Stochastic Regular Approximation of Tree Grammars and Its Application to Faster ncRNA Family Annotation

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Tree Adjoining Grammar (TAG) is a useful grammatical tool to model RNA secondary structures containing pseudoknots, but its time complexity for parsing is not small enough for the practical use. Recently, Weinberg and Ruzzo proposed a method of approximating stochastic context free grammar by stochastic regular grammar and applied it to faster genome annotation of non-coding RNA families. This paper proposes a method for extending their idea to stochastic approximation of TAGs by regular grammars. We will also report some preliminary experimental results on how well we can filter out non candidate parts of genome sequences by using obtained approximate regular grammars.

1. Introduction

Biological sequences contain both of stochastic and structural information. Formal grammars are quite useful tools for modeling, with high accuracy, such stochastic and structural features of biological sequences. Covariance Model\(^7\),\(^{24}\), CM for short, is one of the most successful grammatical model for RNA families, in which stochastic context free grammar is used to model RNA primary and secondary structures. This seminal grammatical technique makes it possible to produce reliable RNA databases\(^{26}\).

A secondary structure of an RNA sequence \(w = a_1 \cdots a_n\) \((a_i \in \{A, C, G, U\})\) is base pairing information between bases in \(w\), which is described as a finite set of integer pairs \((i, j)\) with \(i \\leq j\) \((1 \leq i < j \leq n)\) indicating \(i\)-th and \(j\)-th bases in \(w\), respectively. It is recognized as an important issue to deal with secondary structure of a given RNA sequence, since its structure has strong relation to its biological function. However, one of the difficulties in the CM approach is that it can not model the secondary structure, called pseudoknot, which commonly appear in various RNA molecules and play various roles in biological functions\(^{5,16}\).

When an RNA secondary structure contains base pairs \((i, j)\) and \((i', j')\) such that \(i < i' < j < j'\) or \(i'' < i < i' < j' < j\), it is called a pseudoknot (See Fig. 1). Since the crossing dependency in a pseudoknot can not be represented by context free grammars, in order to deal with pseudonot, it is necessary to prepare a grammatical model whose generative capacity is beyond context freeness. There are several candidate grammars proposed, including Tree Adjoining Grammar (TAG)\(^{13,30,31}\), RNA Pseudoknot Grammar\(^{22}\), Multiple Context Free Grammar\(^{11,12}\), CFG based Parallel Communicating Grammar\(^3\), etc.

A Tree Adjoining Grammar (TAG) is a grammatical device to generate a set of trees rather than a set of strings, which was first proposed by Joshi and Takahashi\(^{10}\). It is known that the string languages generated by TAGs are between context sensitiveness and context freeness, and can model pseudoknotted structures.

Although TAG has enough computational capacity to precisely analyze pseudoknots, its time complexity for parsing is not small enough for the practical use. Recently, Weinberg and Ruzzo proposed a method of approximating stochastic context free grammar by stochastic regular grammar and applied it to faster genome annotation of non-coding RNA families using CM method\(^{32}\). The idea is to use such an approximate stochastic regular grammar to filter total genome sequences and find out candidate positions of the target family. CM is applied only to such candidates, which drastically reduces the total time of annotation of ncRNA families. This paper extends their idea and applied it to approximating stochastic tree grammars by regular grammars. We will also report some preliminary experimental results on how well the obtained approximate regular grammars can be used to filter out non

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The purpose of this paper is to show the effectiveness of the proposed approximation method to filter out non-candidate positions of ncRNA families in total genome sequences. The computational experiment of the ncRNA family annotation by TAG still requires much computation time. But, the authors believe that the technique used in this paper can also be extensively used in the parsing process of TAG itself. Thus, the results reported in this paper is a challenging first step toward the goal where we can effectively and effectively use TAG for the genome annotation with high accuracy.

In Section 2, we will give definitions and notations of tree adjoining grammars and describe how we can model RNA secondary structures including pseudoknots using TAGs. Inspired from the results by Weinberg and Ruzzo, in Section 3, we will propose a method of approximating a given stochastic tree adjoining grammar for RNA modeling to a stochastic regular grammar. We will also explain a method of tuning stochastic parameters. In Section 4, some preliminary experimental results are reported, in which we will show the effectiveness of the proposed method. In Section 5, we will show related works. Finally, Section 6 gives some concluding remarks.

