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Natural Image Matting with Membership Propagation

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We present a semi-supervised technique of object extraction for natural image matting. At first, we present a novel unsupervised graph-spectral algorithm for extraction of homogeneous regions in an image. We next derive a semi-supervised scheme from this unsupervised algorithm. In our method, it is sufficient for users to draw strokes only in one of object and background regions. The semi-supervised optimization problem is solved with an iterative method where memberships are propagated from strokes to their surroundings. We suggest a guideline for placement of strokes by exploiting the same iterative solution process in the unsupervised algorithm. We project the color vectors with the linear discriminant analysis to improve the color discriminability and speed up the convergence of the iterative method. Performance of the proposed method is examined for some images and the results are compared with other methods and ground truth mattes.

1. Introduction

Many techniques have recently been developed for natural image matting where objects are extracted from photographs with natural backgrounds. These matting methods can be used more easily than the standard technique called the chroma-key matting which needs a specialized studio with a blue or green background wall. Natural image matting techniques require no such a special room and are applicable to arbitrarily photographed pictures.

Although the natural image matting techniques can be widely applicable to various images, early methods for it were practically hard to be used because they require users to draw supplementary images called trimaps such as shown in Fig. 1 (a) of which drawing is laborious and time-consuming. So, recently developed methods have become to accept only rough strokes as is illustrated in Fig. 1 (b) instead of complex trimaps. These strokes, also called scribbles, can be drawn easily and quickly by even unskilled users. They are, however, required to be drawn in both of the foreground region and the background area as the same as the trimaps.

We present, in this paper, a similar easily usable matting method by using a technique of semi-supervised extraction of objects. In our method, it is sufficient that strokes are drawn only in either objects or backgrounds. This is owing to that our semi-supervised method is derived from a novel form of unsupervised graph-spectral algorithm of cluster extraction type in contrast to those of cluster partitioning type adopted in almost all matting methods.

Although the basic unsupervised scheme cannot extract a specific object of a single homogeneous region in an image, it can detect inhomogeneous color regions in the image. This detection of inhomogeneous color regions is useful for guiding the placement of strokes in the semi-supervised method derived from the unsupervised algorithm.

In matting methods based on rough strokes such as in Fig.1 (b), places of strokes are crucial for the performance of object extraction. Levin, et al. have recently presented a guiding scheme for placement of strokes in their matting method where strokes are required to be drawn in both of foregrounds and backgrounds as is in other existing methods. Their theoretical analysis of the guiding scheme is excellent, however, their scheme requires for users to examine two eigen-images and the guidance is not so easy to execute practically.

In this paper, we present a similar guiding scheme for our matting method. Our scheme exploits an iterative solution process in an unsupervised object extraction
method. Our guiding scheme utilizes only the first eigen-image and so is simpler
and faster than Levin’s scheme.

2. Unsupervised Extraction of Homogeneous Color Regions

The main scheme proposed in this paper is a method for semi-supervised object
extraction from an image. This semi-supervised matting method is derived from a
novel graph-spectral algorithm for unsupervised cluster extraction from an image.
The distinct point in our method is that our algorithm is a cluster extraction type
in contrast to the cluster partitioning type algorithms\textsuperscript{13},\textsuperscript{14} used in almost all
matting methods. We use here a term clusters as regions in an image with
homogeneous colors.

2.1 Unsupervised Cluster Extraction

Let the similarity between pixels $i$ and $j$ be $s_{ij} = e^{-\|a_i - a_j\|^2/\sigma_a^2 - \|c_i - c_j\|^2/\sigma_c^2}$
where $a_i$ is the spatial position of pixel $i$ and $c_i = [R_i, G_i, B_i]^T$ is its color. We
set $s_{ii} = 0$ as is common in graph spectral methods. We set the free parameters
$\sigma_a$ and $\sigma_c$ also in a popular way of graph spectral methods as $\sigma_a = \bar{D}_a$ where
$\bar{D}_a$ is the average of spatial distances between pixels in the window around
the pixel $i$ and $\sigma_c = \bar{D}_c$ where $\bar{D}_c$ is the average of color distances from every color
of pixels in the strokes to its fifth nearest neighbor colors.

