using LEX. One reason for this difference may have been

Table 1 The RMSDs of racket velocity and acceleration for each extrapolation method.

<table>
<thead>
<tr>
<th></th>
<th>Racket Velocity (m/s)</th>
<th>Racket Acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>REF</td>
<td>LEX</td>
</tr>
<tr>
<td>X-Axis</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Y-Axis</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Z-Axis</td>
<td>0.4</td>
<td>0.6</td>
</tr>
</tbody>
</table>
that although the data of Pezzack, et al., (1977) were used in these studies, the characteristics of their chosen endpoint data were completely different. Smith (1989), who recommended REF, selected a phase similar to that used in the present study for the X-axis, in which the acceleration approached zero. In contrast, Vint and Hinrichs (1996) and Giakas, et al., (1998) chose a phase similar to that used in the present study for the Y-axis, in which the acceleration reached a peak value. The results obtained in the present study were identical to those of previous studies. That is, the acceleration obtained by REF was more consistent with the criterion data than that obtained by LEX for the X-axis, and the acceleration in LES was most similar to the criterion data for the Y-axis (Figure 4, Table 1). These results indicate that the endpoint data is corrected in accordance with the characteristics of each extrapolation method and that it is necessary to choose the proper extrapolation method according to the pattern of the acceleration at the endpoint.

For a flat power serve, it is natural that the velocity of the racket head toward the net would be maximal at impact, i.e., the acceleration would be zero. Omichi (1984) confirmed that the racket acceleration was zero at the moment of impact using an accelerometer. The present study shows that the acceleration criterion data on the X-axis was approximately zero (Figure 4, Figure 5). The differentiated data in REF and LEX were more similar to the criterion data than the differentiated data in LES for the X-axis (Figure 4, Table 1). The reason for this is as follows: REF and LEX have characteristics that produce zero acceleration at impact (Figure 1). That is, REF is associated with zero acceleration at impact because the extrapolation is performed point-symmetrically on the basis of the impact frame. Since LEX yields zero acceleration over the padding points, LEX has the characteristic of creating a slightly higher acceleration at impact. Therefore, the calculated acceleration at impact obtained using REF was closer to zero than that obtained using LEX for all axes. Therefore, REF and LEX are effective in correcting the data in which the acceleration approaches zero at ball impact as the X-axis. In contrast, LES exhibits the characteristic of producing a higher

![Figure 5](http://www.soc.nii.ac.jp/jspe3/index.htm)

**Figure 5** Changes in the differentiated data calculated using extrapolated and non-extrapolated data for the racket in the second shadow serving. Key: ( ) criterion data; ( ) differentiated data using REF; ( ) differentiated data using LEX; ( ) differentiated data using LES.
acceleration at impact than the other extrapolation methods because the padding data were calculated using a fourth-order polynomial. Thus, LES may be a suitable method for correcting data in which the acceleration reaches its peak at impact as the Y-axis; however, it should not be applied to tennis serving data in which the acceleration approaches zero at impact as the X-axis.

Analysis at 200 Hz or less is common in the practical analysis of tennis movement (Elliott et al., 1989; Sprigings et al., 1994; Elliott et al., 1995). In the present study, the effects of the extrapolation methods in the lower sampling time data were examined. Four different sampling time data sets (500, 250 and 200 Hz data), which had the same impact frame obtained from the original 1000 Hz shadow serving displacement data, were set and were padded with extra points after impact using REF and LEX. Extrapolated and non-extrapolated data were smoothed with a recursive Butterworth digital filter and were differentiated by the central difference method. The differentiated data calculated using non-extrapolated data were defined as the criterion data in each sampling time. The differentiated data calculated using the extrapolated data were compared with the criterion data. Table 2 shows the velocity and acceleration at impact on the X-axis obtained using extrapolated and non-extrapolated data for each sampling time. The velocity and acceleration in the analysis at 1000 Hz are also shown in the same table for comparison. The reductions in velocity were observed for all conditions in proportion to the shortening of the sampling time. The acceleration obtained using LEX showed a higher value than the criterion acceleration data from just before impact for all sampling times, and the difference between the criterion velocity data and the velocity in LEX at impact became large based on the reduced sampling time. The result of the decrease in velocity under all conditions indicated that the same effect as filtering was obtained through the shortening of the sampling time (Omichi and Miyashita, 1981). The reason for the high acceleration in LEX is thought to be as follows: In the tennis serve, the racket velocity increases rapidly from just before impact. As the sampling time decreased, the slope between the last two points became higher, and consequently high acceleration was produced at impact in LEX because the padding data in LEX were calculated based on the slope. In contrast, zero acceleration was invariably produced at impact in REF and the velocity in REF at impact was similar to the criterion velocity data for each sampling time. Nunome (1999) reported that the REF proposed by Smith (1989) was expedient in situations in which the acceleration became zero at impact. Thus, in the practical analysis of the tennis serve, REF is the more suitable method for correcting the distortion caused by the collision between the racket and the ball for the X-axis, where the acceleration would be zero at ball impact.

The present study focused on the extrapolation method to correct the endpoint data for the flat tennis serve. Suzuki and Maeda (2003), who determined the acceleration of baseball bat swings using an accelerometer, confirmed that the acceleration toward the swing direction at impact was approximately zero. That is, the REF used in the present study may also be applicable to the motion analysis of the baseball bat swing to remove the influence of the collision between the bat and the ball. If the direction of the axis for the static coordinate system differs from the swing direction, the direction of the axis should be corrected to correspond to the swing direction using the coordinate transformation matrix.

