Industrial Policy and Firm's R&D Choice under Process and Product R&D

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Abstract

This study clarifies how governments' industrial policies affect the firm's R&D choice when firms simultaneously conduct both cost-reducing process and quality-improving product R&D. We found the following results. Under Cournot competition, while output increases when quality improves and/or the production cost decreases, output decreases as the proportion of product R&D becomes higher compared to that of process R&D. Under Bertrand competition, whether prices become higher or lower depends on the degree of fraction of investment in two types of R&D. If firms only conduct one of the two kinds of R&D, this effect does not exist. A government always subsidizes its domestic firm's R&D investments.

Keywords: Product R&D, Process R&D, R&D Policy

1 Introduction

The purpose of this study is to explore the impacts of governments' industrial policies (subsidy or tax) on firms' research and development (hereafter R&D) choices when firms conduct both cost-reducing process and quality-improving product R&D and also of particular concern herein are to clarify the channels through which government policy affects domestic firm's R&D investments as well as those of rival firms. To achieve this object, we especially focus on the R&D investment fraction into process and product R&D, that is, the portfolio of the two types of R&D and how governments' policies affect its domestic and rival firms' total R&D investments. We show that whether the government subsidy increases the proportion of product R&D depends on the costs of two types of R&D. This result coincides with the empirical results. Leibowicz (2018, p.397), for instance, showed that the firm's product R&D effort depends on the cost of R&D. Furthermore, we demonstrate that the policy does not affect the rival firms' R&D fraction, while it strategically affects the rival firm's total R&D investment.

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In almost all the industries such as automobile, transportation equipment, chemical products and electrical products, etc, firms ordinary conduct both cost-reducing process R&D and quality-improving product R&D simultaneously.¹⁾ For example, automobile companies not only invest into product R&D on a constant bases but also at the same time embark process R&D.²⁾ Associated with this firm's R&D activities, it is well known that governments in many countries encourage their domestic firms' R&D performance by several means of industrial polices including direct subsidy/tax policies or indirect supports such as tax incentives.³⁾ These policies play a role to reduce the production costs, raise the production quality and then improve their domestic firms' competitiveness in the world market. In highly competitive international markets which the firms strategically compete against each other, the points which the firms must ponder with the object of their R&D activities are how much amount of total R&D investment investing into which types of R&D effectively and how governments' policies affect the firms' R&D investments. Recent theoretical studies on R&D policies, however, construct the model under only either process or product R&D. Thus, the questions how the government policies affect the firms' R&D choices has not been clear. The contribution of the present study is to give the answer to this question.

Our object is to focus on the relation between government's policy and firm's R&D strategies under Cournot and Bertrand competition. The reasons why we consider both the mode of competition are that as we mention below in this section, from the theoretical point of view, the previous studies clarified that the optimal R&D policy differs depending on the mode of competition. Furthermore, the empirical analysis, for instance, Negassi and Hung 2014 analyzed that the relationship between innovation and the mode of competition. They showed that if the firms compete under Bertrand competition, the degree of competition is strongly correlated with innovation output, on the other hand, the extant of competition is not correlated with innovation output if firms compete under Cournot competition.⁴⁾

Following extant research, we adopt a traditional third-country trade model in an international duopoly. The analysis comprises three stages. In the first stage the governments of both firms determine R&D policies. In the second stage firms choose their product qualities and costs. In the third stage they compete in a third-country under Cournot-quantity and Bertrand-price competition. In particular, in the second stage we consider the case in which the firms endogenously choose both total amount of R&D investments and the R&D fraction attributed to product R&D. We show following results.

First, a government subsidy always increases its domestic firm's total R&D investment, which is maintained irrespective of the mode of competition. Second, whether a government

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- 1) Negassi and Hung 2014 indicate that how many firms in an industry conduct both product and process R&D or either of them.
 - 2) For instance, the cost-reducing R&D includes the change in location of factory for many reasons such as easy access to the transport.
 - 3) See more detail in OECD outlook 2014.
 - 4) Negassi and Hung 2014 took into account the relationship between competition and innovation in two sectors, i.e., a public sector and a civil sector. They showed that in the public sector the firms compete in Cournot competition because in the public sector its product price is higher than the marginal cost and then firms compete under Cournot competition. On the other hand, in the civil sector the difference between price and marginal cost is very small. Then the competition in the market is very intense. That is, the firms compete in Bertrand competition.

subsidy increases the R&D fraction into product R&D depends on the costs of product and process R&D. Third, a government subsidy has no impact on the rival firm's R&D fraction. This implies that the government subsidy impacts on the rival firm's R&D activity only through its total amount of R&D investment. This is because the R&D fractions are actually determined by the R&D costs, which are exogenously given and determined only by domestic factors. Thus, the government subsidy cannot have an effect on rival's those costs. Forth, we also have different results between the mode of competition. These come from the magnitude of R&D fraction. Under Cournot competition, while output increases when quality improves and/or the production cost decreases, output decreases as the proportion of product R&D becomes higher compared to that of process R&D. Under Bertrand competition, prices become higher when the quality improves and production costs increase. An increase in total investment leads to both quality improvements due to product R&D and production cost reductions due to process R&D. Thus, whether prices become higher or lower depends on the degree of fraction of investment in two types of R&D. If firms only conduct one of the two kinds of R&D, this effect does not exist.

Previous studies examining optimal R&D policies can be delineated along two lines: those considering policies relevant to process R&D and those considering policies relevant to product R&D. Qiu and Tao (1998) conclude that governments have incentives to subsidize their domestic firms' process R&D activities when a domestic and a foreign firm engage in R&D collaboration, which maximizes two firms' profits or R&D coordination, which maximizes a weighted sum of their profits. Liao (2007) demonstrates that optimal government R&D tax/subsidy policies change depending on the degree of technology spillover. Kim and Gu (2015) examine the impact of market size difference on government R&D policies and conclude that a country with a large (small) market size and lower (higher) product differentiation decreases (increases) the level of optimal R&D subsidy. Studies concerning product R&D, such as those by Park (2001), Zhou et al. (2002), Jinji (2003), and Jinji and Toshimitsu (2006) apply Hotelling vertically differentiated models. Together, these studies posit that different policies are chosen depending on the mode of competition. Further, studies using horizontal and vertically differentiated models, such as those by Ishii (2014), Toshimitsu (2014), Taba and Ishii (2016), and Taba (2016), result in qualitatively similar outcomes. This implies that the respective governments of horizontally and vertically-differentiated firms have incentives to subsidize.⁵⁾ As mentioned above, however, these studies investigate policies pertaining to either process or product R&D. In actual, empirical, contexts both process and product innovations very often coexist in firms' R&D portfolios (Bacchiega, Lambertini, and Mantovani 2011). Indeed, several extent empirical studies testify to this observation. Brewin, Monchuk, and Partridge (2009) find evidence of a significant interrelation between product and process innovations in the Canadian food processing industry. Scherer (1991), Cohen and Klepper (1996), and Fritsch and Meschede (2001) focus on the relationship between firm size and the choice of process and product R&D. Boone (2000) examines the effects of competitive pressure on a firm's incentives to invest in product and process innovations. Becker and

5) On the analysis of a government's optimal R&D policy when firms conduct only product R&D, we can categorize the model into two types: one uses the Hotelling model, the other employs vertically and horizontally differentiated model. Regarding the difference between two models, see Taba (2016).

Egger (2013) investigate the impacts of process and product R&D on a firm's export propensity.

Through the theoretical analyses, Lin and Saggi (2002) demonstrate the relationship between process and product R&D under Cournot and Bertrand competition and indicate the competition mode in which firms invest more in process or product R&D. Yin and Zuscovitch (1998) analyze the interrelation between firm size and process and product R&D and posit the conditions under which a large or small firm increases or decreases each type of R&D. Among several studies that consider the interconnection between the choice of R&D and market size, market concentration, competition mode and so on, the studies by Rozenkranz (2003) and Saha (2007) investigate the interdependence between consumer preferences and a firm's R&D choice between process and product R&D. Vives (2008) illuminates robust relationships between competitive pressures such as product substitutability, the number of competitors and market size and R&D. Saha (2014) extends the forgoing model with respect to the relationship between a firm's objective function and the decision among process and product R&D. However, they do not investigate governments optimal R&D policies; indeed it seems that few studies have examined optimal R&D policies when firms invest in both process and product R&D.

The rest of the paper is organized as follows. Section 2 delineates assumptions and posits a three-stage game comprising of Cournot and Bertrand competition. Section 3 offers the conclusions and the implications for industrial policy.

2 The Model

In this section, we introduce the model and its assumptions. The model follows a traditional third-country trade model in an international duopoly. The world consists of three countries. Two exporting countries, home and foreign, and one importing country. In each exporting country, a firm and government exist. The firm in each country differs in terms of product quality. In the importing country, the so-called third country, only consumers exist.⁶⁾ The exporting firms compete in the third-country market under Cournot and Bertrand competition. We first discuss the case of Cournot-quantity competition.

2.1 Cournot Competition

Each firm produces output denoted by x and x^* . The parameters associated with the foreign country and its firm are hereafter denoted by “*”. On the demand side, a representative consumer in the third country has preferences for product quality and quantity.⁷⁾ Therefore, the

6) In this setting, we assume that there are no consumers in the exporting countries. However, we can expect the results in the model with consumers. First, under Cournot competition an increment of both R&D is always beneficial for consumers. Because higher product quality and lower marginal cost increase consumer surplus. Under Bertrand competition, on the other hand, an increment of process R&D always makes consumer surplus increase. However, the effects of an increase in product R&D depend on the magnitude of R&D fraction. Because an improvement of product quality increases consumer's utility but also rises price. We believe that the setting yields a number of important insights about industrial policies and firm's incentive to invest R&D.

7) The consumer's utility function is given by $U(x, x^*; q, q^*) = qx + q^*x^* + \frac{m}{2}(x^2 + x^{*2}) - nxx^* + z$ where

inverse demand function they face is given by, respectively,

$$p = q(I, \beta) - mx - nx^*, \quad p^* = q^*(I^*, \beta^*) - mx^* - nx, \quad m > n \quad (1)$$

where $q(I, \beta)$ and $q^*(I^*, \beta^*)$ represent the product quality of production, and I and I^* are the amount of R&D investments. We assume that a parameter $m > 0$ and $n \in (0, m]$, which shows a degree of horizontal product differentiation or product substitution, and also that $m > n$ holds.

On the production side, we presume that the firms conduct both process and product R&D simultaneously. The total amount of investment used in process and product R&D is denoted by I and I^* . The firms invest total investment fraction $\beta(\beta^*) \in (0, 1)$ in product R&D, and total investment fraction $1 - \beta(1 - \beta^*)$ in process R&D, which are endogenously determined. When $\beta(\beta^*)$ tends to 1; the firm invests more in product R&D, when $\beta(\beta^*)$ tends to 0, the firm invests more in process R&D.⁸⁾ Each firm's quality function is given by

$$q(I, \beta) = q_0 + \frac{1}{2}(\beta I)^{\frac{1}{2}}, \quad q^*(I^*, \beta^*) = q_0^* + \frac{1}{2}(\beta^* I^*)^{\frac{1}{2}}, \quad (2)$$

where $q_0(q_0^*) > 0$ represents the initial quality level before the firm invests in product and process R&D, the quality functions are twice differentiable with respect to I and I^* with $\partial q / \partial I > 0$ and $\partial^2 q / \partial I^2 < 0$.

The firm's profit functions are given by

$$\pi = (p - c)x - (r_0 - s)\beta I - (r_1 - s)(1 - \beta)I \quad (3-1)$$

$$\pi^* = (p^* - c^*)x^* - (r_0^* - s^*)\beta^* I^* - (r_1^* - s^*)(1 - \beta^*)I^* \quad (3-2)$$

where $r_0(r_0^*) > 0$ represents the constant investment cost of product R&D and $r_1(r_1^*) > 0$ denotes that of process R&D. $s(s^*)$ is the government's total R&D subsidy (tax) if it takes positive (negative) values.

The marginal production cost functions are specified as follows:

$$c(I, \beta) = c_0 - 2\{(1 - \beta)I\}^{\frac{1}{2}}, \quad c^*(I^*, \beta^*) = c_0^* - 2\{(1 - \beta^*)I^*\}^{\frac{1}{2}}, \quad (4)$$

z is a numéraire good. This quality-argued utility function is widely used in the analysis of horizontal and vertical product differentiated model such as by Häckner (2000), Symeonidis (1999, 2003), Toshimitsu (2014) and Ishii (2014). Here, we apply the version of Ishii (2014).

