Scale-free Property of the Passing Behaviour in a Team Sport

Yuji Yamamoto

Research Center of Health, Physical Fitness, and Sports, Nagoya University
Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601 Japan

[Received May 7, 2009; Accepted September 30, 2009; Published online February 20, 2010]

This study examined the application of network theory to skilled passing behaviour in a team sport by considering small-world and scale-free network models. Using data obtained from a 2006 soccer game in Japan between Japan and Ghana, we counted the number of passes by each player within 5-minute intervals. The structural properties of the passing behaviour, which included a characteristic path length and clustering coefficient, and the degree of distribution were analysed. This showed that the structural property of the passing behaviour represented neither a complete graph nor a random graph; rather, it reflected a small-world or scale-free network. In addition, the probabilities of outgoing and incoming passes reflected links that followed a power-law distribution. Passing behaviour in a soccer match appeared to be similar to behaviour in social networks with smaller vertices in terms of the scale-free property and a self-organising mechanism.

Key word: network theory, soccer, power-law distribution

[International Journal of Sport and Health Science Vol.7, 86-95, 2009]

1. Introduction

Human movement science, especially research on motor control, has been examining coordinative movement patterns with a dynamical system approach. Using a bimanual coordination task, Kelso (1981, 1984) demonstrated the self-organising property of human movement. Kelso asked participants to move their fingers or hands according to the rhythm established by an auditory metronome. The relative angles of both index fingers or the hands were defined as an order parameter or a collective variable, and the frequency of movements was defined as a control parameter. At low frequencies, humans can perform this task in a stable manner only in bistable, in-phase, and antiphase coordination modes. When the participants began in the antiphase mode and the frequency required was increased gradually, the coordinative pattern shifted abruptly to the in-phase mode of coordination at a particular frequency. Based on these experimental results, a theoretical framework, referred to as the HKB model, was developed by Haken et al. (1985); this model owes much to theories of self-organisation and pattern formation in non-equilibrium

systems, especially those focusing on synergetics (Haken, 1983).

The last decade has seen much research on the structure of dyadic and team sports as a dynamical system or self-organising phenomenon. McGarry et al. (1999) analysed squash competition as a dyadic sport and, based on the perturbations and stability characterising the system, concluded that a squash game could be considered a dynamical system. In a similar vein, Palut and Zanone (2005) examined the relative lateral motion of tennis players as a collective variable and found a bistable mode—that is, it included both in-phase and antiphase modes as well as transitions between the two. These studies were based on the two coupling oscillators model, that is, the HKB model (Haken et al., 1985), and on the synergetics of coordinative movements (Haken, 1978). For team sports, Gréhaigne et al. (1997) explored the interactions between the main variables of the components of systems and subsystems in soccer by examining the transitions between configurations of play. Gréhaigne et al. (1997) did not use the HKB model, although their entire system was also composed of dynamic interactions among elements.

Our study represents another approach to exploring the dynamics of team sports. We examined the application of network theory to skilled passing behaviour in a team sport to investigate the merits of the small-world (Watts & Strogatz, 1998) and scale-free (Barabási & Albert, 1999) network models.

1.1. Brief review of network theory

Network theory is rooted in Euler's graph theory (Biggs et al., 1976), which was originally designed to solve the problem of the seven bridges of Königsberg, Prussia (now Kaliningrad, Russia) mathematically. The question was whether it was possible to follow a route that crossed each bridge exactly once. Euler (1736) formulated the problem by graphing Königsberg. First, he eliminated all of the features except the landmasses and the bridges connecting them; next, he replaced each landmass with a dot, called a vertex or node, and each bridge with a line, called an edge or link. The geographic relationships between the landmasses and the distances of the bridges were not considered in this model and only the connectivities of vertices deemed important.

Subsequently, Euler's graph theory (Biggs et al., 1976) has been expanded to include various types of graph, including star, tree, and wheel graphs (See **Figure 1**). Studies by Erdős and Rényi (1959, 1960, 1961), cited by Karoński and Ruciński (1997), elaborated on these graph theories by introducing the graph generation approach. This approach uses a regular graph and a random graph; a graph is regular when all vertices have equal edges and a random graph is reconstructed from a regular graph by random linkages between vertices with a particular probability p . Random graphs can be used to examine the phase transition and percolation of graph properties. In this context, the terms graph and network are synonymous.

The small-world phenomenon refers to the relatively short path between two vertices that occurs in most networks despite their frequently large size. During the 1960s, social psychologist Stanley Milgram examined this phenomenon with a sophisticated experiment (Milgram, 1967) that examined how many people were needed to connect a target person randomly selected by an initial participant with that participant. He found that about six acquaintances connected most pairs of people in the United States. This research, which examined short chains of social acquaintances, gave rise to the notion of "six degrees of separation",

Figure 1 Examples of star, tree, and wheel graph. n : number of vertices, m : number of edges, $\langle k \rangle$: degree, edges per vertex.

although Milgram never actually used this phrase.