Connected components of pixels linked with large $s_{ij}$ are homogeneous color
regions, i.e. clusters. An image generally includes multiple clusters which corre-
spond to objects or sub-regions in an object. The fraction $x_i$ of pixel $i$ belonging
to such clusters can be evaluated with

$$
\begin{align*}
\max & \quad \sum_i \sum_{j \in W_i} s_{ij} x_i x_j \\
\text{subj.to} & \quad \sum_i d_i x_i^2 = 1
\end{align*}
$$

(1)

where $d_i = \max\left\{ \sum_j s_{ij}, \epsilon \right\}$ and $W_i$ is a $(2p + 1) \times (2p + 1)$ square window around
pixel $i$. This equation expresses an unsupervised extraction of a cluster from an
image. If two pixels $i$ and $j$ are connected with large $s_{ij}$, then their $x_i$ and $x_j$
become large simultaneously, i.e. they are extracted as the members in a cluster
which is hence composed of a connected component of such tightly linked pixels.

The novelty in Eq. (1) lies in $d_i = \max\left\{ \sum_j s_{ij}, \epsilon \right\}$ which differs from the basic

graph spectral algorithm of cluster extraction type\textsuperscript{12} where $d_i = 1$ and also
differs from the spectral clustering of cluster partitioning type as is shown in
the next subsection. If we set $\epsilon$ sufficiently large, then $d_i = 1$ and Eq. (1) reduces to
the basic graph spectral algorithm\textsuperscript{12} because 1 in the constraint $\sum_i d_i x_i^2 = 1$
in Eq. (1) has no more means than a constant, i.e., $\sum_i d_i x_i^2 = 1$ in Eq. (1) can be
written as $\sum_i d_i x_i^2 = \text{const.}$ where const. is an arbitrary constant.

The basic graph spectral algorithm with $d_i = 1$\textsuperscript{12}, however, can extract only
spherical clusters. In contrast, its extended form in Eq. (1) can extract arbitrarily-
shaped clusters as is also mentioned in the next subsection.

The solution of Eq. (1) is a stationary point of its Lagrange function:

$$
\begin{align*}
\max & \quad \lambda \sum_i \sum_{j \in W_i} s_{ij} x_i x_j - \lambda \left( \sum_i d_i x_i^2 - 1 \right) \\
\text{subj.to} & \quad \sum_i d_i x_i^2 = \text{const.}
\end{align*}
$$

(2)

Differentiating this function with $x = [x_1, \ldots, x_m]^T$ and set the derivative to
zero, we get the solution of Eq. (2) as a generalized eigenvector of $Sx = \lambda Dx$
where $S = [s_{ij}]$ is the similarity matrix and $D = \text{diag}(d_1, \ldots, d_m)$.

Thus the solution of Eq. (1) is the first eigenvector of the normalized similarity
matrix $D^{-1} S$. Every element in the first eigenvector is nonnegative, hence it can
be used for the membership. If $\epsilon = 0$, then the first, i.e. maximal, eigenvalue is
1. When $\epsilon > 0$, the maximal eigenvalue becomes smaller than 1, however still
nearly equal to 1.

A simple scheme for computing $x$ is the power method:

$$
x_i^{(\xi + 1)} = \frac{\sum_j \tilde{s}_{ij} x_j^{(\xi)}}{\sqrt{\sum_k \tilde{s}_{ik} \left( \sum_j \tilde{s}_{kj} x_j^{(\xi)} \right)^2}}
$$

(3)

where $\tilde{s}_{ij} = s_{ij}/d_i$ and $\xi$ is an iteration counter.

Note that $x_i$ denotes the fraction of datum $i$ belonging to the cluster and is
not the membership itself which must satisfies $\max\{x_i\} = 1$. Therefore, after the
convergence of the iteration of Eq. (3), we normalize $x_i$ as $\tilde{x}_i = x_i/\max_k \{x_k\}$
which is then the membership of pixel $i$ in the cluster. If we incorporate this
normalization into Eq. (1), they are collectively expressed as
max \( \sum_{i} \sum_{j \in W_i} s_{ij} x_i x_j \)
subj.to \( \sum_i d_i x_i^2 = \phi \), \( \max \{x_i\} = 1 \) \( (4) \)

where \( x_i \) takes the values between 0 and 1 in contrast to Eq. (1) where \( x_i \) takes small values \( 0 \leq x_i \ll 1 \).