8) As stated in Levin and Reiss (1989, p. 7) we also consider the firm's R&D activities as follows. In firm's R&D activities, the correspondence between process and product R&D is not so clear, especially because the design and development of improved products often requires accompanying process improves. Thus, we consider the total R&D investment of process and product R&D. Saha (2014) uses essentially the same idea regarding to the total R&D investment. In his analysis the absolute amounts of process and product R&D investment are determined endogenously.

where $c_0(c_0^*) > 0$ is the initial level of production cost. The cost function of each firm is twice differentiable with respect to $I(I^*)$ with $\partial c / \partial I < 0$ and $\partial^2 c / \partial I^2 > 0$.⁹⁾

The government's objective function is defined as its domestic firm's profit net of the subsidy paid to its domestic firm's R&D investment. Thus, we have

$$W = \pi - Is, \quad W^* = \pi^* - I^* s^*. \quad (5)$$

The government's role in this model is to determine an optimal R&D policy for its domestic firm's total R&D investment.

The game consists of three stages. In the first stage, each government decides an optimal R&D policy for its domestic firm's process and product R&D investment activities. In the second stage, they choose the optimal total amount of investment in product and process R&D and the R&D fraction into product R&D. In the third stage, the firms compete against each other under Cournot-quantity or Bertrand-price competition in the third country. We obtain the sub-game perfect Nash equilibrium by solving the game backwards.

2.1.1 The Third-Stage: Quantity Competition

In the third stage, the firms determine their optimal amount of production. Given the quality levels and R&D policies, which are respectively determined in the second and the first stages, firms choose their optimal outputs. From the first-order condition: $\partial \pi / \partial x = 0$ and $\partial \pi^* / \partial x^* = 0$, the firms' equilibrium outputs are obtained by

$$\begin{aligned} x &= \frac{2m\{q(I, \beta) - c(I, \beta)\} - n\{q^*(I^*, \beta^*) - c^*(I^*, \beta^*)\}}{4m^2 - n^2}, \\ x^* &= \frac{2m\{q^*(I^*, \beta^*) - c^*(I^*, \beta^*)\} - n\{q(I, \beta) - c(I, \beta)\}}{4m^2 - n^2} \end{aligned} \quad (6)$$

The second-order conditions hold as $\partial^2 \pi / \partial x^2 = \partial^2 \pi^* / \partial x^{*2} = -2m < 0$. The cross effects hold as $\partial^2 \pi / \partial x^* \partial x = \partial^2 \pi^* / \partial x \partial x^* = -n < 0$. Thus, the stability conditions are satisfied. Eq.(6) shows that output increases when the firm's quality improves and/or the rival firm's production cost rises, but decreases when the rival firm's quality improves and/or the firm's own production cost increases. How does the R&D investment of each firm affect its own output? That is obtained by

$$\begin{aligned} \frac{\partial x}{\partial I} &= \frac{2m\left\{\frac{\partial q}{\partial I} - \frac{\partial c}{\partial I}\right\}}{\Delta}, & \frac{\partial x^*}{\partial I^*} &= \frac{2m\left\{\frac{\partial q^*}{\partial I^*} - \frac{\partial c^*}{\partial I^*}\right\}}{\Delta}, \\ &= \frac{m}{2\Delta I^{\frac{1}{2}}}\left\{\beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}}\right\} > 0, & &= \frac{m}{2\Delta I^{*\frac{1}{2}}}\left\{\beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}}\right\} > 0. \end{aligned}$$

9) With respect to the coefficients of quality and cost functions, we here consider biased coefficients in efficiency between two functions, which is weighted in process R&D. If we use more general coefficient, e.g., $\theta_q, \theta_c \in (0, 1]$, we can obtain outcomes with more precise cases. However, the main conclusion on governments' optimal R&D policies does not change even if we use general coefficients.

where $\partial q / \partial I (\partial q^* / \partial I^*) > 0$, $\partial c / \partial I (\partial c^* / \partial I^*) < 0$ and $\Delta = 4m^2 - n^2$. These imply that an increase in R&D investment always enhances production due to improve product quality and reduce marginal production costs. How does R&D investment influence the rival firm's production? These effects are shown by

$$\begin{aligned} \frac{\partial x}{\partial I^*} &= \frac{-n \left\{ \frac{\partial q^*}{\partial I^*} - \frac{\partial c^*}{\partial I^*} \right\}}{\Delta} & \frac{\partial x^*}{\partial I} &= \frac{-n \left\{ \frac{\partial q}{\partial I} - \frac{\partial c}{\partial I} \right\}}{\Delta}, \\ &= \frac{-n}{4\Delta I^{*\frac{1}{2}}} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} < 0, & &= \frac{-n}{4\Delta I^{\frac{1}{2}}} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} < 0. \end{aligned}$$

The increment in the firm's R&D investment always decreases its rival firm's output because an increase in the rival firm's R&D investment improves its quality ($\partial q / \partial I (\partial q^* / \partial I^*) > 0$) and reduces its marginal cost ($\partial c / \partial I (\partial c^* / \partial I^*) < 0$).

We now examine the impacts of $\beta(\beta^*)$ on each firm's output. The case of the home firm is given by

$$\begin{aligned} \frac{\partial x}{\partial \beta} &= \frac{2m}{\Delta} \left(\frac{\partial q}{\partial \beta} - \frac{\partial c}{\partial \beta} \right) \\ &= \frac{mI^{\frac{1}{2}}}{2\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} \begin{cases} > 0 & \text{if } 0 < \beta < \frac{1}{17} \\ = 0 & \text{if } \beta = \frac{1}{17} \\ < 0 & \text{if } \frac{1}{17} < \beta < 1 \end{cases}. \end{aligned} \quad (7-1)$$

That of the foreign firm is

$$\begin{aligned} \frac{\partial x^*}{\partial \beta^*} &= \left(\frac{\partial q^*}{\partial \beta^*} - \frac{\partial c^*}{\partial \beta^*} \right) \\ &= \frac{mI^{*\frac{1}{2}}}{2\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\} \begin{cases} > 0 & \text{if } 0 < \beta^* < \frac{1}{17} \\ = 0 & \text{if } \beta^* = \frac{1}{17} \\ < 0 & \text{if } \frac{1}{17} < \beta^* < 1 \end{cases}. \end{aligned} \quad (7-2)$$

We have the following lemma.

Lemma 1. *Under Cournot competition when each firm increases the ratio of product R&D, as a result (1) the amount of output increases if $0 < \beta(\beta^*) < 1/17$, (2) the output level remains unchanged if $\beta(\beta^*) = 1/17$, and (3) the amount of output decreases if $1/17 < \beta(\beta^*) < 1$.*

Lemma 1(1) implies that each firm enhances its production if $\partial q / \partial \beta > \partial c / \partial \beta (\partial q^* / \partial \beta^* > \partial c^* / \partial \beta^*)$. In this case, the effect of quality improvement is larger than cost increasing in response to an increase in $\beta(\beta^*)$. Thus, output increases. Lemma 1(2) is the case in which $\partial q / \partial \beta = \partial c / \partial \beta (\partial q^* / \partial \beta^* = \partial c^* / \partial \beta^*)$ holds. In this case, the rate of quality improvement and that of cost increase are the same. Thus, the level of production does not change. Lemma 1(3) is the case where $\partial q / \partial \beta < \partial c / \partial \beta (\partial q^* / \partial \beta^* < \partial c^* / \partial \beta^*)$ holds. This case shows that the increment of production costs is greater than the quality improvement effect, so that output decreases. We can also interpret the results as follows. In response to an increase in the fraction of β , the firms increase their outputs when the ratio of product R&D is relatively small ($0 < \beta(\beta^*) < 1/17$) because increasing the fraction of product R&D enhances the production qualities, which leads to expansion of outputs at the degree to which the increment of the production cost does not decrease outputs. As we will prove below in Appendix A, this case actually corresponds to a situation where the cost of product R&D $r_p(r_p^*)$ is higher than that of process R&D represented by $r_c(r_c^*)$. Thus, originally, firms' investment in product R&D is relatively small. On the other hand, the firms decrease their outputs if the ratio of product R&D is comparatively large ($1/17 < \beta(\beta^*) < 1$) because in this case, although quality is increasing, it is very large to harm the effects of process R&D. Thus, the output decreases. We, next turn to the effects of an increase in firm's product R&D investment fraction on rival firm's output. Changing the foreign firm's R&D fraction, the home firm's production changes such as

$$\begin{aligned} \frac{\partial x}{\partial \beta^*} &= \frac{-n}{\Delta} \left(\frac{\partial q^*}{\partial \beta^*} - \frac{\partial c^*}{\partial \beta^*} \right) \\ &= -\frac{nI^{*\frac{1}{2}}}{4\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\} \begin{cases} > 0 & \text{if } \frac{1}{17} < \beta^* < 1 \\ = 0 & \text{if } \beta^* = \frac{1}{17} \\ < 0 & \text{if } 0 < \beta^* < \frac{1}{17} \end{cases} \end{aligned} \quad (8-1)$$

Applying similar logic, the foreign firm's production changes in response to rival firm R&D fractional changes. That is,

$$\begin{aligned}
\frac{\partial x^*}{\partial \beta} &= \frac{-n}{\Delta} \left(\frac{\partial q}{\partial \beta} - \frac{\partial c}{\partial \beta} \right) \\
&= -\frac{nI^{\frac{1}{2}}}{4\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} \begin{cases} > 0 & \text{if } \frac{1}{17} < \beta < 1 \\ = 0 & \text{if } \beta = \frac{1}{17} \\ < 0 & \text{if } 0 < \beta < \frac{1}{17} \end{cases} .
\end{aligned} \tag{8-2}$$

We have the following lemma.

Lemma 2. *Under Cournot competition when a home (foreign) firm increases the ratio of product R&D, it results in its rival firm's output (1) increasing if $1/17 < \beta(\beta^*) < 1$, (2) decreasing if $0 < \beta(\beta^*) < 1/17$, and (3) remaining unchanged if $\beta(\beta^*) = 1/17$.*

From Lemma 2, whether own increase in the product R&D fraction expand the rival firm's output depends on whether own output increases or not. Lemma 2(1) implies that $\partial q / \partial \beta < \partial c / \partial \beta (\partial q^* / \partial \beta^* < \partial c^* / \partial \beta^*)$. That is, the rate of increase in own production cost becomes higher than that of own quality improvement in response to an increase in the ratio of product R&D, which leads to a decrease in own output, represented in Eqs.(7-1) and (7-2). Thus, this causes the rival firm's output to increase. Lemma 2(2) posits that an increase in $\beta(\beta^*)$ increases own output, which results in decreasing the rival firm's output. Lemma 2(3) is the case in which $\partial q / \partial \beta = \partial c / \partial \beta (\partial q^* / \partial \beta^* = \partial c^* / \partial \beta^*)$ holds. Here, the rate of quality improvement and production cost increase are equal in response to an increase in $\beta(\beta^*)$. Thus, own output does not change, which leads to that the rival firm's output does not change. Now, we turn to the second stage, quality choice.

2.1.2 The Second-Stage: Quality Choice

In the second stage, the firms choose the total amount of investment for process and product R&D. The first-order conditions of the respective firms are given by

$$\begin{aligned}
\frac{\partial \pi}{\partial I} &= \frac{\partial \pi}{\partial x^*} \left\{ \frac{\partial x^*}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial x^*}{\partial c} \frac{\partial c}{\partial I} \right\} + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial \pi}{\partial c} \frac{\partial c}{\partial I} + \frac{\partial \pi}{\partial I} = 0 \\
&= \frac{m^2 x}{\Delta I^{\frac{1}{2}}} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} - (r_b - s)\beta - (r_c - s)(1-\beta) = 0
\end{aligned} \tag{9-1}$$

$$\begin{aligned}\frac{\partial \pi^*}{\partial I^*} &= \frac{\partial \pi^*}{\partial x} \left\{ \frac{\partial x}{\partial q^*} \frac{\partial q^*}{\partial I^*} + \frac{\partial x}{\partial c^*} \frac{\partial c^*}{\partial I^*} \right\} + \frac{\partial \pi^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} + \frac{\partial \pi^*}{\partial c^*} \frac{\partial c^*}{\partial I^*} + \frac{\partial \pi^*}{\partial I^*} = 0 \\ &= \frac{m^2 x^*}{\Delta I^{*\frac{1}{2}}} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} - (r_s^* - s^*)\beta^* - (r_c^* - s^*)(1-\beta^*) = 0\end{aligned}\quad (9-2)$$

The second-order conditions are $\partial^2 \pi / \partial I^2 = (m^2 / 2I\Delta) \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} \left[(m / \Delta) \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} - x / I^{\frac{1}{2}} \right] < 0$ if $x > (mI^{\frac{1}{2}} / \Delta) \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\}$ and $\partial^2 \pi^* / \partial I^{*2} = (m^2 / 2I^*\Delta) \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} \left[(m / \Delta) \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} - x^* / I^{*\frac{1}{2}} \right] < 0$ if $x^* > (mI^{*\frac{1}{2}} / \Delta) \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\}$. The cross effects are $\partial \pi^2 / \partial I^* \partial I = \partial \pi^{*2} / \partial I \partial I^* = (-m^2 n / 4II^*\Delta^2) \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} < 0$. We assume that the second-order and stability conditions hold.