Watts and Strogatz (1998) were the first to report the use of a network model to examine the small-world phenomenon mathematically; additional research has explored the small-world network model extensively (e.g., Dorogovtsev et al., 2001; Kleinberg, 1999, 2000). Indeed, several classic findings related to random graphs indicated that even networks with no definable topological structure exhibited the small-world phenomenon (Sporns et al., 2004). The small-world phenomenon is mathematically expressed as the diameter of the network, which grows logarithmically with the number of vertices (Albert et al., 2000). A similar result can be mapped onto networks with a power-law degree distribution; these networks are known as scale-free networks because power-laws are devoid of a characteristic scale (Barabási & Albert, 1999).

1.2. Some concepts for quantifying the property of complex networks

Current network theory can be used as an analytic tool for examining complex network behaviour. Watts and Strogatz (1998) examined random rewiring procedures as a means of interpolating between regular and random networks. They began with a ring lattice with n vertices and k edges per vertex and rewired each edge with probability p at random. The rewiring procedure could be examined as representing characteristics between a regular graph ($p=0$) and a random graph ($p=1$). They found that small-world networks could be highly clustered in a manner similar to regular lattices, yet have short characteristic path lengths, such as those seen in random graphs (see Appendix 1).

Watts and Strogatz (1998) defined the characteristic path length, $L(p)$, as the number of edges in the shortest

path between two vertices, averaged across all pairs of vertices. Defined in this way, $L(p)$ is a global property, measured by the typical separation between two vertices in the graph (see Appendix 2). The clustering coefficient, $C(p)$, refers to measures of a local property, the cliquishness of a typical neighbourhood. When a vertex, v , has k_v neighbours, then $k_v(k_v-1)/2$ edges could exist between these neighbourhoods. If C_v denotes the fraction of the allowable edges that actually exist, then C is defined as the average of all C_v s (see Appendix 3). Described in terms of these measures, a regular graph has a long path length and a high clustering coefficient. By contrast, a random graph has a short path length and a low clustering coefficient.

The degree distribution refers to the connectivity distribution, $P(k)$, given the probability that a vertex in the network is connected to k other vertices. The small-world network has the same characteristic degree distribution as does the random graph (Albert et al., 2000). Both networks are characterised by a Poisson-like $P(k)$ distribution that peaks at an average $\langle k \rangle$ and decays exponentially for large k (Albert & Barabási, 2002). By contrast, many real networks, such as the World Wide Web and social networks, suggest that $P(k)$ decays as a power law (i.e., $P(k) \sim k^{-\gamma}$); these are referred to as scale-free networks. The power-law in degree distribution suggests that self-organisation phenomena occur in complex networks with a few nodes connected by numerous links, referred to as “hubs” (see Appendix 4).

1.3. Application of network theory to pass behaviour

In team sports involving a ball, if we regard the pass as an edge or link and the players as vertices or nodes in network theory, passing behaviour would be described as an example of complex network behaviour identical to that in social networks, such as the World Wide Web (Albert et al., 1999) and scientific collaborative work (Newman, 2001). These networks have demonstrated short paths of intermediate acquaintances, high clustering coefficients, and power law degree distributions, such as found in small-world and scale-free networks (Albert & Barabási, 2002). The passing behaviour in team sports involving a ball would also be characterised by the property of complex networks assumed to be either completely regular or completely random. However, Buchanan (2002) regarded scale-free networks as aristocratic and small-world networks as egalitarian. An aristocratic network

is characterised by some vertices that are highly connected, called hubs, and illustrates the development of networks with self-organising phenomena with a degree distribution power-law form (Barabási & Albert, 1999).

Some studies have examined team sports as self-organising phenomena (Araújo et al., 2006; Gréhaigne et al., 1997; Passos et al., 2006) and have explored the order and control parameters based on the two coupling oscillators model (Haken et al., 1985) rather than on network theory. In team sports such as soccer, it is said that a good player who controls the flow of the team's offensive play is very important and that such a player will often maintain possession of the ball, allowing other team members to make offensive runs. The presence of momentum in team sports has also been noted (Higham et al., 2005; Gréhaigne et al., 1997; Johnes & Harwood, 2008). In the context of network theory, a good player would be regarded as a hub and an analysis of passing behaviour would contribute to considerations of the self-organising properties, including momentum, of soccer games.