2.2 Relationship with Spectral Clustering
The above scheme is a graph-spectral method of cluster extraction type. Another type is the spectral clustering of cluster partitioning type. The spectral clustering is based on the Laplacian eigenmap expressed by

\[
\min \sum_{i} \sum_{j \in W_i} s_{ij} (x_i - x_j)^2
\]
subj.to \( \sum_i f_i x_i^2 = 1 \) \( (5) \)

where \( f_i = \sum_{j \in W_i} s_{ij} \). Equation (5) is rewritten as

\[
\max \sum_{i} \sum_{j \in W_i} s_{ij} x_i x_j
\]
subj.to \( \sum_i f_i x_i^2 = 1 \) \( (6) \)

which has the same form as Eq. (1) with the only difference that \( d_i = \max \{\sum_j s_{ij}, \epsilon\} \) in Eq. (1) while \( f_i = \sum_j s_{ij} \) in Eq. (6).

The solution of Eq. (6) is the first eigenvector of \( F^{-1} S \) where \( F = \text{diag}(f_1, \ldots, f_m) \) of which eigenvalue is 1. This eigenvector coincides with the solution of Eq. (5) which is the last eigenvector of the normalized Laplacian matrix with the minimal eigenvalue 0. In the spectral clustering such as the normalized cuts, this principal eigenvector is discarded and the second eigenvector is used for the clustering. As is well known, the spectral clustering algorithm can deal with arbitrarily-shaped clusters.

This is the essential difference between our method and the spectral clustering. In our method, the first eigenvector is used as the membership, while the spectral clustering uses the second eigenvector of which some elements are positive and some are negative, hence it cannot be used for the memberships.

Equation (1) reduces to the spectral clustering when \( \epsilon = 0 \) while becomes the basic graph spectral method for cluster extraction if \( \epsilon \) is sufficiently large. Mixing of these two extreme cases leads to Eq. (1) which is a new graph spectral method of cluster extraction type and is able to extract arbitrarily-shaped clusters.

Thus the introduction of \( \epsilon > 0 \) in Eq. (1) is crucial in our method for utilizing its solution as the memberships and extracting clusters of arbitrary shapes. The value of \( \epsilon \) acts as a threshold for the degree \( \sum_j s_{ij} \) of graph node \( i \). If the degree is greater than \( \epsilon \), node \( i \) is included in a homogeneous region. For most images we experimented, its adequate value is \( 1 \leq \epsilon \leq 10 \) hence we set it manually in this range, usually \( \epsilon = 5 \).

2.3 Experimental Example

For instance, Fig. 2(b) illustrates memberships for an image in Fig. 2(a) extracted with the above unsupervised method. We set \( p = 5 \) and set the parameters \( \sigma_a \) and \( \sigma_c \) to the average distances between data as mentioned above. Their values are \( \sigma_a = 10, \sigma_c = 10 \). We set \( \epsilon \) heuristically to the value for adequately dissecting homogeneous regions as \( \epsilon = 5 \). In Fig. 2(a), homogeneous color regions exist in background areas in addition to the object (white peacock). Region boundaries are vague because the membership is extremely fuzzy.

As is seen in this result, this unsupervised method cannot delineate a specific object, e.g. the white peacock. So, this unsupervised scheme is generally insufficient for object extraction. Exceptional cases are images satisfying both the following two conditions:

(1) Objects are composed of only inhomogeneous color regions.
(2) Background colors are homogeneous.

We can extract objects from such exceptional images by using the above un-
supervised method. An image of spider webs in Fig. 3 (a) is such an example of easily extractable objects. Its memberships are shown in Fig. 3 (b) of which negative is the membership of objects since plotted $\tilde{x}_i$ in Fig. 3 (b) is the membership in homogeneous color regions which are backgrounds in this case.