Those conditions in terms of β and β^* are

$$\frac{\partial \pi}{\partial \beta} = \frac{m^2 x I^{\frac{1}{2}}}{\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} - (r_s - r_c)I = 0 \quad (9-3)$$

$$\frac{\partial \pi^*}{\partial \beta^*} = \frac{m^2 x^* I^{*\frac{1}{2}}}{\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\} - (r_s^* - r_c^*)I^* = 0. \quad (9-4)$$

The second-order conditions and the cross effects are contained in Appendix A. In the second stage, we assume that the second-order conditions are always satisfied. However, the cross effects change depending on the product R&D investment fraction. Once stability conditions are satisfied from Eqs.(9-1) and (9-2), Eqs.(9-3) and (9-4), and the cross effects from Eqs.(C4) to (C9) in Appendix A, we can obtain the sub-game perfect Nash equilibrium investments and the investment fractions are obtained as the function of governments policies such as $I = I(s, s^*) (I^* = I^*(s, s^*))$ and $\beta = \beta(s, s^*) (\beta^* = \beta^*(s, s^*))$. These imply that the optimal qualities and costs are shown by functions of $I(I^*)$ and $\beta(\beta^*)$; $q(s, s^*) = q(I(s, s^*), \beta(s, s^*))$ and $c(s, s^*) = c(I(s, s^*), \beta(s, s^*))$.

2.1.3 Comparative Statics under Cournot Competition

We now investigate how the amount of R&D investments and the R&D fractions are affected

by government subsidy changes. From the second-order conditions and the cross effects of second stage, we obtain the following matrix.

$$\begin{vmatrix} \pi_{II} & \pi_{II^*} & \pi_{I\beta} & \pi_{I\beta^*} \\ \pi_{I^*I}^* & \pi_{I^*I^*}^* & \pi_{I^*\beta}^* & \pi_{I^*\beta^*}^* \\ \pi_{\beta I} & \pi_{\beta I^*} & \pi_{\beta\beta} & \pi_{\beta\beta^*} \\ \pi_{\beta^*I}^* & \pi_{\beta^*I^*}^* & \pi_{\beta^*\beta}^* & \pi_{\beta^*\beta^*}^* \end{vmatrix} \begin{vmatrix} dI \\ dI^* \\ d\beta \\ d\beta^* \end{vmatrix} = \begin{vmatrix} -\pi_{Is}ds \\ -\pi_{I^*s}^*ds^* \\ -\pi_{\beta s}ds \\ -\pi_{\beta^*s}^*ds^* \end{vmatrix} \quad (10)$$

where $\pi_{ij}(\pi_{i^*j^*}^*)$ is the abbreviation for $\partial^2\pi/\partial j\partial i(\partial^2\pi^*/\partial j^*\partial i^*)$, $i = I, \beta, j = I, I^*, \beta, \beta^*$ ($i^* = I^*, \beta^*, j^* = I, I^*, \beta, \beta^*$). The Hessian matrix is symmetric and $\pi_{\beta s} = \pi_{\beta^*s}^* = 0$. The determinant of the matrix $|D|$ is positive if $\pi_{II}\pi_{I^*I^*}^* - \pi_{II^*}\pi_{I^*I}^* > 0$.¹⁰⁾

How does the total amount of R&D investments change if the R&D subsidies provided by the governments change? From Eq.(10), with respect to the home country's firm we have

$$\begin{aligned} \frac{\partial I}{\partial s} &= \frac{-\pi_{Is}}{|D|} \left[\pi_{I^*I^*}^* (\pi_{\beta\beta}\pi_{\beta^*\beta^*}^* - \pi_{\beta\beta^*}\pi_{\beta^*\beta}^*) - \pi_{\beta I^*} (\pi_{I^*\beta}\pi_{\beta^*\beta^*}^* - \pi_{I^*\beta^*}\pi_{\beta^*\beta}^*) \right. \\ &\quad \left. + \pi_{\beta^*I^*}^* (\pi_{I^*\beta}\pi_{\beta\beta^*} - \pi_{I^*\beta^*}\pi_{\beta\beta}) \right] \\ &= \frac{\pi_{Is}}{|D|} \left(\frac{m^2}{2\Delta} \right)^3 \frac{I}{I^{*\frac{1}{2}}} x^* (A^*E^* - F^{*2}) \left[\left\{ \frac{m}{\Delta} F^2 - \frac{x}{I^{\frac{1}{2}}} E \right\} \left\{ \frac{m}{\Delta} A^* - \frac{x^*}{I^{*\frac{1}{2}}} \right\} \right. \\ &\quad \left. - \left(\frac{n}{2\Delta} \right)^2 A^* F^2 \right] > 0 \end{aligned} \quad (11-1)$$

where $(A^*E^* - F^{*2}) > 0$ and $[\cdot] > 0$ if $\pi_{II}\pi_{I^*I^*}^* - \pi_{II^*}\pi_{I^*I}^* > 0$. We, hereafter, denote $A^* = \left\{ \beta^{\frac{1}{2}} + \right.$

$$\left. 4(1-\beta^*)^{\frac{1}{2}} \right\}, E^* = \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{3}{2}} + 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{3}{2}} \right\}, \text{ and } F^* = \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\}.$$

Rearranging eq.(11-1), we obtain

$$\frac{\partial I}{\partial s} = \pi_{\beta\beta}\pi_{I^*I^*}^* - (\pi_{\beta I^*})^2 > 0 \quad (11-1.1)$$

With respect to the foreign country's firm we have

10) For existence of an equilibrium, we have to check the naturally ordered principal minors of the matrix D . The detail is contained in Appendix A2.

$$\begin{aligned}\frac{\partial I^*}{\partial s^*} &= \frac{-\pi_{I^*s^*}^*}{|D|} \left[\pi_{II} (\pi_{\beta\beta} \pi_{\beta^*\beta^*}^* - \pi_{\beta\beta^*} \pi_{\beta^*\beta}^*) - \pi_{\beta I} (\pi_{I\beta} \pi_{\beta^*\beta^*}^* - \pi_{I\beta^*} \pi_{\beta^*\beta}^*) \right] \\ &\quad + \pi_{\beta^*I}^* (\pi_{I\beta} \pi_{\beta\beta^*} - \pi_{I\beta^*} \pi_{\beta\beta}) \\ &= \frac{\pi_{I^*s^*}^*}{|D|} \left(\frac{m^2}{2\Delta} \right)^3 \frac{I^*}{I^{\frac{1}{2}}} x (AE - F^2) \left[\left\{ \frac{m}{\Delta} F^{*2} - \frac{x^*}{I^{*\frac{1}{2}}} E^* \right\} \left\{ \frac{m}{\Delta} A - \frac{x}{I^{\frac{1}{2}}} \right\} \right. \\ &\quad \left. - \left(\frac{n}{2\Delta} \right)^2 A F^{*2} \right] > 0\end{aligned}\quad (11-2)$$

where $(AE - F^2) > 0$ and $[\cdot] > 0$ if $\pi_{II} \pi_{I^*I^*}^* - \pi_{II^*} \pi_{I^*I}^* > 0$. Hereafter we denote $A = \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\}$, $E = \left\{ \left(\frac{1}{\beta} \right)^{\frac{3}{2}} + 4 \left(\frac{1}{1-\beta} \right)^{\frac{3}{2}} \right\}$, and $F = \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\}$.

Rearranging eq.(11-2), we obtain

$$\frac{\partial I^*}{\partial s^*} = \pi_{\beta^*\beta^*}^* \pi_{II} - (\pi_{\beta^*I}^*)^2 > 0 \quad (11-2.1)$$

From eqs.(11-1.1) and (11-2.1), we obtain Lemma 3.

Lemma 3. *An increase in a government R&D subsidy enhances its domestic firm's R&D investment irrespective of the magnitude of β and β^* under Cournot competition.*

From eqs. (11-1.1) and (11-2.1), the effects of government's subsidy on the total R&D investment depend on the cross-effects (i.e., $\pi_{\beta I^*}$ and $\pi_{\beta^* I}$). The sign of these effects relies on the magnitude of β and β^* described in Appendix A1 eq.(C8) and eq.(C9). However, eqs. (11-1.1) and (11-2.1) shows that a marginal increase in the government subsidy always makes its domestic firm's total R&D investment increase. This is because given the magnitude of $\beta(\beta^*)$ an increase in total R&D investment always increase the product qualities and decrease the production costs and which leads to increase the outputs irrespective of the magnitude of β and β^* , described in the subsection 2.1.1.

Next we examine the change in rival firm's investment when the government increases its R&D subsidy. Thus,

$$\begin{aligned}\frac{\partial I^*}{\partial s} &= \frac{\pi_{Is}}{|D|} \left[\pi_{I^*I}^* (\pi_{\beta\beta} \pi_{\beta^*\beta^*}^* - \pi_{\beta\beta^*} \pi_{\beta^*\beta}^*) - \pi_{\beta I} (\pi_{I^*\beta}^* \pi_{\beta^*\beta^*}^* - \pi_{I^*\beta^*}^* \pi_{\beta^*\beta}^*) \right] \\ &\quad + \pi_{\beta^*I}^* (\pi_{I^*\beta}^* \pi_{\beta\beta^*} - \pi_{I^*\beta^*}^* \pi_{\beta\beta}) \\ &= \frac{-\pi_{Is}}{|D|} \frac{nm^6}{16\Delta^4} x x^* (AE - F^2) (A^* E^* - F^{*2}) < 0\end{aligned}\quad (12-1)$$

$$\begin{aligned}
\frac{\partial I}{\partial s^*} &= \frac{\pi_{I^*s^*}^*}{|D|} \left[\pi_{II^*} \left(\pi_{\beta\beta} \pi_{\beta^* \beta^*}^* - \pi_{\beta\beta^*} \pi_{\beta^* \beta}^* \right) - \pi_{\beta I^*} \left(\pi_{I\beta} \pi_{\beta^* \beta^*}^* - \pi_{I\beta^*} \pi_{\beta^* \beta}^* \right) \right] \\
&\quad + \pi_{\beta^* I^*}^* \left(\pi_{I\beta} \pi_{\beta\beta^*} - \pi_{I\beta^*} \pi_{\beta\beta} \right) \\
&= \frac{-\pi_{I^*s^*}^*}{|D|} \frac{nm^6}{16\Delta^4} x x^* (AE - F^2) (A^* E^* - F^{*2}) < 0
\end{aligned} \tag{12-2}$$

Eqs.(12-1) and (12-2) show that an increase in the government's subsidy for its domestic firm's R&D investment decreases the rival firm's total R&D investment irrespective of β and β^* . We get Lemma 4.

Lemma 4. *An increase in a government R&D subsidy reduces its rival firm's R&D investment irrespective of β and β^* under Cournot competition.*

The result deeply depends on the sign of $\pi_{II^*} < 0$ and $\pi_{I^*I}^* < 0$. The strategic nature of total R&D investments in the second stage is negative. Thus, Lemma 4 holds. This is because taking into account Lemma 3, it states that a marginal increase in government's subsidy always makes its domestic firm's total R&D investment increase, which leads to enhance domestic firm's output. Furthermore, the strategic nature in the third stage is strategic substitutes under Cournot competition. Therefore, if the rival firm increases its output, the competition is more intense.