This paper applied network theory to the complex, skilled passing behaviour that occurred during an international soccer game. The existence of a power-law structure in the network of passing behaviour might suggest that skilled passing behaviour has self-organising properties.

2. Method

2.1. Material

We sampled a video recording of a soccer game played in Japan, the 2006 Kirin Challenge Cup, a friendly match between Japan and Ghana. According to records kept by the Japan Football Association, the match was 93 minutes long, including 3 extra minutes. Ghana scored at the 73rd minute and won the match with a score of 1 to 0. At halftime, six players from the Japanese team and two players from the Ghanaian team were replaced with substitutes. Japan was in possession of the ball 50.5% of the time.

2.2. Data analysis

During the second half, several players on both teams were replaced. To eliminate the influence of these substitutions on the passing behaviour of each team, we analysed each half separately. For each half of

the match, we noted which players acted as passers and as receivers for each pass. According to this notation system, G and J represented Ghana and Japan, respectively. A single digit referred to our identification number for each player (rather than to the uniform number). Passing behaviours were recorded in bracketed pairings; in each case, the first number indicated the passing player and the second number identified the receiving player. The number of passes per player was counted in 5-minute intervals, which we selected as the optimal time frame for the purposes of this study. We concluded that 3-minute intervals would be too brief to contain a reasonable number of passes and that 15-minute intervals would result in too few segments to allow examination of the momentum in the match. This notation method is illustrated by the tracking of the ball handling during the 5-minute interval between minutes 25 and 30 of the second half, which included the score by Ghana: Corner kick, G6, J3, Goal kick, {G1, G5}, {G5, G8}, {G8, G7}, J6, Throw-in, {G7, G3}, {G3, G11 goal}, Kick-off, G7, {J11, J2}, {G8, G6}, {G6, G11}, {G11, G8}, {G8, G1}, {G1, G9}, {G9, G8}, {G8, G11}, {G11, G3}, Shoot, {J1, J2}, {J2, J7}, {J7, J2}, {G4, G7}, {G7, G10}, J11, Foul.

In describing passing behaviour in network terms, we regarded each player as a vertex and each pass as an edge. Using these parameters, we drew a ring lattice to represent each 5-minute interval for each team (see **Figure 2**). To examine the global and local properties of the network of passing behaviour, we calculated the characteristic path length, L , and the clustering coefficients, C , as defined by Watts and Strogatz (1998). These values were calculated for each team for each 5-minute interval following techniques outlined in *Combinatorica in Mathematica* (Pemmaraju & Skiena, 2003). The diameter, D , and the numbers of edges, m , within each graph were calculated to clarify the characteristics of the network. The diameter of a graph is the maximum eccentricity of any vertex, and the eccentricity of a vertex, v , is the length of the shortest path from v to some other vertex.

We also analysed the probability distributions for the connectivities of the vertices or the players $P(k)$ as a means of examining the power-law scaling proposed by Barabási and Albert (1999) as reflecting the scale-free network model. For each 5-minute interval, the frequency of touching the ball was counted separately for each player; these tabulations included both sending and receiving a pass, identified as outgoing and

Figure 2 Network graphs of 25-30 minutes in the second half, for Japan and Ghana, leading to a goal by Ghana.

incoming links, respectively. A log-log plot was applied to the frequency of ball touching and the number of players—that is, to connectivity to k other vertices and the probability of each connectivity within each team for each half of the match.

3. Results

3.1. Frequency of passes

During the first half of the match, 402 passes were completed, with Ghana passing 205 times and Japan passing 197 times. The player who touched the ball most frequently handled the ball 84 times during the first half; this included sending the ball 44 times and receiving it 40 times. By contrast, the player who had the least contact with the ball touched it only nine times (see **Table 1**). During the second half of the match, 348 passes were made, with Ghana passing 197 times and Japan passing 151 times. The player who touched the ball most frequently passed and received the ball 71 times during this period, whereas the player with the least contact with the ball touched it only six times (see **Table 2**).

3.2. Structural properties of pass behaviour as a network

Table 3 shows the characteristic path length, L , clustering coefficient, C , diameter, D , number of edges, m , and edges per vertex, $\langle k \rangle$, for each 5-minute interval and for each team. **Figure 3a** shows the frequency of passes (i.e., the number of edges, m) in each 5-minute interval.

This table presents each time interval for each team separately. To calculate the characteristic path length, L , and the clustering coefficient, C , we regarded players as vertices and passes between two players as edges. We were unable to calculate the value of L and C with

Table 1 Frequency of sending (Out) and receiving (In) the ball for each player in the 1st half of the match.