3. Semi-supervised Extraction of Objects

We next derive a semi-supervised cluster extraction technique from the above unsupervised method. In our semi-supervised matting method, it is sufficient for strokes to be drawn in only one of the object and the background. If we draw strokes in the object, then the object is extracted directly. Conversely if we draw strokes in the background areas, then the background is extracted, from which we can get the object as the complement of the background. Users can freely select regions in which he/she draws strokes simpler and easier. This is favorable for some images, for instance, including thin line objects such as the web in Fig. 3.

3.1 Semi-supervised Cluster Extraction

Let $T$ be an area, that is a subset of pixels, of strokes drawn by a user. We fix the value of $x_i$ to 1 at pixels $i \in T$. Then the normalization constraint in Eq. (1) or in Eq. (4) becomes unnecessary and the Lagrange multiplier $\lambda$ in Eq. (2) can be fixed to an arbitrary value. The most appropriate setting is $\lambda = 1$ because the maximal eigenvalue $\lambda$ in Eq. (2) corresponding to the solution of Eq. (1) is 1 as was mentioned in Section 2.1.

If we fix $\lambda$ in Eq. (2) to 1, then Eq. (2) becomes

$$\max_x \sum_{i \in T} \sum_{j \in W_i} s_{ij} x_i x_j - \sum_{i \in \bar{T}} d_x x_i^2$$

We solve this equation also with the iteration:

$$x_i^{(\xi+1)} = \sum_{j \in W_i} \tilde{s}_{ij} x_j^{(\xi)} \quad (i \notin T) \quad (8)$$

As this iteration proceeds, the membership $x_i$ propagates from stroke area $T$ to its surroundings. We call this process the membership propagation which is the main routine in our matting method.

Note that $x_i$ in Eq. (7) takes the values between 0 and 1, i.e. this $x_i$ is the membership itself, as the same as in Eq. (4).

3.2 Relationship with Label Propagation

This membership propagation resembles the label propagation popularly used for semi-supervised learning for pattern recognition \(^{15} ,^{16}\). The label propagation is derived from the Laplacian eigenmap in Eq. (5). Since the minimal eigenvalue of the Laplasian is 0 as was mentioned in Section 2.2, the semi-supervised form of Eq. (5) becomes

$$\min \sum_i \sum_{j \in W_i} s_{ij} (x_i - x_j)^2$$

which is the equation for the label propagation which is equivalent to a random walk \(^{16}\) used for the image matting \(^{6}\).

Thus the label propagation or random walk is a semi-supervised form of the spectral clustering of cluster partitioning type such as the normalized cuts. The label function $x_i$ is harmonic \(^{16}\). The regression of harmonic functions needs labels to be given in both of the foreground and the background. On the other hand, our membership propagation given by Eq. (7) is derived from the graph spectral method of cluster extraction type given by Eq. (1). The membership function is not harmonic and the membership propagation corresponds to a lazy random walk. Our method allows labels to be given only at one of the foreground and the background regions.

3.3 Projection of Color Vector

In the random walk matting \(^{6}\), color vectors are projected with the locality preserving projections (LPP) which is an unsupervised dimensionality reduction method, hence neglects the supervisory information given by user’s strokes. We utilize that information here with the supervised dimensionality reduction method. We adopt the linear discriminant analysis (LDA) with which we project colors to 1-dimensional subspace. The procedure is summarized as follows:

$$x_i^{(\xi+1)} = \sum_{j \in W_i} \tilde{s}_{ij} x_j^{(\xi)} \quad (i \notin T) \quad (8)$$
(1) Dilate strokes to surrounding areas with similar colors.
(2) Set a dilated region as one class and the remaining area as the other class and execute the standard LDA for two classes to compute a projection vector $q$. By using the obtained $q$, project the color of every pixel to $f_i = q^T c_i$.
(3) By using $f_i$, construct the pixel similarity $s_{ij} = e^{-\|a_i - a_j\|^2/\sigma_i^2 - (f_i - f_j)^2/\sigma_f^2}$ and compute $x_i$ by using Eq. (8).

The dilation in the first step is a popular morphological filtering technique. By this dilation, the strokes are broadened to get the area $T$ as wide as possible. This broadening of strokes increases the stability and reliability of the LDA. Before projecting the colors with LDA, we transform the color space from RGB to more uniform CIELAB space.