We are particularly concerned with the effect of a government's subsidy on its domestic firm's R&D fraction. This effect for each firm is given by

$$\begin{aligned}
\frac{\partial \beta}{\partial s} &= \frac{-\pi_{Is}}{|D|} \left[\pi_{I^*I}^* \left(\pi_{\beta I^*} \pi_{\beta^* \beta^*}^* - \pi_{\beta\beta^*} \pi_{\beta^* I^*}^* \right) - \pi_{\beta I} \left(\pi_{I^*I^*}^* \pi_{\beta^* \beta^*}^* - \pi_{I^* \beta^*}^* \pi_{\beta^* I^*}^* \right) \right] \\
&\quad + \pi_{\beta^* I}^* \left(\pi_{I^*I^*}^* \pi_{\beta\beta^*} - \pi_{I^* \beta^*}^* \pi_{\beta I} \right) \\
&= \frac{-\pi_{Is}}{|D|} \frac{II^{*1} x^*}{AA^*} \frac{m^2}{2\Delta} (A^* E^* - F^{*2}) (\pi_{II} \pi_{I^*I^*}^* - \pi_{II^*} \pi_{I^*I}^*) F
\end{aligned} \tag{13-1}$$

where $(\pi_{II} \pi_{I^*I^*}^* - \pi_{II^*} \pi_{I^*I}^*) > 0$. The sign of eq.(13-1) depends on the magnitude of β . Thus,

$$\frac{\partial \beta}{\partial s} \begin{cases} < 0 & \text{if } 0 < \beta < \frac{1}{17} \quad (n_b > n_c) \\ = 0 & \text{if } \beta = \frac{1}{17} \quad (n_b = n_c) \\ > 0 & \text{if } \frac{1}{17} < \beta < 1 \quad (n_b < n_c) \end{cases} \tag{13-2}$$

The effect on the foreign firm's R&D fraction is given by

$$\begin{aligned}\frac{\partial \beta^*}{\partial s^*} &= \frac{-\pi_{I^*}^*}{|D|} \left[\pi_{II} \left(\pi_{\beta I^*} \pi_{\beta^* \beta}^* - \pi_{\beta \beta} \pi_{\beta^* I^*}^* \right) - \pi_{\beta I} \left(\pi_{II^*} \pi_{\beta^* \beta}^* - \pi_{I \beta} \pi_{\beta^* I^*}^* \right) \right] \\ &\quad + \pi_{\beta^* I}^* \left(\pi_{II^*} \pi_{\beta \beta} - \pi_{I \beta} \pi_{\beta I^*} \right) \end{aligned} \quad (14-1)$$

$$= \frac{-\pi_{I^*}^*}{|D|} \frac{I^2 I^* x}{AA^*} \frac{m^2}{2\Delta} (AE - F^2) (\pi_{II} \pi_{I^*}^* - \pi_{II^*} \pi_{I^*}^*) F^*$$

where $(\pi_{II} \pi_{I^*}^* - \pi_{II^*} \pi_{I^*}^*) > 0$. The sign of Eq.(14-1) depends on the magnitude of β^* . Thus,

$$\frac{\partial \beta^*}{\partial s^*} \begin{cases} < 0 & \text{if } 0 < \beta^* < \frac{1}{17} \quad (r_b^* > r_c^*) \\ = 0 & \text{if } \beta^* = \frac{1}{17} \quad (r_b^* = r_c^*) \\ > 0 & \text{if } \frac{1}{17} < \beta^* < 1 \quad (r_b^* < r_c^*) \end{cases} \quad (14-2)$$

Eqs.(13-2) and (14-2) show how an increasing R&D subsidy affects the ratio of R&D. If β is relatively small (large) $0 < \beta(\beta^*) < 1/17$ ($1/17 < \beta(\beta^*) < 1$), the firms do not (do) expand the product R&D fraction. We obtain Proposition 1.

Proposition 1. *An increase in government's subsidy causes the domestic firm's proportion of product R&D to decrease if $0 < \beta(\beta^*) < 1/17$, increase if $1/17 < \beta(\beta^*) < 1$, and remain unchanged if $\beta(\beta^*) = 1/17$.*

Proposition 1 is very intuitive. These results are closely related with the R&D costs $r_c(r_c^*)$ and $r_b(r_b^*)$ derived from Eq.(C1.1) in Appendix A. The government subsidy makes its domestic firm invest into cheaper R&D. In the case, for instance, where if product R&D is more costly than process R&D: $r_b > r_c$ ($r_b^* > r_c^*$), the firm invests in more process R&D. However, the effects on the firm's output may be counterintuitive because in the case where if $0 < \beta < 1/17$, the effect of an increase in the government subsidy on the ratio of product R&D is negative, i.e., $\partial \beta / \partial s < 0$, on the other hand, an increase in the ratio of product R&D makes its domestic firm's output increase, i.e., $\partial x / \partial \beta > 0$. This is because the government subsidy affects on β with R&D cost in the firm's profit, i.e., $-r_b \beta I$.

Furthermore, we are particularly interested in the change in the rival firm's investment fractions when the government provides more subsidy to its domestic firm. This is described below.

$$\begin{aligned}
\frac{\partial \beta}{\partial s^*} &= \frac{\pi_{I^*s^*}}{|D|} \left[\pi_{II} \left(\pi_{\beta I^*} \pi_{\beta^* \beta^*}^* - \pi_{\beta \beta^*} \pi_{\beta^* I^*}^* \right) - \pi_{\beta I} \left(\pi_{II^*} \pi_{\beta^* \beta^*}^* - \pi_{I \beta^*} \pi_{\beta^* I^*}^* \right) \right. \\
&\quad \left. + \pi_{\beta^* I}^* \left(\pi_{II^*} \pi_{\beta \beta^*} - \pi_{I \beta^*} \pi_{\beta I^*}^* \right) \right] \\
&= \frac{-\pi_{I^*s^*}}{|D|} \frac{nm^2}{4\Delta^2} \left(\frac{m^2}{2\Delta} \right)^2 \frac{I^{\frac{1}{2}}}{I^{\frac{1}{2}}} AJF \left\{ \left(A^* G^* - J^* F^{*2} \right) - \left(A^* C^* - J^* F^{*2} \right) \right\} \\
&= 0
\end{aligned} \tag{15-1}$$

$$\text{where } J = \left[\frac{m}{\Delta} A - \frac{x}{I^{\frac{1}{2}}} \right], J^* = \left[\frac{m}{\Delta} A^* - \frac{x^*}{I^{*\frac{1}{2}}} \right], G^* = \left[\frac{m}{\Delta} F^{*2} - \frac{x^*}{I^{*\frac{1}{2}}} E^* \right].$$

$$\begin{aligned}
\frac{\partial \beta^*}{\partial s} &= \frac{\pi_{Is}}{|D|} \left[\pi_{I^*I^*}^* \left(\pi_{\beta I^*} \pi_{\beta^* \beta^*}^* - \pi_{\beta \beta^*} \pi_{\beta^* I^*}^* \right) - \pi_{\beta I} \left(\pi_{I^*I^*}^* \pi_{\beta^* \beta^*}^* - \pi_{I^* \beta^*} \pi_{\beta^* I^*}^* \right) \right. \\
&\quad \left. + \pi_{\beta^* I}^* \left(\pi_{I^*I^*}^* \pi_{\beta \beta^*} - \pi_{I^* \beta^*} \pi_{\beta I^*}^* \right) \right] \\
&= \frac{-\pi_{Is}}{|D|} \frac{nm^2}{4\Delta^2} \left(\frac{m^2}{2\Delta} \right)^2 \frac{I^{\frac{1}{2}}}{I^{*\frac{1}{2}}} AA^* F^* \left[\begin{aligned} &\left\{ \left(\frac{n}{2\Delta} \right)^2 A^* F^2 - J^* G \right\} \\ &-\left\{ \left(\frac{n}{2\Delta} \right)^2 A^* F^2 - J^* G \right\} \end{aligned} \right] \\
&= 0
\end{aligned} \tag{15-2}$$

where $G = \left[\frac{m}{\Delta} F^2 - \frac{x}{I^{\frac{1}{2}}} E \right]$. Eqs.(15-1) and (15-2) illustrate that an increase in the government's R&D subsidy has no effect on the rival firm's R&D fractions. These results suggest that the government's subsidy influences the rival firm's strategy of R&D fraction only through its total R&D investment described in Eqs.(12-1) and (12-2). Then, we obtain Lemma 5.

Lemma 5. *An increase in a government's R&D subsidy has no impact on its rival firm's R&D fractions under Cournot competition.*

This is because the strategic relationship between β, β^*, I and I^* denoted in Eq.(C2a) to Eq.(C9) in Appendix A depends on the degree of $\beta(\beta^*)$ and also this magnitude is closely tied with the R&D costs represented in Eq.(C1.1) in Appendix A. In consideration of this relationship, an increase in a government's subsidy should affect the rival firm's R&D costs. However, clearly these costs are exogenously given for the firm so that the rival country cannot affect these costs. Thus, the government's subsidy has no impact on the rival firm's R&D fraction. This is the significant difference of the subsidy effects on $I(I^*)$ and $\beta(\beta^*)$.

2.1.4 The First-Stage: Optimal R&D Policy

Now we turn to the first stage in which the governments of foreign and home firms decide their optimal R&D policies. From the first-order conditions $\partial W / \partial s = 0$ and $\partial W^* / \partial s^* = 0$ for

maximizing their domestic aggregate surplus. The sub-game perfect Nash equilibrium policy for the home government is given by

$$\frac{\partial W}{\partial s} = \frac{\partial \pi}{\partial x^*} \left[\frac{\partial I^*}{\partial s} \left(\frac{\partial x^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} + \frac{\partial x^*}{\partial c^*} \frac{\partial c^*}{\partial I^*} \right) + \frac{\partial \beta^*}{\partial s} \left(\frac{\partial x^*}{\partial q^*} \frac{\partial q^*}{\partial \beta^*} + \frac{\partial x^*}{\partial c^*} \frac{\partial c^*}{\partial \beta^*} \right) \right] - s \cdot \frac{\partial I}{\partial s}$$

where $\partial \beta^* / \partial s = 0$ from Lemma 5. Evaluating at $s = 0$, thus

$$\left. \frac{\partial W}{\partial s} \right|_{s=0} = \frac{\partial I^*}{\partial s} \left(\frac{\partial q^*}{\partial I^*} \frac{\partial x^*}{\partial q^*} + \frac{\partial c^*}{\partial I^*} \frac{\partial x^*}{\partial c^*} \right) \frac{\partial \pi}{\partial x^*} > 0. \quad (16-1)$$

That of the foreign government is

$$\frac{\partial W^*}{\partial s^*} = \frac{\partial \pi^*}{\partial x} \left[\frac{\partial I}{\partial s^*} \left(\frac{\partial x}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial x}{\partial c} \frac{\partial c}{\partial I} \right) + \frac{\partial \beta}{\partial s^*} \left(\frac{\partial x}{\partial q} \frac{\partial q}{\partial \beta} + \frac{\partial x}{\partial c} \frac{\partial c}{\partial \beta} \right) \right] - s^* \cdot \frac{\partial I^*}{\partial s^*}$$

where $\partial \beta / \partial s^* = 0$ from Lemma 5. Evaluating at $s^* = 0$, thus

$$\left. \frac{\partial W^*}{\partial s^*} \right|_{s^*=0} = \frac{\partial I}{\partial s^*} \left(\frac{\partial q}{\partial I} \frac{\partial x}{\partial q} + \frac{\partial c}{\partial I} \frac{\partial x}{\partial c} \right) \frac{\partial \pi^*}{\partial x} > 0. \quad (16-2)$$

Proposition 2. *Both governments have incentives to subsidize their domestic firms' R&D investments when the firms conduct both process and product R&D with endogenous choice of investment fractions.*

The intuition behind this proposition is the following. The firms in both countries gains from an R&D subsidies because an increase in total R&D investments leads to enhance domestic firms' outputs. Furthermore, at the quality choice stage the strategic nature is strategic substitute with regard to firms' investments ($\pi_{II^*} < 0$ and $\pi_{I^*I} < 0$). An increase in total R&D investment has two effects. One is that marginal increase in total R&D investment improves quality and reduces production cost, which lead to increase output. On the other hand, as described in Eqs.(15-1) and (15-2), an increase in government's R&D subsidy has no impact on the rival firm's R&D investment fractions. The ratio of R&D is determined only by domestic factors (i.e., R&D costs r_b, r_c). Thus, both governments determine their optimal R&D policies, letting the domestic firm's output enhance and the rival firm's profit decrease by lowering the rival firm's investment $\partial I^* / \partial s < 0$ ($\partial I / \partial s^* < 0$), which in turn impacts the rival firm's quality and production costs; these then impact on production, and change in the amount of production affects the rival firm's profits.

2.2 Bertrand Competition

The basic framework under Bertrand Competition is the same as that for Cournot Competition described above with the following exception.

The direct demand functions for each product are derived by utility maximization subject

to a budget constraint. Thus,

$$x = \frac{mq - nq^* + np^* - mp}{m^2 - n^2}, \quad x^* = \frac{mq^* - nq + np - mp^*}{m^2 - n^2} \quad (17)$$

We suppose that the quality functions for each firm are the same as Eq.(2) and the cost functions are identical to Eq.(4). Thus, the profit functions are represented by Eq.(3-1) and Eq.(3-2) and the social welfare functions are defined as in Eq.(5).