1st half									
Japan					Ghana				
No	Pos.	Out	In	sum	No	Pos.	Out	In	sum
1	GK	15	9	24	1	GK	6	3	9
4	DF	21	18	39	2	DF	5	4	9
5	DF	20	20	40	4	DF	24	15	39
8	DF	13	12	25	5	DF	20	15	35
3	MF	23	19	42	6	DF	20	16	36
6	MF	18	17	35	7	MF	17	19	36
7	MF	24	27	51	8	MF	44	40	84
9	MF	22	18	40	10	MF	30	35	65
10	MF	20	22	42	11	MF	25	26	51
11	FW	6	15	21	9	FW	10	19	29
2	FW	15	20	35	3	FW	4	13	17
sum		197	197	394	sum		205	205	410

Table 2 Frequency of sending (Out) and receiving (In) the ball for each player in the 2nd half of the match.

2nd half									
Japan					Ghana				
No	Pos.	Out	In	sum	No	Pos.	Out	In	sum
1	GK	12	6	18	1	GK	10	4	14
4	DF	9	4	13	2	DF → FW	15	13	28
5	DF	17	17	34	4	DF	17	10	27
8	DF	12	12	24	5	DF	11	9	20
3	MF	10	9	19	6	DF	12	12	24
6	MF → MF	20	13	33	7	MF	22	26	48
7	MF → MF	27	30	57	8	MF	36	35	71
9	MF → FW	1	5	6	10	MF	32	32	64
10	MF → MF	19	21	40	11	MF → MF	21	20	41
11	FW → MF	14	21	35	9	FW	8	15	23
2	FW → FW	10	13	23	3	FW	13	21	34
sum		151	151	302	sum		197	197	394

precision for intervals in which some players did not touch the ball. In these cases, we disregarded the isolated vertex (i.e., the player who had not touched the ball during the interval in question) and calculated approximate values for L and C .

During the first half of the match, the mean characteristic path length, L , was 1.76 for Ghana and 1.52 for Japan; the clustering coefficient, C , was 0.24 for Ghana and 0.20 for Japan. During the second half, L was 1.56 for Ghana and 1.84 for Japan, and C was 0.19 for Ghana and 0.27 for Japan. **Figure 3b** shows the change in the clustering coefficient during the match.

Ghana won the match with a score of 1 to 0, and **Figure 3** also shows the goal by Ghana.

The mean characteristic path length measures a global property of the graph (i.e., the separation between two vertices or players). In this study, the characteristic path length, L , ranged from 1.33 to 2.67, indicating that each player could connect with every other player in fewer than three passes. A complete graph including all possible edges would show a value for L of 1.0. **Table 3** presents a random graph based on rewiring each edge with a probability of $p=1$ (See Appendix 5). All values of observed characteristic path

Table 3 Structural properties of networks in each 5-min intervals for each team.

Time	Japan ($n=11$)							Ghana ($n=11$)						
	L	L_{rand}	C	C_{rand}	D	m	$\langle k \rangle$	L	L_{rand}	C	C_{rand}	D	m	$\langle k \rangle$
0-5	2.02	3.25	0.53	0.19	4	23	2.09	1.56	2.39	0.19	0.25	3	30	2.73
5-10	1.52	6.40	0.11	0.13	2	16	1.45	1.38	1.93	0.59	0.31	2	38	3.45
10-15	1.56	1.89	0.32	0.32	3	39	3.55	1.61	3.07	0.62	0.20	3	24	2.18
15-20	1.93		0.23	0.07	4	9	0.82	1.40	9.94	0.10	0.12	2	14	1.27
20-25	2.67		0.00	0.08	6	10	0.91			0.00	0.02		2	0.18
25-30	1.95	7.73	0.29	0.12	3	15	1.36	1.64	1.76	0.29	0.36	3	43	3.91
30-35	1.51	1.79	0.37	0.35	3	42	3.82	1.64	4.39	0.60	0.16	3	19	1.73
35-40	2.18		0.00	0.08	4	10	0.91	1.43	4.39	0.48	0.16	2	19	1.73
40-45	1.42	2.18	0.36	0.27	2	33	3.00	1.48	6.40	0.22	0.13	2	16	1.45
45-50	2.09	3.46	0.34	0.18	4	22	2.00	1.56	3.46	0.30	0.18	3	22	2.00
50-55	1.87	3.71	0.28	0.17	4	21	1.91	1.64	3.71	0.74	0.17	3	21	1.91
55-60	1.94	7.73	0.38	0.12	4	15	1.36	1.78	2.25	0.49	0.26	3	32	2.91
60-65	1.33		0.00	0.03	2	4	0.36	1.79	6.40	0.16	0.13	3	16	1.45
65-70	1.67	2.57	0.42	0.23	3	28	2.55	1.98	3.71	0.31	0.17	3	21	1.91
70-75	1.50		0.00	0.03	2	4	0.36	1.71	7.73	0.61	0.12	3	15	1.36
75-80	1.72	2.39	0.72	0.25	3	30	2.73	1.54	2.39	0.11	0.25	3	30	2.73
80-85	1.33		0.78	0.04	2	5	0.45	1.60	6.40	0.05	0.13	3	16	1.45
85-93	1.61	3.46	0.35	0.18	2	22	2.00	1.50	3.07	0.38	0.20	3	24	2.18