2. Tree Adjoining Grammar for RNA Secondary Structure

2.1 Tree Adjoining Grammar

A TAG is a grammatical device for generating trees. Let us consider a tree labeled with symbols in the alphabet $V = N \cup \Sigma$ where $N$ and $\Sigma$ are disjoint. Symbols in $N$ are called nonterminals, and those in $\Sigma$ are called terminals. By $\tau_V$, we denote the set of trees whose internal and leaf nodes are labeled by symbols in $N$ and $V$, respectively. A TAG $G$ is defined by $G = (V, C, A)$, where $V$ is an alphabet, and $C$ and $A$ are finite subsets of $\tau_V$ such that every $t \in C$ does not have a leaf node with a label in $N$, every $t \in A$ has exactly one leaf node with a label in $N$. Elements of $C$ and $A$ are called center trees and adjunct trees, respectively. Elements of $C \cup A$ are called elementary trees. By definition, the leaf node of an adjunct tree whose label is nonterminal is determined uniquely and is called a foot node. Furthermore, an additional requirement for an adjunct tree is that the label of the foot node should be equivalent to that of the root node. The path from the root to the foot node of an adjunct tree is called a spine. It is often the case that some constraints are associated with each node $n$ of elementary trees. In this paper, we will consider the following constraints:

1. Null Adjoining (NA): No adjunct trees can be adjoined at $n$.
2. Obligating Adjoining (OA): A member of the set $A$ must be adjoined at $n$.

A node without NA constraint is said to be active. These constraints play an important role in defining the derivation process of tree grammars.

Let $G = (V, C, A)$ be a TAG. An adjunct tree $\alpha$ is adjoinable to a tree $\gamma_1 \in \tau_V$, if $\gamma_1$ has an active internal node $n$ whose label is the same as that of the root node of $\alpha$. The operation to adjoin $\alpha$ on $n$ of $\gamma_1$ is defined by the following procedure:

**Split:** Split $\gamma_1$ at $n$ into $\gamma_1'$ and $\gamma_1''$.

**Merge:** Combine $\gamma_1''$ and $\alpha$ by identifying $n$ of $\gamma_1''$ and the root node of $\alpha$, and then combine again the resultant tree and $\gamma_1'$ by identifying the foot node of $\alpha$ and the root node (i.e. $n$) of $\gamma_1$ (See Fig. 2).

We write $\gamma_1 \vdash_G \gamma_2$ if there is an adjunct tree $\alpha \in A$ which is adjoinable to $\gamma_1$ and $\gamma_2$ is obtained by adjoining $\alpha$ to $\gamma_1$. By $\vdash^*_G$, we denote the reflexive and transitive closure of $\vdash_G$. We define:

$$\tau(G) = \{ t \in \tau_V \mid t_0 \vdash^*_G t, t \text{ does not contain a node with OA constraint and } t_0 \in C. \}$$
\[ L(G) = \{ Y(t) \in \Sigma^* \mid t \in \tau(G) \} \]

where \( Y(t) \) is the yield of a tree \( t \), which is defined as the string consisting of labels of leaf nodes of \( t \) ordered in depth-first and left-to-right.

2.2 \( TAG_{RNA} \)

In this paper, we will focus on a subclass of \( TAGs \), called \( TAG_{RNA} \), which was proposed by one of the authors in order to model RNA secondary structures including pseudoknots\(^{13,30,31}\). Although the generative capacity of \( TAG_{RNA} \) is strictly weaker than that of \( TAGs \), it can model various types of existing RNA secondary structures.

A center tree of \( TAG_{RNA} \) should be of the form represented in Fig. 4(a), where \( S \in N \) and \( \lambda \) is an empty string. An active node is indicated by the symbol ‘\( \ast \)’. A center tree used in a \( TAG_{RNA} \) always has only two nodes root and leaf. The root node is an active node and the label of the leaf node is always \( \lambda \). An adjunct tree of \( TAG_{RNA} \) should be of the form represented in Fig. 4(b) or Fig. 4(c), where \( e_1, e_2, e_3, e_4 \in \Sigma \cup \{ \circ \} \) and \( X, Y, Z \in N \).