The game consists of three stages. In the first stage, the governments of both firms determine their R&D policies. In the second stage, the firms choose their product qualities by determining the amount of total investment into R&D. In the third stage, the firms compete under price competition in a third-country market.

2.2.1 The Third Stage: Price Competition

In the third stage, firms determine their prices so as to maximize their profits. From the first order conditions $\partial \pi / \partial p = 0$ and $\partial \pi^* / \partial p^* = 0$, the sub-game equilibrium prices are given by

$$\begin{aligned} p &= \frac{(2m^2 - n^2)q - mnq^* + m(2mc + nc^*)}{4m^2 - n^2}, \\ p^* &= \frac{(2m^2 - n^2)q^* - mnq + m(2mc^* + nc)}{4m^2 - n^2} \end{aligned} \quad (18)$$

The second order and stability conditions are satisfied: $\pi_{pp} = \pi_{p^*p^*}^* = -2m / (m^2 - n^2) < 0$, $\pi_{pp^*} = \pi_{p^*p}^* = n / (m^2 - n^2) > 0$, and $\pi_{pp}\pi_{p^*p^*}^* - \pi_{pp^*}\pi_{p^*p}^* > 0$. In the price competition stage, the relationship between the prices of both firms is strategic complements. In addition, own quality improvements increase own prices and cause the rival firm's price to decrease. The incremental production costs increases both firms' prices. The effect of R&D investment on firm's own prices is, respectively, given by

$$\begin{aligned} \frac{\partial p}{\partial I} &= \frac{(2m^2 - n^2) \frac{\partial q}{\partial I} + 2m^2 \frac{\partial c}{\partial I}}{\Delta} \\ &= \frac{(2m^2 - n^2) \beta^{\frac{1}{2}} - 8m^2 (1 - \beta)^{\frac{1}{2}}}{4I^{\frac{1}{2}} \Delta} \geq 0 \Leftrightarrow \beta \geq \frac{(8m^2)^2}{(8m^2 + (2m^2 - n^2)^2)}, \\ \frac{\partial p^*}{\partial I^*} &= \frac{(2m^2 - n^2) \frac{\partial q^*}{\partial I^*} + 2m^2 \frac{\partial c^*}{\partial I^*}}{\Delta} \\ &= \frac{(2m^2 - n^2) \beta^{*\frac{1}{2}} - 8m^2 (1 - \beta^*)^{\frac{1}{2}}}{4I^{*\frac{1}{2}} \Delta} \geq 0 \Leftrightarrow \beta^* \geq \frac{(8m^2)^2}{(8m^2 + (2m^2 - n^2)^2)}. \end{aligned}$$

This effect of each firm depends on the degree of $\beta(\beta^*)$. An increase in own R&D investment causes price increases if $\beta(\beta^*) > (8m^2)^2 / (8m^2 + (2m^2 - n^2)^2)$, and conversely, decreases if $\beta(\beta^*) < (8m^2)^2 / (8m^2 + (2m^2 - n^2)^2)$. This is because a quality improvement raises price whilst a cost reduction lowers price. These effects cancel each other out when $\beta(\beta^*) = (8m^2)^2 / (8m^2 + (2m^2 - n^2)^2)$. The impacts of investment on the rival firm's price are

$$\frac{\partial p^*}{\partial I} = \frac{-mn \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\}}{4\Delta I^{\frac{1}{2}}} < 0, \quad \frac{\partial p}{\partial I^*} = \frac{-mn \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\}}{4\Delta I^{*\frac{1}{2}}} < 0.$$

In contrast to the own firm R&D investment effect, the R&D investment always lowers the rival firm's price since an increase in total R&D investment leads to quality improvements and lower production costs more or less. The impact of increment of β on its own price is given by

$$\frac{\partial p}{\partial \beta} = I^{\frac{1}{2}} \left[\frac{(2m^2 - n^2)}{4\Delta} \left(\frac{1}{\beta} \right)^{\frac{1}{2}} + 2m^2 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right] > 0. \quad (19-1)$$

That of β^* is

$$\frac{\partial p^*}{\partial \beta^*} = I^{*\frac{1}{2}} \left[\frac{(2m^2 - n^2)}{4\Delta} \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} + 2m^2 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right] > 0. \quad (19-2)$$

In contrast to the case of Cournot competition, an increase in the ratio of product R&D investment always derives its own price up irrespective of the degree of β . In turn, if the ratio of process R&D becomes greater, the price lowers. These results are in sharp contrast to the case of Cournot competition described in Eqs.(7-1) and (7-2), which depend on the degree of the R&D fractions.

Further, if the foreign firm increases the ratio of product R&D, how do home firm prices change? That is described as

$$\frac{\partial p}{\partial \beta^*} = -\frac{mnI^{*\frac{1}{2}}}{4\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\} \begin{cases} > 0 & \text{if } \frac{1}{17} < \beta^* < 1 \\ = 0 & \text{if } \beta^* = \frac{1}{17} \\ < 0 & \text{if } 0 < \beta^* < \frac{1}{17} \end{cases}. \quad (20-1)$$

If the home firm increases the ratio of product R&D, how do foreign firm prices change? That is shown by

$$\frac{\partial p^*}{\partial \beta} = -\frac{mnI^{\frac{1}{2}}}{4\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} \begin{cases} > 0 & \text{if } \frac{1}{17} < \beta < 1 \\ = 0 & \text{if } \beta = \frac{1}{17} \\ < 0 & \text{if } 0 < \beta < \frac{1}{17} \end{cases} \quad (20-2)$$

Unlike the case of own-price impact, that on the rival firm depends on the magnitude of rival firm's R&D fractions. When a home (foreign) firm increases the ratio of product R&D, it results in its rival firm's price becoming (1) higher if $1/17 < \beta < 1$, (2) lower if $0 < \beta < 1/17$ and remaining unchanged if $\beta = 1/17$. The results can interpret as follows. In the price competition, each firm's price is strategic complements. Thus, if the home firm's price increases (decreases), the rival firm also takes a strategy to raise (reduce) its own price.

2.2.2 The Second-Stage: Quality Choice

In the second stage, the firms choose their quality through product R&D and their production costs by investing in process R&D. The first-order conditions of the firms, respectively, are

$$\frac{\partial \pi}{\partial I} = \frac{m(2m^2 - n^2)(p - c)}{2(m^2 - n^2)I^{\frac{1}{2}}\Delta} \left\{ \beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right\} + (r_b - s)\beta - (r_c - s)(1 - \beta) = 0 \quad (21-1)$$

$$\begin{aligned} \frac{\partial \pi^*}{\partial I^*} &= \frac{m(2m^2 - n^2)(p^* - c^*)}{2(m^2 - n^2)I^{*\frac{1}{2}}\Delta} \left\{ \beta^{*\frac{1}{2}} + 4(1 - \beta^*)^{\frac{1}{2}} \right\} \\ &+ (r_b^* - s^*)\beta^* - (r_c^* - s^*)(1 - \beta^*) = 0 \end{aligned} \quad (21-2)$$

The second-order conditions are $\pi_{II} < 0$ if $(p - c) > (2m^2 - n^2)I^{\frac{1}{2}} \left\{ \beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right\} / 2\Delta$

and $\pi_{I^*I^*}^* < 0$ if $(p^* - c^*) > (2m^2 - n^2)I^{*\frac{1}{2}} \left\{ \beta^{*\frac{1}{2}} + 4(1 - \beta^*)^{\frac{1}{2}} \right\} / 2\Delta$, the slopes of the reaction

functions are $\pi_{II^*} = \pi_{I^*I}^* = -m^2n(2m^2 - n^2) / 8\Delta^2(m^2 - n^2)I^{\frac{1}{2}}I^{*\frac{1}{2}} \left\{ \beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right\} \left\{ \beta^{*\frac{1}{2}} \right.$

$\left. + 4(1 - \beta^*)^{\frac{1}{2}} \right\} < 0$. The relationship between I and I^* is strategic substitutes. Then the stability

condition for sub-game perfect Nash equilibrium is satisfied, which is $\pi_{II}\pi_{I^*I^*}^* - \pi_{II^*}\pi_{I^*I}^* = \psi > 0$. From Eq.(21-1), Eq.(21-2) and the stability conditions, we get the equilibrium

investments as $I = I(s, s^*)$ and $I^* = I^*(s, s^*)$. The first-order conditions in terms of β and β^* are given by

$$\frac{\partial \pi}{\partial \beta} = \frac{m(2m^2 - n^2)(p - c)I^{\frac{1}{2}}}{2(m^2 - n^2)\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta} \right)^{\frac{1}{2}} \right\} - I(r_b - r_c) = 0 \quad (22-1)$$

$$\frac{\partial \pi^*}{\partial \beta^*} = \frac{m(2m^2 - n^2)(p^* - c^*)I^{*\frac{1}{2}}}{2(m^2 - n^2)\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta^*} \right)^{\frac{1}{2}} \right\} - I^*(r_b^* - r_c^*) = 0 \quad (22-2)$$

The second-order conditions and the cross effects are contained in Appendix B. In the second stage, we assume that the second-order conditions are always satisfied. However, the cross effects change depending on the process and product R&D investment fractions. Those effects directly affect governments' optimal R&D policies. Once stability conditions are satisfied from Eqs.(21-1) and (21-2), Eqs.(22-1) and (22-2) and the cross effects, we can obtain the sub-game perfect Nash equilibrium investments and the investment fractions are obtained as $I = I(s, s^*) (I^* = I^*(s, s^*))$ and $\beta = \beta(s, s^*) (\beta^* = \beta^*(s, s^*))$. These imply that the optimal

qualities and costs are given by the functions of $I(I^*)$ and $\beta(\beta^*)$: $q(s, s^*) = q(I(s, s^*), \beta(s, s^*))$, $q^*(s, s^*) = q^*(I^*(s, s^*), \beta^*(s, s^*))$, $c(s, s^*) = c(I(s, s^*), \beta(s, s^*))$ and $c^*(s, s^*) = c^*(I^*(s, s^*), \beta^*(s, s^*))$.

2.2.3 Comparative Statics under Bertrand Competition

We now approach the corresponding impacts of respective governments' R&D subsidies on their firms' R&D investments. Similar to the case of Cournot competition, we can define the Hessian matrix below.

$$\begin{vmatrix} \phi_{II} & \phi_{II^*} & \phi_{I\beta} & \phi_{I\beta^*} \\ \phi_{I^*I}^* & \phi_{I^*I^*}^* & \phi_{I^*\beta}^* & \phi_{I^*\beta^*}^* \\ \phi_{\beta I} & \phi_{\beta I^*} & \phi_{\beta\beta} & \phi_{\beta\beta^*} \\ \phi_{\beta^*I}^* & \phi_{\beta^*I^*}^* & \phi_{\beta^*\beta}^* & \phi_{\beta^*\beta^*}^* \end{vmatrix} \begin{vmatrix} dI \\ dI^* \\ d\beta \\ d\beta^* \end{vmatrix} = \begin{vmatrix} -\phi_{Is} ds \\ -\phi_{I^*s^*}^* ds^* \\ -\phi_{\beta s} ds \\ -\phi_{\beta^*s^*}^* ds^* \end{vmatrix} \quad (23)$$

where $\phi_{ij}(\phi_{ij}^*)$ is the abbreviation for $\partial^2 \pi / \partial j \partial i (\partial^2 \pi^* / \partial j^* \partial i^*)$, $i = I, \beta$, $j = I, I^*, \beta, \beta^*$ ($i^* = I^*, \beta^*, j^* = I, I^*, \beta, \beta^*$) and the determination is that Hessian matrix $\mathcal{D}$ is a symmetric matrix. The determinant of $|\mathcal{D}| > 0$ if $\phi_{II}\phi_{I^*I^*}^* - \phi_{II^*}\phi_{I^*I}^* > 0$ for keeping stable conditions. In addition, $\phi_{\beta s} = \phi_{\beta^*s^*}^* = 0$.

By using Eq.(23), let's examine the effects of an increase in respective governments' R&D subsidies on their own firms' investment. That of the home firm is given by

$$\frac{\partial I}{\partial s} = \frac{\phi_{Is}}{|\mathcal{D}|} \left(\frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)} \right)^3 \frac{(p^* - c^*)I}{I^{*\frac{1}{2}}} (A^*E^* - F^{*2}) \left\{ K^*Z - \left(\frac{n}{2\Delta} \right)^2 A^*F^2 \right\} > 0. \quad (24-1)$$

where $(A^*E^* - F^{*2}) > 0$ and $\{\cdot\} > 0$ if $\phi_{II}\phi_{I^*I^*}^* - \phi_{II^*}\phi_{I^*I}^* > 0$, and $K^* = \left[\frac{(2m^2 - n^2)}{2\Delta} A^* \right.$
 $\left. - \frac{(p^* - c^*)}{I^{*\frac{1}{2}}} \right]$, $Z = \left[\frac{(2m^2 - n^2)}{2\Delta} F^* - \frac{(p - c)}{I^{\frac{1}{2}}} E \right]$.