L : characteristic path length, L_{rand} : characteristic path length for a random graph, C : clustering coefficient
 C_{rand} : clustering coefficient for a random graph, D : diameter, m : the number of edges, $\langle k \rangle$: edges per vertex

Figure 3 Frequency of the sum of passes (a) and clustering coefficients (b) in each time interval.

length L were smaller than L_{rand} , the predicted characteristic path length for a random graph (calculated using the formula $\ln(n)/\ln(k)$, which applies to a random graph with a probability of $p=1$ only if $k \gg 1$) (Watts & Strogatz, 1998). Albert et al. (1999) showed that the average shortest path between two vertices, d (referred to as characteristic path length, L , in this article), could be estimated using the formula $\langle d \rangle = 0.35 + 2.06 \log(N)$. In this case, when $N=11$ (the

number of players on a soccer team), the predicted value for L was 2.49, a figure approximating the L s observed in this study. This result suggested that passing behaviour in the soccer game did not reflect a random graph or a complete graph; rather, it had characteristics of a more structured network, such as a small-world or scale-free network.

The clustering coefficient, C , measures a local property of the graph, the cliquishness of a typical

Figure 4 The distribution function of connectivities for each 5-min interval in the first and second halves of the match. a: outgoing and incoming links in the first half for Japan. The line is $\gamma_{out}=1.09$, the dashed line is $\gamma_{in}=1.56$. b: the first half for Ghana. The line and dashed line correspond to $\gamma_{out}=1.35$, $\gamma_{in}=1.37$. c: the second half for Japan. The line and dashed line correspond to $\gamma_{out}=1.58$, $\gamma_{in}=1.64$. d: the second half for Ghana. The line and dashed line correspond to $\gamma_{out}=1.70$, $\gamma_{in}=1.56$.

neighbourhood. In our study, the clustering coefficient, C , ranged from 0.00 to 0.78. A complete graph including all possible edges would show a value for C of 1.0. **Table 3** presents a random graph based on rewiring each edge with probability $p=1$ (See Appendix 5). In many cases, the values obtained were larger than C_{rand} , the predicted clustering coefficient for a random graph (calculated using $\langle k \rangle/n$ as a random graph with $p=1$) (Watts & Strogatz, 1998). Therefore, the clustering coefficients of these graphs also indicated that passing behaviour was not random, but was highly clustered.

3.3. Degree distribution

The scale-free property of a graph is characterised by a power-law degree distribution (Albert et al., 1999; Barabási & Albert, 1999). To examine the scale-free property of passing behaviour in the soccer game, we analysed the probability that a given player, k , was sending, $P_{out}(k)$, or receiving, $P_{in}(k)$, the ball.

The probability distribution of the connectivities in these data can be described by the equation $P(k) \approx k^{-\gamma}$, with $\gamma_{out}=1.09$ ($r^2=.956$), $\gamma_{in}=1.56$ ($r^2=.834$) for Japan and $\gamma_{out}=1.35$ ($r^2=.591$), $\gamma_{in}=1.37$ ($r^2=.422$) for Ghana in the first half, and with $\gamma_{out}=1.58$ ($r^2=.939$), $\gamma_{in}=1.64$ ($r^2=.922$) for Japan and $\gamma_{out}=1.70$ ($r^2=.848$), $\gamma_{in}=1.56$ ($r^2=.885$) for Ghana in the second half (see **Figure 4**).

These results show that the passing behaviour in the soccer match was almost characterised by the scale-free

property of a self-organising mechanism found in social networks with smaller vertices, whereas the coefficients of determination for Ghana in the first half were relatively low.

4. Discussion

In this study, we applied network theory to the complex, skilled passing behaviour in a soccer match. An international soccer game was analysed at 5-minute intervals, and certain structural properties of the network and degree distribution were examined.