If \( e_i \) for \( i=1,2,3,4 \) is ‘\( \ast \)’, \( e_i \) is said to be a blank node (the blank node indicates that the node doesn’t exist.). We call an adjunct tree of Fig. 4(b) a \textbf{type-A} adjunct tree which has the root node, the foot node, one active node on the spine, and four nodes (leaf or blank). In a type-A adjunct tree, the root node has three child nodes. These are located down left (the upper-left node of the active node), just under (the active node on the spine), and down right (the upper-right node of the active node) of the root node respectively. The active node also has three child nodes. These are located down left, just under (the foot node), and down right of the active node, respectively. We call an adjunct tree of Fig. 4(c) a \textbf{type-B} adjunct tree which has the root node, the foot node, and two active nodes. One of two active nodes is on the spine, and the other is located at either upper-left, upper-right, down-left or down-right of the active node on the spine. The latter active node has a leaf node as a child, whose label is always \( \lambda \). In this paper, the latter active node is called a \textit{branch active node}.

For convenience of the discussion in the sequel, we introduce notations of elementary trees of \( TAG_{RNA} \) as follows:

By \( [S^* - \lambda] \), we denote a center tree of \( TAG_{RNA} \) whose root label is \( S \in N \). By \( [X \rightarrow Y^*(e_1,e_2,e_3,e_4)] \), we denote a type-A adjunct tree of \( TAG_{RNA} \) such that root label is \( X \in N \), active node label is \( Y \in N \), the upper-left node label is \( e_1 \), the down-left is \( e_2 \), the down-right is \( e_3 \), and the upper-right is \( e_4 \), respectively. By \( [X \rightarrow Y^*(Z^*,\circ,\circ)] \), we denote a type-B adjunct tree of \( TAG_{RNA} \), where \( X \in N \) is the label of the root, \( Y \in N \) is the label of the active node on the spine, and \( Z \in N \) is the label of the branch active node.

A \( TAG_{RNA} \) can be used to model an RNA secondary structure including pseudoknots. Asakawa characterized the class of RNA secondary structures represented by \( TAG_{RNA} \). He proved that \( TAG_{RNA} \) pseudoknotted structure can be characterized by a W-shaped structure (see Fig. 5), where base pairs are represented by dotted horizontal lines. Almost all of the pseudoknot structures in Rfam can be modeled by a \( TAG_{RNA} \) with some exceptions.

For instance, let us consider an RNA sub-
sequence of \( w = AGACUU \) with a secondary structure \{ (1,5), (2,4), (3,6) \}. The set of elementary trees to model this RNA is given by \((c) [X_1^* - \lambda], (a_1) [X_1 \rightarrow X_2^* (A, U, \circ, \circ)], (a_2) [X_2 \rightarrow X_3^* (G, C, \circ, \circ)] \) and \((a_3) [X_3 \rightarrow X_4^* (\circ, A, U, \circ)] \). Its derivation process is shown in Fig. 6.

### 2.3 Automatic Construction of Stochastic TAG

Takakura, et al., developed a system for generating stochastic TAG from multiple alignment data of RNA sequences with secondary structure information\(^{28},29\). This system is based on the algorithm developed by Asakawa which decides whether or not a given secondary structure can be modeled by a TAG\(^3\). By modifying this algorithm and collecting stochastic information of primary sequences in the alignment data, Takakura developed an efficient system for generating stochastic TAG which models the given alignment data. This system can generate stochastic TAG in \(O(n^2m)\), where \( n \) is the maximum length of RNA sequence and \( m \) is the number of RNA sequences in the given alignment data\(^9\).

Although we have not yet developed a learning algorithm of stochastic TAG like that of stochastic CFG in CM method, the simple implementation of collecting stochastic information from the alignment data works fairly well for the unknown data classification\(^{28}\). We will use Takakura’s system for the experiments in Section 4.

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* Theoretically, we can develop an algorithm which runs in time \(O(nm)\), but the current implementation is \(O(n^2m)\).

### 3. Approximating Tree Grammars by Regular Grammars

#### 3.1 Speed-up by Filtering

Although \( \text{TAG}_{RNA} \)'s can effectively model RNA secondary structures including pseudoknots, its time complexity for parsing is \(O(n^5)\) \((n \) is the length of input string\(^{31}\), which makes it hard to apply \( \text{TAG}_{RNA} \)'s to the search of functional RNAs in large genome databases. In order to overcome the difficulty, we will propose a method for making a \( \text{TAG}_{RNA} \) search efficient without loss of its accuracy based on Weinberg’s idea. The method constructs, from a stochastic \( \text{TAG}_{RNA} \), a stochastic regular grammar (SREG), which approximates the given \( \text{TAG}_{RNA} \) and filters genome databases. Stochastic parsing by \( \text{TAG}_{RNA} \) is applied only to candidates of functional RNAs which pass through the filtering process by the SREG. Since SREG parsing is much faster than \( \text{TAG}_{RNA} \) parsing, the good approximation of \( \text{TAG}_{RNA} \) by SREG makes the database search much faster, even if we consider the time of approximating \( \text{TAG}_{RNA} \).