The projection vector $q$ in the second step is calculated as follows:
1) We transform the color space from RGB to CIELAB as $c_i = [l_i, a_i, b_i]^T$.
2) We next calculate their averages $\bar{c}_1$ and $\bar{c}_2$ in each class, and centralize $c_i$ in each class as $\tilde{c}_{i1} = c_i - \bar{c}_1$ and $\tilde{c}_{i2} = c_i - \bar{c}_2$.
3) Covariance matrices $\bar{A}_1$ and $\bar{A}_2$ of colors for each class are given by averaging the covariance matrices at each pixel: $\bar{A}_{ij} = \bar{c}_{ij}^T \bar{c}_{ij}$ and $\bar{A}_{2i} = \bar{c}_{2i}^T \bar{c}_{2i}$. We finally average these two covariance matrices as $\tilde{A} = (\bar{n}_1 \bar{A}_1 + \bar{n}_2 \bar{A}_2) / (\bar{n}_1 + \bar{n}_2)$ where $\bar{n}_1$ and $\bar{n}_2$ are the numbers of pixels in each class.
4) The projection vector $q$ is given by $q = A^{-1}p$ where $p = \bar{c}_1 - \bar{c}_2$. This concludes the LDA procedure.

Projection of each color $c_i$ by the vector $q$ obtained with this LDA increases the discriminability between two classes, i.e. the object and the background. Therefore the similarity $s_{ij}$ in the above third step is more suitable for the object extraction than the original $s_{ij}$ computed directly from the pixel colors $c_i$.

If we use the original color space $c_i$, then we get an unsatisfactory matte shown in Fig. 4 (b) for the image in Fig. 2 (a) even if we draw a long stroke in the background as is shown in Fig. 4 (a). The projection of the color space with the LDA is effective for our method. Similar effectiveness of color space projection has also been reported for the random walk matting which uses the LPP for the projection.

Another role of the LDA in our method is acceleration of the convergence of Eq. (8). Since our method is iterative, setting of initial values is crucial for the computational time needed for the convergence of $x_i$. We use the above LDA also for this setting of initial values of $x_i$ as $x_i^{(0)} = 1/(1 + e^{-\gamma (f_i - \delta)})$ which is a rough estimation of the membership $x_i$ from the projected value $f_i$. This sigmoidal function enhances the separation between two regions and serves for fast convergence of the iteration. Additionally, in order to speed up the computation of each step of the iteration of Eq. (8), we use a fast algorithm based on an approximate spatial decomposition of $s_{ij}$.

Furthermore in order to save the computation, we stop the iteration at pixels where $x_i$ becomes greater than 0.99 or below 0.01, and the iteration is continued only at the remaining pixels.

4. Placement of Strokes

Through the above iteration, the membership propagates easily within homogeneous color regions whereas they are hardly infiltrated into inhomogeneous areas. Therefore, initial strokes must be drawn on each inhomogeneous region in addition to homogeneous ones within a prescribed area (either object or background) to be extracted.

4.1 Guideline for Drawing Strokes

Since inhomogeneous segments are usually narrow, it is easy to draw strokes as bridging over or touching into them. Thus the guideline is stated as: Choose one of the object and the background regions and draw strokes in the inhomogeneous areas in the selected region in the way that they touch a homogeneous area.

A little amount of strokes are sufficient owing to the effective propagation of memberships as described above. Especially in Eq. (8), rough and sparse strokes are sufficient owing to high propagation capability of memberships due to broad windows $W_i$ in our method. This broad window is another new point utilized in our method.
Note that the strokes should not be drawn in boundaries between objects and backgrounds. If the strokes are drawn there, then the membership propagates into both of the object and the background areas and we fail to extract the object.