The values of the characteristic path length, L , in this study ranged from 1.33 to 2.67. If passing behaviour constituted a complete graph containing all possible edges, then L would be 1, which is lower than the value found here. If passing behaviour constituted a random graph, then L would have a higher value than that observed in this study. The clustering coefficient, C , ranged from 0.00 to 0.78. If the number of passes in 5-minute intervals were very low (e.g., fewer than ten passes), a low clustering coefficient, C , would be observed; that is, triangles would not be formed easily in the network. However, when the number of passes in a 5-minute interval exceeded ten, the clustering coefficient, C , was found to be greater than C_{rand} , a random graph with $p=1$. The results of these analyses of the characteristic path length, L , and clustering coefficient, C , suggest that the structural property of the observed passing behaviour was not a complete

graph or a random graph, but rather a small-world or scale-free network, as proposed by Watts and Strogatz (1998) and Barabási and Albert (1999).

To examine the scale-free property of the passing behaviour in the soccer match, we analysed the probabilities of player k having outgoing and incoming links: $P_{out}(k)$ and $P_{in}(k)$, respectively. The probability distribution of the connectivities can be described by the equation $P(k) \approx k^{-\gamma}$, with values ranging from 1.09 to 1.70 in this study. Previous research showed that the degree exponents from a scale-free network such as the World Wide Web were typically $\gamma \geq 2$ (Albert et al., 1999); however, those from scale-free spatial networks of biological origin have been found to be $P(k) \propto k^{-1}$ (Hermann et al., 2003). Park et al. (2005) suggested that two classes of model exhibited the scale-free property with and without growth of the network. According to these authors, the scale-free property of the network might arise through a self-organising mechanism via the natural evolution of links in the network, even without any growth mechanism. Our results suggested that passing behaviour in the soccer match had the scale-free property with a self-organising mechanism, as expected in a social network with smaller vertices. In other words, the existence of a power-law structure in the network of passing behaviour might suggest that skilled passing behaviour has self-organising properties.

The degree exponents in this study were lower than were those derived from such scale-free networks as social networks, indicating that the power-law tail in this study was shorter than that in many social networks. This finding suggests that the maximum number of edges connected to one vertex was relatively low, but that some vertices had a relatively high number of edges; therefore, more than one player in a team made the plays. Indeed, attack vulnerability would be high if only one player made all the plays; in other words, the team would not function when that player could not play successfully. The scale-free property with a power-law degree distribution such as that of the World Wide Web has shown high degrees of tolerance against random attacks or errors. However, when the attack removed the most densely connected vertices, the diameter or characteristic path length of the scale-free network increased rapidly, indicating high vulnerability (Albert et al., 2000). Since both error tolerance and self-organising properties are required in team sports, passing behaviour would reflect redundant wiring within the functional web consisting of several

active players. This functional network property would be typical for team sports that involve a ball and fewer vertices usually constrained by spatial-temporal factors.

Note that the greatest contrast between the two teams appeared in the cluster coefficient describing the interval in which Ghana scored. During this interval, Ghana had high cliquishness, but Japan had low connectivity. The results suggest that the probabilities of passing behaviour were characterised by outgoing and incoming links that followed a power-law distribution as a complex property of a scale-free network. That is, some players (or vertices) acted as highly connected hubs, although most players had a low degree of connection. Taken together, the results suggest that the highly skilled team had these rich-get-richer hubs, like an aristocratic network; however, the difference in connectivity between teams for a particular duration would affect scoring in a team sport.

The number of vertices or players in the soccer match was fixed, and this number was smaller than that in many other network systems that have been investigated using network theory (e.g., the Internet or *Caenorhabditis elegans* [a roundworm]). However, further applications of network models to team sports might reveal emergent or self-organising properties, such as game momentum. In addition, comparisons using different skill levels are necessary to confirm the validity of applying network theory to team games.

Acknowledgements

The author would like to thank to Takashi Yoshino for his assistance with the program for network analysis. The work reported in this article have been supported in part by Grant-in-Aid for Scientific Research (A) 20240060 from the Japan Society for the Promotion of Science.

Reference

- Albert, R. & Barabási, A. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics*, **74**, 47-97.
- Albert, R., Jeong, H., & Barabási, A. (1999). Diameter of the world-wide web. *Nature*, **401**, 130-131.
- Albert, R., Jeong, H., & Barabási, A. (2000). Error and attack tolerance of complex networks. *Nature*, **406**, 378-382.
- Araújo, D., Davids, K., & Hristovski, R. (2006). The ecological dynamics of decision making in sport. *Psychology of Sport and Exercise*, **7**, 653-676.
- Barabási, A. & Albert, R. (1999). Emergence of scaling in random networks. *Science*, **286**, 509-512.
- Biggs, N. L., Lloyd, E. K., & Wilson, R. J. (1976). *Paths. Graph theory 1736-1936*. Clarendon Press: London.
- Buchanan, M. (2002). *Nexus: small worlds and groundbreaking science of networks*. W. W. Norton & Company, Inc.: New York.
- Dorogovtsev, S. N., Mendes, J. F. F., & Samukhin, A. N. (2001).