Let \( P(w \mid G) \) be the maximum probability of all derivations of \( G \) which generate \( w \). An approximate SREG \( G_{reg} \) for a \( \text{TAG}_{RNA} \) \( G_{tag} \) should satisfy:

\[
P(w \mid G_{reg}) \geq P(w \mid G_{tag}), \text{ (for any } w)\]

in order to guarantee that we do not loose any candidate functional RNAs at the filtration stage.

We will describe the construction of an approximate SREG for a \( \text{TAG}_{RNA} \) satisfying the above constraint in two steps:

1. Construct a regular grammar from a stochastic \( \text{TAG}_{RNA} \) without considering probabilistic parameters.
2. Determine the probability of each production rule of an SREG.

#### 3.2 Regular Approximation of a Stochastic \( \text{TAG}_{RNA} \)

The approximation method proposed by Weinberg and Ruzzo\(^{32}\) is essentially the same as that by Nederhof\(^{17}\). Although it is possible to apply CFG approximation methods by Harbusch or Poller\(^8\),\(^{19}\) to a given TAG and then apply Nederhof’s regular approximation to it, we will extendedly apply regular approximation technique similar to Nederhof’s\(^{17}\) directly to TAGs for its simplicity.
We introduce some technical terms before showing the method of constructing an approximate SREG for a given $TAG_{RNA}$. A node with nonterminal symbol $Z$ is said to be an initial node if there exists no $\gamma \in A$ such that $\gamma = [X \to Z*(a, b, c, d)]$ for some $a, b, c, d \in \Sigma \cup \{\circ\}$ and some $X \in N$ with $X \neq Z$. Let $\beta$ be an adjunct tree. By $UL(\beta)$ ($DL(\beta)$, $DR(\beta)$, $UR(\beta)$, respectively), we denote the substring of $Y(\beta)$ which is located at the upper-left ($UL$) (down-left ($DL$), down-right ($DR$), upper-right ($UR$), respectively) segment from the active node in $\beta$.

In order to construct a regular approximation of a $TAG_{RNA}$ we will convert an adjunct tree $\beta$ into four regular production rules each corresponding to one of segments $UL(\beta)$, $DL(\beta)$, $DR(\beta)$, $UR(\beta)$. For instance, consider a type-A adjunct tree denoted by $[X \to Y*(a, b, c, d)] (X, Y \in N, a, b, c, d \in \Sigma \cup \{\circ\})$. Production rules corresponding to $\beta$ are as follows:

$$
\begin{align*}
X_{UL} &\to aY_{UL}, \ Y_{DL} \to bX_{DL}, \\
X_{DR} &\to cY_{DR}, \ Y_{UR} \to dX_{UR}.
\end{align*}
$$

However, these production rules are independent and not connected to each other. We need to construct special production rules which connect these production rules. If the node denoted by $X$ is an initial node, we construct a special production rule which connects $DL$ and $DR$ segments: $X_{DL} \to X_{DR}$. In this paper, we call this special production rule a connecting production rule.

From the behavior of the adjoin operation, we know that $DL$ and $DR$ segments of an initially applied adjunct tree are located consecutively in the yield of a resultant tree. Thus, we only have to construct a special production rule which connects $DL$ and $DR$ segments according to the label of an initial node.

Second, we show how to construct production rules in the case of a type B adjunct tree. Let $\beta$ be a type B adjunct tree denoted by $[X \to Y*(Z*, \circ, \circ, \circ)] (X, Y, Z \in N)$. Production rules corresponding to $\beta$ are as follows:

$$
\begin{align*}
X_{UL} &\to Z_{UL}, \ Z_{UR} \to Y_{UL}, \\
Y_{DL} &\to X_{DL}, \ X_{DR} \to Y_{DR}, \\
Y_{UR} &\to X_{UR}.
\end{align*}
$$

Because the branch active node (in this case, the node denoted by $Z$) is always an initial node, we also construct a connecting production rule: $Z_{DL} \to Z_{DR}$.