Table 2 The velocity and acceleration on the X-axis at impact obtained using extrapolated and non-extrapolated data for sampling times of 1000, 500, 250, and 200 Hz.

<table>
<thead>
<tr>
<th>Racket</th>
<th>Velocity (m/s)</th>
<th>Acceleration (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CRITERION</td>
<td>REF</td>
</tr>
<tr>
<td>1000Hz</td>
<td>46.3</td>
<td>45.6</td>
</tr>
<tr>
<td>500Hz</td>
<td>44.3</td>
<td>43.6</td>
</tr>
<tr>
<td>250Hz</td>
<td>41.7</td>
<td>41.0</td>
</tr>
<tr>
<td>200Hz</td>
<td>40.8</td>
<td>40.1</td>
</tr>
<tr>
<td></td>
<td>CRITERION</td>
<td>REF</td>
</tr>
<tr>
<td>1000Hz</td>
<td>179.9</td>
<td>-7.4</td>
</tr>
<tr>
<td>500Hz</td>
<td>143.6</td>
<td>-1.7</td>
</tr>
<tr>
<td>250Hz</td>
<td>88.9</td>
<td>0.1</td>
</tr>
<tr>
<td>200Hz</td>
<td>83.0</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Since the proper extrapolation method changes according to the acceleration pattern at the endpoint, further research into extrapolation methods for other types of movement, such as the tennis volley, must be conducted.

4.2. Application of REF to 200 Hz Data in the Actual Serving

As mentioned above, the practical analysis of tennis movement was commonly analyzed at 200 Hz or less. In the analysis at 200 Hz, a maximum of one image can be captured during ball contact because the contact duration of the racket and ball is 2 - 5 ms (Watanabe and Ikegami, 1977). Therefore, researchers are not always certain about which frame should be selected as the start frame for extrapolation. In the present study, we investigated how the racket velocity at impact changed through REF, when the start frame to extrapolate was shifted. Figure 6 shows the possible start frames from which the analysis at 200 Hz could be extrapolated. The impact frame (IMP) is the same frame that was used as the impact frame when performing the analysis.

![Figure 6](image-url)
at 1000 Hz. B1, B2, B3 and B4 are the frames that occur 1 ms, 2 ms, 3 ms and 4 ms before IMP, respectively. Similarly, A1, A2, A3 and A4 are the frames that occur 1 ms, 2 ms, 3 ms and 4 ms after IMP, respectively. We selected nine different 200 Hz displacement data, which had nine different start frames from the original 1000 Hz actual serving data (Figure 6). Each raw displacement data series was extrapolated by REF, and the original 1000 Hz actual serving data was padded with extra points after impact using REF for comparison. Both 200 Hz and 1000 Hz extrapolated data were smoothed with a recursive Butterworth digital filter and differentiated by the central difference method. In the present study, only racket-differentiated data on the X-axis, which included the influence of the collision with the ball, were considered.

When a start frame is selected for analysis to extrapolate at 200 Hz, uncertainty arises between each pair of frames: B4 or A1, B3 or A2, B2 or A3 and B1 or A4. Table 3 shows the racket velocity on the X-axis at impact and the ratio of 200 Hz data to 1000 Hz data. The racket velocity at impact decreased from 10.1 - 21.5% in the analysis at 200 Hz compared to the velocity in the analysis at 1000 Hz (48.5 m/s). We then compared the racket velocities of each pair frame. Consequently, a high ratio was observed in frames B1, A1, A2 and A3 rather than in frames A4, B4, B3 and B2, although A1, A2 and A3 were ball contact frames. However, from the 200 Hz image, it is difficult to decide whether the image was frame B1 or B2, and A3 or A4. Therefore, in the analysis at 200 Hz, the final frame before impact should be selected as the start frame for extrapolation when no image is captured during ball contact. If an image of ball contact can be obtained, then the final frame before impact and the ball contact frame are set as pseudo-start frames for extrapolation, and the frame for which the higher racket velocity is obtained should be selected.

5. Conclusions

The present study investigated the distortions of three-dimensional kinematic data during the flat power serve in tennis using existing extrapolation methods. The results provide the following valuable information for correcting the distortions produced by the contact between the racket and the ball by extrapolation methods:

1) Because the racket displacement data for the swing direction axis included the influence of the collision with the ball, the extrapolation method should be applied on this axis to remove the influence of the collision. In contrast, since the displacement data for the other axes were only slightly affected by the collision with the ball, no extrapolation method should be applied on these axes and the displacement data before and after impact should be smoothed together without extrapolation.

2) In the practical analysis of the flat power serve, REF is the most suitable method for correcting the distortion produced by the collision with the ball for the swing direction axis among the extrapolation methods used in the present study.

3) In the analysis at 200 Hz, the final frame before impact should be selected as the start frame for extrapolation when no image is captured during ball contact. When a ball contact image can be captured, the final frame before impact and the ball contact frame are set as pseudo-start frames for extrapolation and the frame in which the higher racket velocity is obtained should be selected.

References


Extrapolation Method in Power Tennis Serve


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Membership in Learned Societies:
• Japan Society of Physical Education, Health and Sport Sciences
• Japanese Society of Biomechanics
• Japan Society on Tennis Science
• The Japanese Society of Physical Fitness and Sports Medicine