Rearranging Eq.(24-1), we get

$$\pi_{\beta\beta}\pi_{I^*I^*}^* - (\pi_{\beta I^*})^2 > 0. \quad (24-1.1)$$

Similarly, that of the foreign firm is

$$\frac{\partial I^*}{\partial s^*} = \frac{\phi_{I^*s^*}^*}{|\mathcal{D}|} \left(\frac{m(2m^2 - n^2)}{4(m^2 - n^2)\Delta} \right)^3 \frac{(p - c)I^*}{I^{\frac{1}{2}}} (AE - F^2) \left\{ KZ^* - \left(\frac{n}{2\Delta} \right)^2 AF^{*2} \right\} > 0. \quad (24-2)$$

where $(AE - F^2) > 0$ and $\{\cdot\} > 0$ if $\{\cdot\} > 0$ if $\phi_{II}\phi_{I^*I^*}^* - \phi_{II^*}\phi_{I^*I}^* > 0$, and $K = \left[\frac{(2m^2 - n^2)}{2\Delta} \right.$

$$\left. A - \frac{(p - c)}{I^{\frac{1}{2}}} \right]$$
, $Z^* = \left[\frac{(2m^2 - n^2)}{2\Delta} F^* - \frac{(p^* - c^*)}{I^{*\frac{1}{2}}} E^* \right]$.

Rearranging Eq.(24-2), we get

$$\pi_{\beta^*\beta^*}\pi_{II} - (\pi_{\beta^*I}^*)^2 > 0. \quad (24-2.1)$$

Lemma 6. *An increase in a government's R&D subsidy enhances its domestic firm's R&D investment under Bertrand competition.*

As same as in the case of Cournot competition, an increase in the government subsidy makes the total R&D investment increase irrespective of the magnitude of $\beta(\beta^*)$. This is because given the magnitude of $\beta(\beta^*)$ an increase in the total R&D investment always raises the production quality and reduces the production costs. However, the effects on the price in response to increase the total R&D investment rely on the ratio of the product R&D investment. In the case of $\partial p / \partial I > 0$, the amount of product R&D is larger than that of process

R&D, the effect of higher price on the profit is larger than that of output reduction caused by increase the production cost. In the case of $\partial p / \partial I < 0$, the reduction of production cost, the amount of process R&D is larger than that of product R&D, is large enough and the price decrease but the output increases, which leads to increase firm's profit.

The impact of subsidy increase, due to home government policy intervention, on the rival firm's R&D investment is shown as

$$\frac{\partial I^*}{\partial s} = \frac{-\pi_{Is}}{|\mathcal{D}|} \cdot \frac{m^4 n (2m^2 - n^2)^3}{128 \Delta^4 (m^2 - n^2)^2} (p^* - c^*) (p - c) (A^* E^* - F^{*2}) (AE - F^2) < 0. \quad (25-1)$$

Whereas the effect of foreign government policy on the rival firm's R&D investment is given by

$$\frac{\partial I^*}{\partial s^*} = \frac{-\pi_{I^*s^*}}{|\mathcal{D}|} \cdot \frac{m^4 n (2m^2 - n^2)^3}{128 \Delta^4 (m^2 - n^2)^2} (p - c) (p^* - c^*) (AE - F^2) (A^* E^* - F^{*2}) < 0. \quad (25-2)$$

We obtain Lemma 7.

Lemma 7. *An increase in a government's R&D subsidy decreases the rival firm's R&D investment under Bertrand competition.*

We can explain Lemma 7, by taking into account the effects of the government's R&D subsidy on its domestic firm's aggregate R&D investment shown in Eqs.(24-1.1) and (24-2.1) and the strategic nature in the second stage calculated in Eq.(B2) in Appendix B is strategic substitute ($\pi_{II} < 0$ and $\pi_{I^*I^*}^* < 0$). Thus, the government subsidy backs up its domestic firm's competitive advantage in international markets.

We are highly interested in the influence of governments' R&D subsidy on their domestic firms' investment fractions. The case of the home country is

$$\frac{\partial \beta}{\partial s} = \frac{-\pi_{Is}}{|\mathcal{D}|} \frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)} \frac{II^{*\frac{1}{2}}}{AA^*} (p^* - c^*) (A^* E^* - F^{*2}) (\phi_{II} \phi_{I^*I^*}^* - \phi_{II^*} \phi_{I^*I}^*) F \quad (26-1)$$

where we assume $(\phi_{II} \phi_{I^*I^*}^* - \phi_{II^*} \phi_{I^*I}^*) > 0$. The effect of a government's subsidy on its domestic firm's investment fractions depends on the magnitude of β . That is,

$$\frac{\partial \beta}{\partial s} \begin{cases} < 0 & \text{if } 0 < \beta < \frac{1}{17} \quad (n_b > n_c) \\ = 0 & \text{if } \beta = \frac{1}{17} \quad (n_b = n_c) \\ > 0 & \text{if } \frac{1}{17} < \beta < 1 \quad (n_b < n_c) \end{cases} \quad (26-2)$$

That of the foreign government is

$$\frac{\partial \beta^*}{\partial s^*} = \frac{-\pi_{I^*s^*}^*}{|\mathcal{D}|} \frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)} \frac{I^{\frac{1}{2}} I^*}{AA^*} (p - c)(AE - F^2)(\phi_{II}\phi_{I^*I^*}^* - \phi_{II^*}\phi_{I^*I}^*)F^* \quad (26-3)$$

where we assume $(\phi_{II}\phi_{I^*I^*}^* - \phi_{II^*}\phi_{I^*I}^*) > 0$. The sign relies on the degree of β . Thus, we have

$$\frac{\partial \beta^*}{\partial s^*} \begin{cases} < 0 & \text{if } 0 < \beta^* < \frac{1}{17} \quad (r_{\delta}^* > r_{\epsilon}^*) \\ = 0 & \text{if } \beta^* = \frac{1}{17} \quad (r_{\delta}^* = r_{\epsilon}^*) \\ > 0 & \text{if } \frac{1}{17} < \beta^* < 1 \quad (r_{\delta}^* < r_{\epsilon}^*) \end{cases} \quad (26-4)$$

Proposition 3. *An increase in a government's R&D subsidy causes its domestic firm's product R&D ratio to (1) rise if $1/17 < \beta(\beta^*) < 1$, (2) remain unchanged if $\beta(\beta^*) = 1/17$, and (3) fall if $0 < \beta(\beta^*) < 1/17$.*

This result is very intuitive. As same as the case of Cournot competition, the effects on the proportion of product R&D in response to an increase in the government subsidy depends on the R&D costs determined exogenously. We can explain the results in the same way of Proposition 1.

We now consider how the rival firm's investment fractions change when the government increases R&D subsidy.

When the home government increases its R&D subsidy to its domestic firm's R&D investment that effects on the foreign firm's R&D fraction is described as

$$\begin{aligned} \frac{\partial \beta}{\partial s^*} &= \frac{-\pi_{I^*s^*}^*}{|\mathcal{D}|} \frac{m^4 n (2m^2 - n^2)^3}{128 \Delta^4 (m^2 - n^2)^2} \frac{I^{*\frac{1}{2}}}{I^{\frac{1}{2}}} AKF \left\{ (A^* A^* - K^* F^{*2}) - (A^* Z^* - K^* F^{*2}) \right\} \\ &= 0. \end{aligned} \quad (26-5)$$

$$\text{where } K = \left[\frac{(2m^2 - n^2)}{2\Delta} A - \frac{(p - c)}{I^{\frac{1}{2}}} \right], \quad K^* = \left[\frac{(2m^2 - n^2)}{2\Delta} A^* - \frac{(p^* - c^*)}{I^{*\frac{1}{2}}} \right], \quad \text{and } Z^* =$$

$$\left[\frac{(2m^2 - n^2)}{2\Delta} F^{*2} - \frac{(p^* - c^*)}{I^{*\frac{1}{2}}} E^* \right]. \quad \text{That effect of the foreign government on the home firm's R\&D fraction is}$$

$$\begin{aligned} \frac{\partial \beta^*}{\partial s} &= \frac{-\pi_{Is}}{|\mathcal{D}|} \frac{m^4 n (2m^2 - n^2)^3}{128 \Delta^4 (m^2 - n^2)^2} \frac{I^{\frac{1}{2}}}{I^{*\frac{1}{2}}} AA^* F^* \left\{ \left(\frac{m^2 n}{4 \Delta^2} A^* F^2 - ZK^* \right) - \left(\frac{m^2 n}{4 \Delta^2} A^* F^2 - ZK^* \right) \right\} \\ &= 0. \end{aligned} \quad (26-6)$$

where $Z = \left[\frac{(2m^2 - n^2)}{2\Delta} F^2 - \frac{(p-c)}{I^{\frac{1}{2}}} E \right]$. Then, we obtain Lemma 10.

Lemma 8. *An increase in a government's R&D subsidy has no impact on the rival firm's R&D fraction under Bertrand competition.*

We can state that the government's R&D policy affects the rival firm's R&D activities only through the total investment I or I^* even if we endogenously determine the R&D fraction. This result is similar to the case of Cournot competition discussed in subsection 2.1.3. We can explain the results in the same way of Proposition 1.

2.2.4 The First-Stage: Optimal R&D Policy under Bertrand Competition

From the first-order conditions in the first-stage: $\partial W / \partial s = 0$ and $\partial W^* / \partial s^* = 0$, we get optimal R&D policies as follows.

$$\begin{aligned} \frac{\partial W}{\partial s} &= \frac{\partial I^*}{\partial s} \left[\frac{\partial \pi}{\partial p^*} \left(\frac{\partial p^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} + \frac{\partial p^*}{\partial c^*} \frac{\partial c^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right] \\ &\quad + \frac{\partial \beta^*}{\partial s} \left[\frac{\partial \pi}{\partial p^*} \left(\frac{\partial p^*}{\partial q^*} \frac{\partial q^*}{\partial \beta^*} + \frac{\partial p^*}{\partial c^*} \frac{\partial c^*}{\partial \beta^*} \right) + \frac{\partial \pi}{\partial q^*} \frac{\partial q^*}{\partial \beta^*} \right] - s \cdot \frac{\partial I}{\partial s} \end{aligned}$$

By evaluating at $s = 0$ and taking into account $\partial \beta^* / \partial s = 0$.

$$\begin{aligned} \left. \frac{\partial W}{\partial s} \right|_{s=0} &= \frac{\partial I^*}{\partial s} \left[\frac{\partial \pi}{\partial p^*} \left(\frac{\partial p^*}{\partial q^*} \frac{\partial q^*}{\partial I^*} + \frac{\partial p^*}{\partial c^*} \frac{\partial c^*}{\partial I^*} \right) + \frac{\partial \pi}{\partial q^*} \frac{\partial q^*}{\partial I^*} \right] \\ &= -\frac{\partial I^*}{\partial s} \frac{m^2 n (p-c)}{2(m^2 - n^2) I^{*\frac{1}{2}} \Delta} \left(\beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right) > 0 \end{aligned} \quad (27-1)$$

That of the foreign government is

$$\begin{aligned} \frac{\partial W^*}{\partial s^*} &= \frac{\partial I}{\partial s^*} \left[\frac{\partial \pi}{\partial p} \left(\frac{\partial p}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial p}{\partial c} \frac{\partial c}{\partial I} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial I} \right] \\ &\quad + \frac{\partial \beta}{\partial s^*} \left[\frac{\partial \pi}{\partial p} \left(\frac{\partial p}{\partial q} \frac{\partial q}{\partial \beta} + \frac{\partial p}{\partial c} \frac{\partial c}{\partial \beta} \right) + \frac{\partial \pi}{\partial q} \frac{\partial q}{\partial \beta} \right] - s \cdot \frac{\partial I^*}{\partial s^*} \end{aligned}$$

By evaluating at $s = 0$ and taking into account $\partial \beta^* / \partial s = 0$.

$$\begin{aligned}
\left. \frac{\partial W^*}{\partial s^*} \right|_{s^*=0} &= \frac{\partial I}{\partial s^*} \left[\frac{\partial \pi^*}{\partial p} \left(\frac{\partial p}{\partial q} \frac{\partial q}{\partial I} + \frac{\partial p}{\partial c} \frac{\partial c}{\partial I} \right) + \frac{\partial \pi^*}{\partial q} \frac{\partial q}{\partial I} \right] \\
&= -\frac{\partial I}{\partial s^*} \frac{m^2 n (p^* - c^*)}{2(m^2 - n^2) I^{\frac{1}{2}} \Delta} \left(\beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right) > 0
\end{aligned} \tag{27-2}$$

From Eqs.(27-1) and (27-2), we can obtain the main results.