In cases of $[X \to Y*(\circ, Z*, \circ, \circ)]$, $[X \to Y*(\circ, \circ, Z*, \circ)]$, $[X \to Y*(\circ, \circ, \circ, Z*)]$, production rules are introduced in a similar way.

Third, we show how to construct special production rules which connect $UL$ and $DL$ segments, $DR$ and $UR$ segments respectively. $UL$ and $DL$ segments, and $DR$ and $UR$ segments are separated at the active node which was introduced at the final step of the derivation. Thus, we only have to construct special production rules according to the label of such a terminating node $X$: $X_{UL} \to X_{DL}$, $X_{DR} \to X_{UR}$.

Also, we call these special production rules connecting production rules.

### 3.3 Constraints on Stochastic Parameters

In our RNA secondary structure analysis system, a 0th-order Hidden Markov model is used as a random model to be compared with a stochastic $TAG_{RNA}$ model, in order to judge whether a given RNA sequence can be a member of the family. Let $P(w \mid G_{rand})$ be the probability of an RNA subsequence $w$ generated by a random model $G_{rand}$. Let $P(w \mid G_{tag})$ be the probability of $w$ generated by a $TAG_{RNA} G_{tag}$.

We use a random model as threshold, i.e., $w$ belongs to an RNA family modeled by $G_{tag}$, if the following inequality Eq. (4) holds:

$$
P(w \mid G_{tag}) \geq P(w \mid G_{rand}).
$$

In order to construct a stochastic regular grammar $G_{reg}$ which approximates $G_{tag}$, it is important to assign probabilities so that Eq. (1) holds, since $P(w \mid G_{rand}) > P(w \mid G_{reg})$ and Eq. (1) imply $P(w \mid G_{rand}) > P(w \mid G_{tag})$, which means that the inequality $P(w \mid G_{rand}) > P(w \mid G_{reg})$ can be used at the filtration stage.

In this subsection, we use the logarithm of the probability instead of the probability itself. We will show how to construct probabilities of production rules satisfying Eq. (1). Consider a type-A adjunct tree $\beta$ denoted by $[X \to Y*(a, b, c, d)]$ and let $L_1$ be the logarithm of the probability of $\beta$. According to our algorithm, we obtain the production rules shown in Eq. (2) from $\beta$. Let $\ell_1$, $\ell_2$, $\ell_3$ and $\ell_4$ be the logarithm of probabilities of these production rules, respectively. In order to satisfy Eq. (1), it suffices to use $\ell_1$, $\ell_2$, $\ell_3$ and $\ell_4$ satisfying the following inequality constraint:

$$
\ell_1 + \ell_2 + \ell_3 + \ell_4 \geq L_1.
$$

The probability of connecting production rules is always assigned to be 1.

Next, we consider a type-B adjunct tree $\gamma$ denoted by $[X \to Y*(Z*, \circ, \circ, \circ)]$. Let $L_2$ be the
logarithm of the probability of $\gamma$. We obtain the production rules shown in Eq. (3) from $\gamma$. In a similar manner as in the case of type-A adjunct tree, the problem is reduced to the one to find $L_5$, $L_6$, $L_7$, $L_8$ and $L_9$ satisfying:

$$L_5 + L_6 + L_7 + L_8 + L_9 \geq L_2,$$

where $L_5$, $L_6$, $L_7$, $L_8$ and $L_9$ correspond to the logarithm of the probabilities of $X^{UL} \rightarrow Z^{UL}$, $Y^{UR} \rightarrow X^{UR}$, respectively. We construct a set of inequality constraints for all adjunct trees and denote it by $CN$.

### 3.4 Improving Stochastic Parameters

In this subsection, we will show a method to improve the filtering efficiency. The probabilistic parameters satisfying the set $CN$ of constraints always meet the filtration condition (Eq. (1)). However, we aim to remove candidates that cannot be a member of the family modeled by $G_{tag}$ as many as possible. Following the works by Weinberg and Ruzzo, we will propose a method for tuning probabilistic parameters based on nonlinear programming to meet such a requirement.

Parameter tuning method proposed in this subsection uses two grammar models, $G_{tag}$ and $G_{rand}$ below, in order to improve the approximation accuracy of an SREG obtained from $G_{tag}$ as in the previous subsection. Thus, it is different from Kullback-Leibler distance minimization approach\(^{18}\).