4.2 Detection of Inhomogeneous Area

This guideline for placement of strokes requires detection of inhomogeneous regions in an image. Fortunately, as was shown in Section 2.3, the unsupervised algorithm in Section 2.1 can be utilized for this purpose. However its full convergence needs many iteration steps and obtained eigen-images are too fuzzy to visually detect inhomogeneous regions. For instance, convergence to Fig. 2(b) requires 300 iterations of Eq. (3). In order to save computational time, we set \( \varepsilon \) in \( d_i \) excessively large for inhomogeneous regions to be extracted sufficiently and quickly, this fast convergence enables us to stop the iteration of Eq. (3) early before its full convergence.

Figure 5 illustrates 10-th iterant \( x_i^{(10)} \) for Fig. 2(a) binarized as \( x_i = 1 \) if \( x_i^{(10)} > 0.001 \) else \( x_i = 0 \). Black areas in Fig. 5 are inhomogeneous regions.

4.3 Drawing of Strokes

The procedure for drawing strokes is summarized as:

1) We execute the unsupervised algorithm and display the binarized 10-th iterant \( x_i^{(10)} \).
2) We choose one of the object and the background regions where strokes can be drawn easily.
3) We draw some strokes in every major black area in the selected region in the way that they touch a white area.

An example of strokes placed in the background area following this procedure is shown in Fig. 6(a) from which the membership propagates into the whole background area. Obtained membership of the object (peacock) are illustrated in Fig. 6(b) which is better than Fig. 4(b) even though the total area of strokes is lesser in Fig. 6(a) than Fig. 4(a).

Other examples of strokes are shown in the left column in Fig. 7 where the right column shows binarized \( x_i^{10} \). Figures 7(c) and (e) were experimented with other existing methods \( {3,4} \). The number of pixels marked in Figs. 7(c) and (e) is smaller than other methods \( {3,4} \).

Memberships obtained with the strokes shown in Fig. 7 are illustrated in Fig. 8 where the left column shows initial values and the right are converged ones. Thus we can get mattes by using our method with strokes shorter and lesser than other methods where they must be drawn in both objects and backgrounds.
5. Experiments

Mattes extracted with our method are compared with those obtained with the method of Wang, et al. \(^3\) and of Levin, et al. \(^4\) for the images shown in Fig. 9 where the upper left is the image of a girl in Fig. 7 (c) and the upper right is the image of fire flames in Fig. 7 (e), lower three images are new ones. Extracted mattes are shown in Fig. 10 where the left are the mattes obtained with our method, the middle are those of Wang, et al. \(^3\) and the right are those of Levin, et al. \(^4\). The results for the first example of a girl are nearly the same for all methods. For the second image of fire flames, the method of Levin, et al. fails to extract the flame at the upper left side. For the third image of rabbit doll, the method of Wang, et al. and that of Levin, et al. extract extra portions at the middle left in the background. Finally the wheel and the ball in the last two images are extracted crisply with our method. This tendency of crispness of the matte which is often observed in our method, however, deteriorates transparent mattes or gradual boundaries as is revealed below.

Finally, mattes obtained with our method are shown in Fig. 11 for all the im-
Fig. 11 Benchmark images.

ages in the homepage “http://juew.org/data/data.htm” of Jue Wang. In Fig. 11, the original color images are shown in the left column, the mattes obtained with our method are shown in the middle column and the ground truth mattes given in the homepage are shown in the right column. Our method is inclined to blur fine structures such as thin hair lines and discretize the transparency or gradual change of mattes at hair peripheries or see-through objects. The performance of our method also deteriorates if close colors exist in both objects and backgrounds. The last three images in Fig. 11 are such examples which are hard for our method to extract objects in them. We interactively increase strokes in these images.

6. Conclusion

We have presented a semi-supervised image matting method and a guiding scheme for the placement of strokes for the method. Some features of our method are summarized as:

1. Membership propagation over holes or gaps owing to broad windows.
2. Strokes are sufficient to be drawn in either object areas or backgrounds.
3. Facilitation of object extraction by projection of colors with the LDA.
4. Effective initial values for membership propagation.
5. Simple guidance for placement of strokes.

Although the second feature of sufficiency for strokes drawn only in one of objects and background is a merit in our method, it deteriorates the accuracy of the mattes near transparent boundaries. Especially our method is hard to extract objects including colors close to those in backgrounds. Refinement and improvement for coping with these difficulties in our method are future subjects.

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