Proposition 4. *Both governments have incentives to subsidize their domestic firms' R&D investments.*

The results are qualitatively the same as per the case of Cournot competition. Both governments determine their optimal R&D policies by considering the effect of the subsidy on rival firm's R&D investment denoted by $\partial I^* / \partial s < 0$ ($\partial I / \partial s^* < 0$), and that on domestic firm's price via impacts of domestic firm's quality and cost represented in the square brackets of the denominator in Eq.(27-1) and Eq.(27-2), respectively.

2.3 The Effects of R&D Subsidies on Product Quality and Production Cost

We have already obtained the effects of $s(s^*)$ on $I(I^*)$ and $\beta(\beta^*)$ in the above analysis. We are further interested in the direct influence of subsidies on product qualities. We already know that the results under both competition modes are qualitatively identical and also those are maintained between the foreign and the home firms. To simplify the analysis, we only show the case of the home firm.

$$\frac{\partial q}{\partial s} = \frac{\partial q}{\partial I} \frac{\partial I}{\partial s} + \frac{\partial q}{\partial \beta} \frac{\partial \beta}{\partial s} \geq 0 \tag{28}$$

This implies that an increase in the government's subsidy affects its own firm's quality through two directions: one is aggregate R&D investment and the other is the R&D fraction for product R&D. We should consider this effect more precisely because the impact of the subsidy on the product R&D fraction ($\partial \beta / \partial s$) depends on the degree of β , while $\partial I / \partial s$ is always positive as proved in Eqs.(11-1, -2) and (24-1, -2). Thus, we take into account the following cases.

Case 1: $\partial \beta / \partial s < 0$ if $0 < \beta < 1/17$.

In this case, an increase in the government subsidy reduces its firm's product R&D fraction. We have to consider the following three subcases.

Case 1-1: $\beta(\partial I / \partial s) > I | \partial \beta / \partial s | \Leftrightarrow \partial q / \partial s > 0$

This binary relation shows that the increment of the investment into product R&D through change in aggregate investment is larger than the reduction of that through the change in the product R&D fraction. In this case, an increase in the government subsidy improves product quality.

Case 1-2: $\beta(\partial I / \partial s) < I | \partial \beta / \partial s | \Leftrightarrow \partial q / \partial s < 0$.

This binary relation shows that the increment of the investment into product R&D through aggregate investment is smaller than the reduction of that through the change in the product R&D fraction. Consequently, an increase in the government subsidy impacts negatively on product quality.

Case 1-3: $\beta(\partial I / \partial s) = I |\partial \beta / \partial s| \Leftrightarrow \partial q / \partial s = 0$.

This case posits that the increase in the investment into product R&D through the change in total investment is equal to the reduction of that through the change in the product R&D fraction. As a result, there is no change in the amount of investment into product R&D. Therefore, an increase in the government subsidy leaves product quality remains unchanged.

Case 2: $\partial \beta / \partial s > 0$, if $1/17 < \beta < 1$. The following always holds.

$$\beta \frac{\partial I}{\partial s} + I \frac{\partial \beta}{\partial s} > 0 \Leftrightarrow \frac{\partial q}{\partial s} > 0.$$

In this case, an increase in the government subsidy always expands the investment into product R&D. Consequently, an increase in the government subsidy improves firm's product quality.

Case 3: $\partial \beta / \partial s = 0$, if $\beta = 1/17$.

$$\beta \frac{\partial I}{\partial s} > 0 \Leftrightarrow \frac{\partial q}{\partial s} > 0.$$

In this case, an increase in the government subsidy affects on the quality through only the change in aggregate investment because the subsidy does not alter the product R&D function. An increase in the subsidy always increases the aggregate investment. Therefore, the subsidy improves product quality.

Proposition 5. *An increase in government's subsidy causes its domestic firm's product quality to improve if $\beta(\partial I / \partial s) > I |\partial \beta / \partial s|$, conversely diminish if $\beta(\partial I / \partial s) < I |\partial \beta / \partial s|$, and remain unchanged if $\beta(\partial I / \partial s) = I |\partial \beta / \partial s|$.*

Proposition 5 expresses that product quality degrades if the absolute value of the change in the R&D investment fraction is larger than that of aggregate investment, while the amount of product R&D expands, the total investment decreases, which in turn leads to a decrease in the amount of investment into product R&D. Therefore, we can state that an increase in the government subsidy improves product quality when the amount of the investment into product R&D increases through the increment aggregate investment rather than the expansion of the product R&D fraction

As well as considering impacts on product quality, we can obtain production cost effects. By differentiating cost function Eq.(4) with respect to s , we have

$$\frac{\partial c}{\partial s} = \frac{\partial c}{\partial I} \frac{\partial I}{\partial s} + \frac{\partial c}{\partial \beta} \frac{\partial \beta}{\partial s} \geq 0. \quad (29)$$

As per the case with quality, we can reach the following results.

Proposition 6. *An increase in government's R&D subsidy causes its domestic firm's production cost to rise if $I|\partial\beta/\partial s| > (1-\beta)\partial I/\partial s$, reduce if $I|\partial\beta/\partial s| < (1-\beta)\partial I/\partial s$, and remain unchanged if $I|\partial\beta/\partial s| = (1-\beta)\partial I/\partial s$.*

In response to an increase in the government R&D subsidy, and in accordance with the case of quality change described above, production cost decreases when the investment into process R&D through the change in total R&D investment is larger than that through the change in R&D investment fractions.

The corresponding effects on the rival firm's product quality and production cost are easily obtained by applying the forgoing logic. However, the rival government's subsidy has zero influence on the firm's R&D investment fractions, as proved in Eq.(15-1, -2) and (26-5, 26-6) so that we only take into account the effect on total R&D investment. The induced results show that an increase in the rival government's R&D subsidy causes the firm's product quality to deteriorate and the firm's production costs to rise.

3 Conclusions and Policy Implications

This study explores governments' optimal research and development (R&D) policies when firms invest in both process R&D and product R&D simultaneously. We develop a model based on the third-country trade model in an international duopoly and construct a three-stage game. In the analysis, we considered that firms endogenously choose the total amount of R&D investments and the ratio of R&D investment in process and product R&D. The analysis revealed that governments have incentives to subsidize their domestic firms' R&D activities. In addition, we also clarify the mechanism whereby government policy affects the rival firm's R&D investments. The analysis suggests that such policy affects the rival firm's R&D effort only through the firm's R&D investment, not through the R&D fractions. This is because these fractions are closely related to R&D costs, which are determined exogenously. This result holds under any general quality and cost functions and irrespective of the mode of competition.

In the analysis we assume specific quality and cost functions. Employing more general functions, we can still obtain qualitatively similar results, but the analysis becomes more complex. Furthermore, we can consider the more extensive four stage case where, for instance, in the third or fourth stage total R&D investment and R&D fractions are determined respectively. In so doing, we have to account for specific functions for total R&D investment in terms of the R&D fractions or the R&D fractions with respect to total R&D investment. However, it particularly arduous to specify these functions.

Various improvements to this modeling endeavor could be suggested. For example, although we assume that R&D policy is implemented over the total R&D investment, as indicated in the study by Yin and Zuscovitch (1998), we must clearly distinguish the R&D investments or expenditures with respect to process and product R&D because total expenditures (investments) on R&D conceal the qualitative differences among R&D activities. Accordingly, model variants may yield different outcomes.

As a policy implication we can refer to the following. Whether or not the government

subsidy stimulates firm's R&D investments has long been controversial issue, i.e., crowding-in and -out effects. There are no crowding-out effects in this analysis because in the setting the government subsidy always makes firm's total R&D investment increase. However, some controversial issues exist. For example, it is said that the amount of quality-improving investment is relatively smaller than that of cost-reducing process R&D in Japan (NISTEP 2019). From this paper, if the grounds on which the one of the reasons for this fact is based on the high cost of product R&D, first government should directly subsidize the product R&D not the total R&D as long as possible, second the government should subsidize more factors used in R&D such as skilled labors and provide the environment in which the firms can easily employ the factors of R&D. In fact, NISTEP 2019 shows that firms consider employment, social security, tax system, and competitive environment as innovation costs. Therefore, it might be necessary for the government to reform the laws and the regulations related to the firm's innovation.

Appendix A

In this section we show the second- and cross-effects in the second stage of Cournot competition. The profit function of the home firm is defined as $\pi(I, I^*, \beta, \beta^*) = \pi(x(\cdot), x^*(q^*(I^*, \beta^*), q(I, \beta), c^*(I^*, \beta^*), c(I, \beta)), q(I, \beta), c(I, \beta), I, \beta)$. That of the foreign firm is stated as $\pi^*(I^*, I, \beta^*, \beta) = \pi^*(x^*(\cdot), x(q(I, \beta), q^*(I^*, \beta^*), c(I, \beta), c^*(I^*, \beta^*)), q^*(I^*, \beta^*), c^*(I^*, \beta^*), I^*, \beta^*)$.

The first-order conditions of $I(I^*)$ and $\beta(\beta^*)$ are obtained in eq.(9-1)(eq.9-2) and eq.(9-3)(eq.9-4). From eq.(9-3), we obtain the following relationship between β and the R&D costs: $r_b(r_b^*)$ and $r_c(r_c^*)$.

$$\frac{m}{\Delta} \frac{x}{I^{\frac{1}{2}}} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} = r_b - r_c \quad (C1)$$

We can get the following relationship.

$$\begin{aligned}
 \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} &> 0 \Leftrightarrow 0 < \beta < \frac{1}{17} \text{ if } r_b - r_c > 0 \\
 \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} &= 0 \Leftrightarrow \beta = \frac{1}{17} \text{ if } r_b = r_c \\
 \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} &< 0 \Leftrightarrow \frac{1}{17} < \beta < 1 \text{ if } r_b - r_c < 0
 \end{aligned} \tag{C1.1}$$

where we assume that $\beta \in (0,1)$.

Following is the proof of the relationship between the R&D fraction and the R&D costs, presented in eq.(C1) and eq.(C1.1). If the right hand side of eq.(C1) ($r_b - r_c$) is constant, it must be unchanged when the variables x and I change. To show this rearranging the eq.(C1) as follows.

$$\frac{x}{I^{\frac{1}{2}}} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} = (r_b - r_c) \frac{\Delta}{m} \tag{C1.2}$$

In this equation, the right hand side is constant and the left hand side is changeable when such as s and s^* change. Let us consider the effects of change in s^* on $x/I^{\frac{1}{2}}$. The effect can be written by

$$\begin{aligned}
 \frac{\partial}{\partial s^*} \left(\frac{x}{I^{\frac{1}{2}}} \right) &= \frac{\partial}{\partial s^*} \left(\log x - \frac{1}{2} \log I \right) \\
 &= \frac{1}{x} \frac{\partial x}{\partial s^*} - \frac{1}{2I} \frac{\partial I}{\partial s^*} \\
 &= 0.
 \end{aligned} \tag{C1.3}$$

Eq.(C1.3) shows that an increase in the subsidy of the foreign country has no effect on $x/I^{\frac{1}{2}}$. In other words, the magnitude of changes in denominator and numerator are the same against the change in the rival government's subsidy. This result is consistent with eq.(15-1) and eq.(15-2), which represent that the effect of an increase in the subsidy of rival government on its own R&D fraction is zero. That is, the rival government's subsidy does not affect on the R&D fraction. On the other hand, when the subsidy of its own government s changes, all the variables in the left hand side change as the equation holds. Thus, we can state that R&D fraction is tied with its R&D costs, which implies that the R&D fraction is only determined in domestic factors.