For $w \in L(G_{reg})$, let $Pr(w \mid G_{reg})$ be the probability of $w$ generated by $G_{reg}$. Let $\pi$ be a derivation path to generate $w$ by $G_{reg}$. Let $Pr(w \mid \pi)$ be the probability of generating $w$ based on the path $\pi$. Let $\Pi_w$ be the set of all paths for generating $w$. Then, we have:

$$Pr(w \mid G_{reg}) = \sum_{\pi \in \Pi_w} Pr(w \mid \pi).$$

We use $G_{rand}$ as a random model for whole genome sequences (0th order HMM based on a, g, c, u frequencies). We define the objective function $OB$ as follows:

$$OB = \sum_{w \in L(G_{reg})} Pr(w \mid G_{reg}) Pr(w \mid G_{rand}).$$

By minimizing $OB$ subject to the set $CN$ of constraints, the number of candidate RNAs might be reduced.

The structure of the grammar $G_{reg}$ can be explicitly represented by converting it to a corresponding automaton $M$ such that $L(G_{reg}) = L(M)$, where each production rule $S_i \rightarrow xS_j$ is transformed into a transition from $S_i \rightarrow S_j$ with a label $x$. The approximate regular grammar $G_{reg}$ has a simple linear structure if we neglect self-loops of the form $S_i \rightarrow xS_i$.

For easy computation of $OB$, we will restrict $\Pi_w$ to the set of paths $\pi_w$ such that each self-loop is contained at most once in $\pi_w$.

Then, we can efficiently compute $OB$ with a dynamic programming method, as follows:

$$OB(S_i) = OB(S_{i-1}) \times \left( \sum_{S_{i-1} \rightarrow xS_i \in R} P(S_{i-1} \rightarrow xS_i) Pr_{rand}(x) \right) \times \left( 1 + \sum_{S_i \rightarrow xS_{i} \in R} P(S_i \rightarrow xS_i) Pr_{rand}(x) \right),$$

where $R$ is the set of production rules of $G_{reg}$, and $Pr_{rand}(x)$ is the probability of the occurrence of base $x$ in the genome sequence. $OB(S_i)$ is the value of $OB$ corresponding to the set of paths from an initial state to the state $S_i$. Thus, $OB = OB(S_{end})$ holds for end state $S_{end}$ of $G_{reg}$. The time complexity for building $OB$ is $O(n)$. In order to solve this minimization problem, we use a nonlinear optimization solver CFSQP\(^{14}\). Note that the probabilities $P(S_{i-1} \rightarrow xS_i)$’s are variables in this optimization problem. In conclusion, we can improve stochastic parameters by solving the following minimization problem:

**Minimize:**

$$OB = \sum_{w \in L(G_{reg})} Pr(w \mid G_{reg}) Pr(w \mid G_{rand}).$$

**Subject to:**

set of constraints in $CN$.

### 4. Experimental Results

The purpose of this paper is to show the effectiveness of the proposed approximation method for annotating RNA family in genome sequences. In this section, we will show some experimental results on how such approximation grammars can be used to reduce drastically the candidate positions of the RNA families.

The procedure of each experiment is given as follows:

1. Get alignment data from the Rfam database\(^ {26}\).

2. Generate a $TAG_{RNA}$ $G_1$ that models RNA family by the system developed by Takakura\(^ {28}\).

3. Generate an SREG $G_2$ approximating $G_1$ by the proposed method, whose time
Table 1 Experimental Results: The column Name is an RNA family name. The column ID is accession number in Rfam database. The value Length is the length of genome sequence. The column Start shows the position where the RNA family appears. The column cp gives the frequency of candidate positions. The value ratio is defined as cp/Length, i.e., filtering efficiency. Each column step \( i \) \((i = 2, 3, 4, 5)\) shows the execution time of each step. Est. \( \text{TAG}_{RNA} \) column shows rough estimations of the execution time by a \( \text{TAG}_{RNA} \). In each ID, the upper row is the result of optimized SREG, the lower is the result without optimization.