- Giant strongly connected component of directed networks. *Physical Review E*, **64** 025101, 1-5.
- Erdős, P. & Rényi, A. (1959). On random graphs I. *Publicationes Mathematicae Debrecen*, **6**, 292-297.
- Erdős, P. & Rényi, A. (1960). On the evolution of random graphs. *Publications of the Mathematical Institute of the Hungarian Academy of Science*, **5**, 17-64.
- Erdős, P. & Rényi, A. (1961). On the evolution of random graphs. *Bulletin of the Institute of International Statistics*, **38**, 343-347.
- Euler, L. (1736). Solutio problematis ad geometriam situs pertinentis (The solution of a problem relating to the geometry of position). *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, **8**, 128-140.
- Gréhaigne, J.-F., Bouthier, D., & David, B. (1997). Dynamic-system analysis of opponent relationships in collective actions in soccer. *Journal of Sports Sciences*, **15**, 135-149.
- Haken, H., Kelso, J. A. S., & Bunz, H. (1985). A theoretical model of phase transitions in human hand movements. *Biological Cybernetics*, **51**(5), 347-356.
- Haken, H. (1978). *Synergetics: an introduction, non-equilibrium phase transitions and self-organization in physics, chemistry and biology* (2nd edition). Springer: Berlin.
- Haken, H. (1983). *Advanced synergetics: instability hierarchies of self-organizing systems and devices*. Springer: Berlin.
- Hermann, C., Barthélemy, M., & Provero, P. (2003). Connectivity distribution of spatial networks. *Physical Review E*, **68** 026128, 1-6.
- Higham, A., Harwood, C., & Cale, A. (2005). *Momentum in soccer*. Coachwise Ltd.: Leeds, UK.
- Johnes, M. I. & Harwood, C. (2008). Psychological momentum within competitive soccer: players' perspective. *Journal of Applied Sport Psychology*, **20**, 57-72.
- Karoński, M. & Ruciński, A. (1997). The origins of the theory of random graphs. In R. L. Graham & J. Nešetřil (Eds.), *The mathematics of Paul Erdős I*, 311-336. Springer-Verlag: Berlin.
- Kelso, J. A. S. (1981). On the oscillatory basis of movement. *Bulletin of the Psychonomic Society*, Vol. 18, p. 63.
- Kelso, J. A. S. (1984). Phase transitions and critical behavior in human bimanual coordination. *American Journal of Physiology*, **246**, R1000-R1004.
- Kleinberg, J. M. (1999). The small-world phenomenon: an algorithmic perspective. *Cornell Computer Science Technical Report*, **99-1776**, 1-14.
- Kleinberg, J. M. (2000). Navigation in a small world. *Nature*, **406**, 845.
- McGarry, T., Khan, M. A., & Franks, I. M. (1999). On the presence and absence of behavioural traits in sports: an example from championship squash match-play. *Journal of Sports Sciences*, **17**, 297-311.
- Milgram, S. (1967). The small-world problem. *Psychology Today*, **1**, 61-67.
- Newman, M. E. (2001). The structure of scientific collaboration. *Proceedings of the National Academy of Sciences of the United States of America*, **98**, 404-409.
- Palut, Y. & Zanone, P.-G. (2005). A dynamical analysis of tennis: concepts and data. *Journal of Sports Sciences*, **23**, 1021-1032.
- Park, K., Lai, Y.-C., & Ye, N. (2005). Self-organized scale-free networks. *Physical Review E*, **72** 026131, 1-5.
- Passos, P., Araújo, D., Davids, K., Gouveia, L., & Serpa, S. (2006). Interpersonal dynamics in sport: the role of artificial neural networks and 3-D analysis. *Behavior Research Methods*, **38**, 683-691.
- Pemmaraju, S. & Skiena, S. (2003). *Computational discrete mathematics: combinatorics and graph theory with Mathematica*. Cambridge University Press: Cambridge, UK.
- Sporns, O., Chialvo, D. R., Kaiser, M., & Hilgetag, C. C. (2004). Organization, development and function of complex brain networks. *Trends in Cognitive Sciences*, **8**, 418-425.
- Watts, D. J. & Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature*, **393**, 440-442.