Similarly, these conditions of foreing firm are specified by $*$ hold. We assume that the second-order conditions are negative to satisfy the stability condition.

$$\frac{\partial^2 \pi}{\partial I^2} = \frac{m^2}{2\Delta I} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} \left[\frac{m}{\Delta} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} - \frac{x}{I^{\frac{1}{2}}} \right] < 0 \quad (C2a)$$

$$\frac{\partial^2 \pi^*}{\partial I^{*2}} = \frac{m^2}{2\Delta I^*} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} \left[\frac{m}{\Delta} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} - \frac{x^*}{I^{*\frac{1}{2}}} \right] < 0 \quad (C2b)$$

$$\frac{\partial^2 \pi}{\partial \beta^2} = \frac{m^2 I}{2\Delta} \left[\frac{m}{\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\}^2 - \left\{ \left(\frac{1}{\beta} \right)^{\frac{3}{2}} + 4 \left(\frac{1}{1-\beta} \right)^{\frac{3}{2}} \right\} \frac{x}{I^{\frac{1}{2}}} \right] < 0. \quad (C3a)$$

$$\frac{\partial^2 \pi^*}{\partial \beta^{*2}} = \frac{m^2 I^*}{2\Delta} \left[\frac{m}{\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\}^2 - \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{3}{2}} + 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{3}{2}} \right\} \frac{x^*}{I^{*\frac{1}{2}}} \right] < 0 \quad (C3b)$$

The cross-effects of I and I^* are

$$\frac{\partial^2 \pi}{\partial I^* \partial I} = \frac{\partial^2 \pi^*}{\partial I \partial I^*} = \frac{-nm^2}{4\Delta^2 I^{\frac{1}{2}} I^{*\frac{1}{2}}} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} < 0. \quad (C4)$$

The cross-effects of β and β^* are

$$\frac{\partial^2 \pi}{\partial \beta^* \partial \beta} = \frac{\partial^2 \pi^*}{\partial \beta \partial \beta^*} = \frac{-nm^2 I^{\frac{1}{2}} I^{*\frac{1}{2}}}{4\Delta^2} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\}. \quad (C5)$$

Unlike the case of the total investments I and I^* shown by eq.(C4), we cannot clearly determine the sign of these conditions. Those depend on the magnitude of β and β^* . The cross effects of I and β are derived as

$$\frac{\partial^2 \pi}{\partial \beta \partial I} = \frac{\partial^2 \pi}{\partial I \partial \beta} = \frac{m^2}{2\Delta} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} \left[\frac{m}{\Delta} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} - \frac{x}{I^{\frac{1}{2}}} \right]. \quad (C6)$$

We also cannot certainly determine the sign. It relies on the fraction of β . The sign the cross-effects of I^* and β^* are

$$\frac{\partial^2 \pi^*}{\partial \beta^* \partial I^*} = \frac{\partial^2 \pi^*}{\partial I^* \partial \beta^*} = \frac{m^2}{2\Delta} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\} \left[\frac{m}{\Delta} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} - \frac{x^*}{I^{*\frac{1}{2}}} \right],$$

(C7)

where the sign depends on the magnitude of β^* . The cross-effects of I and β^* are determined as

$$\frac{\partial^2 \pi}{\partial \beta^* \partial I} = \frac{\partial^2 \pi^*}{\partial I \partial \beta^*} = \frac{-nm^2 I^{*\frac{1}{2}}}{4\Delta^2 I^{\frac{1}{2}}} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\}, \quad (C8)$$

where the sign depends on the magnitude of β^* . The cross effects of I^* and β are

$$\frac{\partial^2 \pi}{\partial I^* \partial \beta} = \frac{\partial^2 \pi^*}{\partial \beta \partial I^*} = \frac{-nm^2 I^{\frac{1}{2}}}{4\Delta^2 I^{*\frac{1}{2}}} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\}, \quad (C9)$$

where the sign depends on the degree of β .

Appendix A2

The proof of existence for an equilibrium is to show that the matrix D is negative definite. The first principal minor of D is $|\pi_{II}| < 0$, the second principal minor of D is $\begin{vmatrix} \pi_{II} & \pi_{II^*} \\ \pi_{I^*I}^* & \pi_{I^*I^*}^* \end{vmatrix} > 0$ if

and only if $\text{sgn}(r_0 - r_c) = \text{sgn} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta} \right)^{\frac{1}{2}} \right\}$, and $\text{sgn}(r_0^* - r_c^*) = \text{sgn} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1-\beta^*} \right)^{\frac{1}{2}} \right\}$, the third principal minor of D is

$\begin{vmatrix} \pi_{II} & \pi_{II^*} & \pi_{I\beta} \\ \pi_{I^*I}^* & \pi_{I^*I^*}^* & \pi_{I^*\beta}^* \\ \pi_{I\beta} & \pi_{\beta I^*} & \pi_{\beta\beta} \end{vmatrix} < 0$, and the forth principal minor of D is

$\begin{vmatrix} \pi_{II} & \pi_{II^*} & \pi_{I\beta} & \pi_{I\beta^*} \\ \pi_{I^*I}^* & \pi_{I^*I^*}^* & \pi_{I^*\beta}^* & \pi_{I^*\beta^*}^* \\ \pi_{I\beta} & \pi_{\beta I^*} & \pi_{\beta\beta} & \pi_{\beta\beta^*} \\ \pi_{\beta^*I} & \pi_{\beta^*I^*} & \pi_{\beta^*\beta} & \pi_{\beta^*\beta^*} \end{vmatrix} > 0$ if and only if $\pi_{II}\pi_{I^*I^*}^* - \pi_{II^*}\pi_{I^*I}^* > 0$.

Appendix B

In this section we show the second- and cross- effects in the second stage of Bertrand competition. The profit function of the home firm is defined as $\pi(I, I^*, \beta, \beta^*) = \pi(p(\cdot), p^*(q^*(I, \beta), q(I, \beta), c^*(I^*, \beta^*), c(I, \beta)), q(I, \beta), q^*(I^*, \beta^*), c(I, \beta), I, \beta, s)$. That of the for-

each firm is $\pi^*(I^*, I, \beta^*, \beta) = \pi^*(p^*(\cdot), p(q(I, \beta), q^*(I^*, \beta^*), c(I, \beta), c^*(I^*, \beta^*)), q(I, \beta), q^*(I^*, \beta^*), c^*(I^*, \beta^*), I^*, \beta^*, s^*)$.

The first-order conditions with respect of $I(I^*)$: $\partial\pi/\partial I = 0$ and $\partial\pi^*/\partial I^* = 0$ are given in eq.(21-1) and eq.(21-2). Those conditions in terms of $\beta(\beta^*)$: $\partial\pi/\partial\beta = 0$ and $\partial\pi^*/\partial\beta^* = 0$ are in eq.(22-1) and eq.(22-2). The second-order conditions in terms of β and β^* are

$$\frac{\partial^2\pi}{\partial\beta^2} = \frac{m(2m^2 - n^2)I}{4\Delta(m^2 - n^2)} \left[\begin{aligned} &\frac{(2m^2 - n^2)}{2\Delta} \left\{ \left(\frac{1}{\beta}\right)^{\frac{1}{2}} - 4\left(\frac{1}{1-\beta}\right)^{\frac{1}{2}} \right\} \\ &- \frac{(p-c)}{I} \left\{ \left(\frac{1}{\beta^3}\right)^{\frac{1}{2}} + 4\left(\frac{1}{(1-\beta)^3}\right)^{\frac{1}{2}} \right\} \end{aligned} \right] < 0, \quad (B1a)$$

$$\frac{\partial^2\pi^*}{\partial\beta^{*2}} = \frac{m(2m^2 - n^2)I^*}{4\Delta(m^2 - n^2)} \left[\begin{aligned} &\frac{(2m^2 - n^2)}{2\Delta} \left\{ \left(\frac{1}{\beta^*}\right)^{\frac{1}{2}} - 4\left(\frac{1}{1-\beta^*}\right)^{\frac{1}{2}} \right\} \\ &- \frac{(p^* - c^*)}{I^*} \left\{ \left(\frac{1}{\beta^{*3}}\right)^{\frac{1}{2}} + 4\left(\frac{1}{(1-\beta^*)^3}\right)^{\frac{1}{2}} \right\} \end{aligned} \right] < 0, \quad (B1b)$$

$$\frac{\partial^2\pi}{\partial I^2} = \frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)I} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} \left[\frac{(2m^2 - n^2)}{2\Delta} \left\{ \beta^{\frac{1}{2}} + 4(1-\beta)^{\frac{1}{2}} \right\} - \frac{(p-c)}{I^{\frac{1}{2}}} \right] < 0, \quad (B1c)$$

$$\frac{\partial^2\pi^*}{\partial I^{*2}} = \frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)I^*} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} \left[\frac{(2m^2 - n^2)}{2\Delta} \left\{ \beta^{*\frac{1}{2}} + 4(1-\beta^*)^{\frac{1}{2}} \right\} - \frac{(p^* - c^*)}{I^{*\frac{1}{2}}} \right] < 0, \quad (B1d)$$

where we use $(r_b - r_c) = \frac{m(2m^2 - n^2)}{2\Delta(m^2 - n^2)} \frac{p-c}{I^{\frac{1}{2}}} \left\{ \left(\frac{1}{\beta}\right)^{\frac{1}{2}} - 4\left(\frac{1}{1-\beta}\right)^{\frac{1}{2}} \right\}$ from eq.(28-1) and

$$(r_{\delta}^* - r_{\epsilon}^*) = \frac{m(2m^2 - n^2)}{2\Delta(m^2 - n^2)} \frac{p^* - c^*}{I^{*\frac{1}{2}}} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta^*} \right)^{\frac{1}{2}} \right\} \text{ from eq.(28-2).}$$

The cross-effects of I and I^* are

$$\frac{\partial^2 \pi}{\partial I^* \partial I} = \frac{\partial^2 \pi^*}{\partial I \partial I^*} = \frac{-m^2 n (2m^2 - n^2)}{8\Delta^2 (m^2 - n^2) I^{\frac{1}{2}} I^{*\frac{1}{2}}} \left\{ \beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right\} \left\{ \beta^{*\frac{1}{2}} + 4(1 - \beta^*)^{\frac{1}{2}} \right\} < 0. \quad (\text{B2})$$

The cross-effects of β and β^* are respectively,

$$\begin{aligned} \frac{\partial^2 \pi}{\partial \beta^* \partial \beta} &= \frac{\partial^2 \pi^*}{\partial \beta \partial \beta^*} \\ &= \frac{-m^2 n (2m^2 - n^2) I^{\frac{1}{2}} I^{*\frac{1}{2}}}{8\Delta^2 (m^2 - n^2)} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta} \right)^{\frac{1}{2}} \right\} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta^*} \right)^{\frac{1}{2}} \right\}. \end{aligned} \quad (\text{B3})$$

The cross-effects between β and I are

$$\begin{aligned} \frac{\partial^2 \pi}{\partial \beta \partial I} &= \frac{\partial^2 \pi}{\partial I \partial \beta} \\ &= \frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta} \right)^{\frac{1}{2}} \right\} \\ &\quad \left[\frac{(2m^2 - n^2)}{2\Delta} \left\{ \beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right\} - \frac{(p - c)}{I^{\frac{1}{2}}} \right]. \end{aligned} \quad (\text{B4})$$

The cross-effects between β^* and I^* are

$$\begin{aligned} \frac{\partial^2 \pi^*}{\partial I^* \partial \beta^*} &= \frac{\partial^2 \pi^*}{\partial \beta^* \partial I^*} = \frac{m(2m^2 - n^2)}{4\Delta(m^2 - n^2)} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta^*} \right)^{\frac{1}{2}} \right\} \\ &\quad \left[\frac{(2m^2 - n^2)}{2\Delta} \left\{ \beta^{*\frac{1}{2}} + 4(1 - \beta^*)^{\frac{1}{2}} \right\} - \frac{(p^* - c^*)}{I^{*\frac{1}{2}}} \right]. \end{aligned} \quad (\text{B5})$$

The cross-effects between I and β^* are

$$\frac{\partial^2 \pi}{\partial \beta^* \partial I} = \frac{\partial^2 \pi^*}{\partial I \partial \beta^*} = \frac{-m^2 n (2m^2 - n^2) I^{\frac{1}{2}}}{8\Delta^2 (m^2 - n^2) I^{\frac{1}{2}}} \left\{ \beta^{\frac{1}{2}} + 4(1 - \beta)^{\frac{1}{2}} \right\} \left\{ \left(\frac{1}{\beta^*} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta^*} \right)^{\frac{1}{2}} \right\}. \quad (\text{B6})$$

The cross-effects between I^* and β are

$$\frac{\partial^2 \pi}{\partial I^* \partial \beta} = \frac{\partial^2 \pi^*}{\partial \beta \partial I^*} = \frac{-m^2 n (m^2 - n^2) I^{\frac{1}{2}}}{8\Delta^2 (m^2 - n^2) I^{\frac{1}{2}}} \left\{ \left(\frac{1}{\beta} \right)^{\frac{1}{2}} - 4 \left(\frac{1}{1 - \beta} \right)^{\frac{1}{2}} \right\} \left\{ \beta^{*\frac{1}{2}} + 4(1 - \beta^*)^{\frac{1}{2}} \right\}. \quad (\text{B7})$$

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