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<th>End</th>
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<th>ratio</th>
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<th>Step 3</th>
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<th>Step 5</th>
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<tr>
<td>X79047</td>
<td>7501</td>
<td>7303</td>
<td>7423</td>
<td>68</td>
<td>0.88%</td>
<td>0.23s</td>
<td>2m69s</td>
<td>500m65s</td>
<td>1230days</td>
<td></td>
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<tr>
<td>HDV_rybozyme</td>
<td>AF309420</td>
<td>1876</td>
<td>855</td>
<td>773</td>
<td>38</td>
<td>2.3%</td>
<td>0.14s</td>
<td>0.19s</td>
<td>18.9s</td>
<td>44m19s</td>
<td>30days</td>
</tr>
<tr>
<td>M1075</td>
<td>1882</td>
<td>886</td>
<td>774</td>
<td>41</td>
<td>2.4%</td>
<td>0.18s</td>
<td>16.5s</td>
<td>44m35s</td>
<td>307days</td>
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<tr>
<td>M21012</td>
<td>1879</td>
<td>884</td>
<td>772</td>
<td>6</td>
<td>2.6%</td>
<td>0.17s</td>
<td>10.1s</td>
<td>43m54s</td>
<td>307days</td>
<td></td>
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<tr>
<td>Tombus_3IV</td>
<td>AJ067422</td>
<td>4744</td>
<td>4852</td>
<td>4744</td>
<td>44</td>
<td>0.92%</td>
<td>0.14s</td>
<td>0.14s</td>
<td>10.9s</td>
<td>120m26s</td>
<td>1083days</td>
</tr>
<tr>
<td>M29270</td>
<td>4701</td>
<td>4813</td>
<td>4701</td>
<td>43</td>
<td>0.91%</td>
<td>0.18s</td>
<td>9.8s</td>
<td>125m38s</td>
<td>99days</td>
<td></td>
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<td>X51456</td>
<td>2200</td>
<td>2110</td>
<td>2200</td>
<td>2125</td>
<td>0.96%</td>
<td>0.17s</td>
<td>11.3s</td>
<td>58m8s</td>
<td>486days</td>
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<tr>
<td>Tyumo_RNA-like</td>
<td>AJ069633</td>
<td>1379</td>
<td>1297</td>
<td>1379</td>
<td>52</td>
<td>2.3%</td>
<td>0.12s</td>
<td>0.19s</td>
<td>15.3s</td>
<td>22m46s</td>
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<tr>
<td>M30763</td>
<td>2962</td>
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<td>31</td>
<td>5.0%</td>
<td>0.14s</td>
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<td>D07776</td>
<td>1255</td>
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<td>35</td>
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<td>0.17s</td>
<td>12.5s</td>
<td>32m31s</td>
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<td></td>
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<td></td>
<td>1097</td>
<td>866</td>
<td></td>
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<td></td>
<td>12m39s</td>
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</table>

Complexity for building an SREG is \( O(n) \), where \( n \) is the size of \( G_1 \).

(4) Tune probabilistic parameters of \( G_2 \) by using nonlinear programming method, for which we use a solver CFSQP.

(5) Filter RNA sequences by using the SREG \( G_2 \), whose time complexity is \( O(lm^2) \), where \( l \) is the length of total genome sequence, and \( m \) is the size of \( G_2 \).

(6) Check how much portion of candidate positions are remained.

The purpose of these experiments is to show efficiency and accuracy of our method, not to find new members of target RNA families. We used six RNA families including pseudoknots for this experiment: Corona_FSE, Corona_pk3, Entero_orI, HDV_rybozyme, Tombus_3IV, Tyumo_RNA-like. Table 1 shows RNA data and execution time statistics of these experiments.

In order to emphasize on the importance of tuning stochastic parameters, and show the effectiveness of the method proposed in Subsection 3.4, for a given stochastic TAG, we generated two approximation grammars, one of which has stochastic parameters after optimization, and the other has parameters without optimization*. The experimental results are summarized in Table 1.

* Non optimized parameters are obtained by finding feasible solutions (satisfying \( CN \)) using the software CFSQP.
The experiments show that SREGs with optimized parameters performs much better than those with non-optimized parameters, which shows the effectiveness of the proposed method in Subsection 3.4. The filtering ratios with optimized SREGs show that the proposed method can have potential ability to drastically reduce the total time for annotating ncRNA families.

5. Related Works

There are so many algorithms proposed for predicting RNA secondary structures including pseudoknots for a given linear RNA sequence by Akutsu\(^1\), Rivas\(^{22}\), Reeder\(^{20}\), Ren\(^{21}\), Ruan\(^{23}\), Uemura\(^{31}\), etc. Although these standard prediction algorithms provide the basis for the analysis of given RNA sequences, they can not be directly used to find the location of a target RNA family in a complete genome sequence, which is the coverage of this paper.