Name:
Yuji Yamamoto

Affiliation:
Research Center of Health, Physical Fitness,
and Sports, Nagoya University

Address:

Furo-cho, Chikusa-ku, Nagoya, Aichi 464-8601, Japan

Brief Biographical History:

1985-86 Research Associates at the Department of Sports Science, Tsukuba University
1986-87 Assistant Professor at the Chukyo Women's University
1987-95 Assistant Professor at the Research Center of Health, Physical Fitness, and Sports, Nagoya University
1995-2004 Associate Professor at the Research Center of Health, Physical Fitness, and Sports, Nagoya University
2001 PhD, University of Tsukuba
2004- Professor at the Research Center of Health, Physical Fitness, and Sports, Nagoya University

Main Works:

- Continuous hitting movements modeled from the perspective of dynamical systems with temporal input. *Human Movement Science*, 19(3), 341-371, 2000.
- An alternative approach to the acquisition of a complex motor skill: Multiple movement training on tennis strokes. *International Journal of Sport and Health Science*, 2, 169-179, 2004.
- Dynamical system approach to human complex movement, University of Tokyo Press, 2005.
- Brain responses to meaningless postures as measured by near-infrared spectroscopy, *Neuroreport*, 19, 1411-1415, 2008.
- Duration of mentally simulated movement before and after a golf shot, *Perceptual and Motor Skills*, 108, 327-338, 2009.

Membership in Learned Societies:

- Japan Society of Physical Education, Health and Sport Sciences
- Japanese Society of Sport Psychology
- Japanese Cognitive Science Society
- The Japanese Psychological Association

Appendix 1 Rewiring procedure in a small-world network

Start with a ring lattice with n vertices in which every vertex is connected to its first K neighbours ($K/2$ on either side). Next, randomly rewire each edge of the lattice with probability p such that self-connections and duplicate edges are excluded. This process introduces $pnK/2$ long-range edges that connect vertices that otherwise would be joined to different neighbours. By varying p , one can closely monitor the transition between order ($p=0$) and randomness ($p=1$).

Appendix 2 Characteristic path length (L)

A global property of the network can be quantified by the typical separation between two vertices in the graph, known as the characteristic path length, L . The characteristic path length is the average number of edges needed to trace the two vertices by the shortest path when two arbitrary vertices are chosen from the network. When l_{ij} denotes the number of edges needed for the shortest path between vertex i and vertex j , the mean path length, L_i , for the vertex, i , can be calculated. This calculation would apply to all vertices, with the characteristic path length denoted as follows:

$$L = \frac{1}{n} \sum_{i=1}^n L_i, \quad L_i = \frac{1}{n-1} \sum_{j \neq i} l_{ij} \quad (\text{A.1})$$

$L(p)$ denotes the characteristic path length in a small-world network rewired with probability p .

The maximum value of the shortest paths in all pairs is the diameter (D), which also represents a quantification of a global property of the network.

Appendix 3 Clustering coefficient (C)

The clustering coefficient (C) quantifies the cliquishness of vertices in a network where cluster refers to three vertices connected to each other by three edges. When a vertex, v_i , has k_i edges, then $k_i(k_i-1)/2$ edges could exist between these neighbourhoods. If E_i denotes the number of triangles, or clusters that actually exist, the clustering coefficient can be defined as equation A.2. If a vertex v_i has no edge or only one edge, the clustering coefficient is defined as $C_i=0$. Therefore, the mean clustering coefficient was defined as follows:

$$C = \frac{1}{n} \sum_{i=1}^n C_i, \quad C_i = \frac{2E_i}{k_i(k_i-1)} \quad (\text{A.2})$$

$C(p)$ denotes the clustering coefficient in a small-world network rewired with probability p .

Appendix 4 Degree distribution

A degree is the number of edges of a vertex. Not all that edges in a network have the same number of degrees in the real world. The range in degrees is characterised by a distribution function, $P(k)$, that gives the probability that a randomly selected vertex has exactly k edges. Since the edges are placed randomly in a random graph, the majority of vertices have approximately the same number of degrees, close to the average degree $\langle k \rangle$ of the network. The degree distribution of a random graph is a Poisson distribution with a peak at $P(\langle k \rangle)$. In addition, the small-world network shows a Poisson distribution (Albert & Barabási, 2002). However, in many complex networks, the degree distribution has a power-law tail, $P(k) \sim k^{-\gamma}$. Such networks are known as scale-free, and the vertex with many degrees is known as a hub (Barabási & Albert, 1999).

Appendix 5 Characteristic path length and clustering coefficient in a random graph

The characteristic path length (L_{rand}) and clustering coefficient (C_{rand}) in a random graph that rewired each edge completely randomly with $p=1$ for large n vertices was defined as follows (Watts & Strogatz, 1998):

$$L_{rand} \sim \frac{\ln n}{\ln \langle k \rangle}, \quad C_{rand} \sim \frac{\langle k \rangle}{n} \quad (\text{A.3})$$

$\langle k \rangle$ denotes edges per vertex, called degrees, and n shows the number of vertices.