Grammatical models are quite useful tools for improving the accuracy of predicting an RNA family location\(^{7,11,13,22,24,28,31}\). This paper proposed a grammatical technique for speeding up such analysis by incorporating the approximation theory of formal grammars. In this sense, in order to improve the accuracy of the analysis, it is important to obtain statistical information of a target RNA family. Thus, we need a reliable alignment data of RNA secondary structures. Recent advances in alignment algorithms of RNA secondary structures including pseudoknots will provide such a reli-

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**Fig. 7** Corona\(_{\text{FSE}}\).

**Fig. 8** Corona\(_{\text{pk3}}\).

**Fig. 9** Entero\(_{\text{OriR}}\).

**Fig. 10** HDV\(_{\text{ribozyme}}\).

**Fig. 11** Tombus\(_{\text{3IV}}\).

**Fig. 12** Tymo\(_{\text{RNA-like}}\).
able alignment data \(^6,9,15,27\). The most related is by Weinberg and Ruzzo, who proposed a method of approximating stochastic context free grammar by stochastic regular grammar and applied it to faster genome annotation of non-coding RNA families \(^{32,33}\). Although CM method is quite effective for modeling and predicting RNA families, it cannot model pseudoknotted structures. This work extends Weinberg and Ruzzo’s approach to the case of stochastic TAGs by which we can model pseudoknots.

There are several grammar models proposed other than TAGs for modeling RNA secondary structures including pseudoknots, RNA Pseudoknot Grammars \(^{22}\), Multiple Context Free Grammars \(^{11,12}\), CFG based Parallel Communicating Grammars \(^3\), etc. As far as the authors’ knowledge, the current work is the first attempt of applying approximation method to the grammars capable of modeling pseudoknots. The experimental results of this work suggest that similar approximation method might be quite effective also for the grammars other than TAGs.

Weinberg and Ruzzo’s method has two steps. In the first step, they apply regular approximation method to stochastic TAGs. Second, they improve stochastic parameters. The approximation method in the first step proposed by them is almost equivalent to Nederhof’s method \(^{17}\). Nederhof proposed approximation of CFG by REG based on Recursive Transition Network. Furthermore, we know that there are some works on approximating TAGs by CFGs \(^8,19\). Based on these works and Nederhof’s works, we can approximate a given TAG by a CFG, and then approximate the obtained CFG by a REG. But, in the current paper, we apply regular approximation technique similar to Nederhof’s method directly to TAGs for its simplicity.

In the second step, we improve stochastic parameters. The tuning method in the second step is different from Nederhof’s method \(^{18}\). Nederhof proposed a method for training finite automaton on SCFG so that Kullback-Leibler distance between them could be minimal. But, we do not use Kullback-Leibler distance for tuning probability parameters, since its direct application might lose some candidate locations of an RNA family which can be found by \(\text{TAG}_{RNA}\). We use two grammar models \(G_{\text{reg}}\) and \(G_{\text{rand}}\) and apply nonlinear programming method in order to optimize parameters of \(G_{\text{reg}}\) with the guarantee that we do not loosen such locations found by \(\text{TAG}_{RNA}\). It might be a theoretically interesting future research topic to extend Kullback-Leibler distance minimization method so that it might not lose candidate locations which can be found by the target grammar.

6. Conclusions and Future Works

Inspired from the work by Weinberg and Ruzzo \(^{32,33}\), we developed a method for approximating a given stochastic tree adjoining grammar to stochastic regular grammar and applied it to faster genome annotation of ncRNA families. Parameter tuning based on an optimization technique is applied in order to improve the filtering efficiency. Preliminary experimental results were reported and the effectiveness of the proposed method was verified by these experiments.

Although we succeeded in filtering out non candidate positions of RNA families effectively, the parsing time efficiency of TAG itself is still computationally heavy for the practical use. In actuality, such parsing time inefficiency is currently a big research obstacle against the attempt of applying these mildly context sensitive grammars to RNA structure modeling. But, after the experience of these experiments, the authors believe that this kind of grammar approximation method would be quite effective also for the efficient parsing itself. We are now developing such an approximation method for stochastic TAG parsing.

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