International Portfolios, Trade Imbalances and the Exchange Rate

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Abstract

Establishing a two-country, two-good, two-money, two-bond continuous-time dynamic portfolio model of international investor, this paper firstly demonstrates that since monies as well as bonds issued in different countries are not perfectly substitutive for each other none of them should be generally ignored in analyzing the exchange rate determination. Then, it presents the general equilibrium conditions of international asset markets considering goods trade imbalances and derives the equilibrium exchange rate. Finally it investigates the effects of large and marginal changes in money supplies, bond supplies and goods trade imbalances on the exchange rate.

Introduction

As the volume of international financial trade has increased tremendously in comparison with the amount of international goods trade, the effect of financial trade on the exchange rate has been recognized as more dominantly significant than that of goods trade. With regard to this point P. Kuori (1988) has said:

Marketable world wealth can be counted in trillions of dollars ; even a small shift in asset preferences can lead to a capital transfer that is very large relative to what can be effected through the current account.

Thus, a lot of international portfolio models have been exploited to examine the roles of portfolio rules of international investors in exchange rate determination, and they have greatly contributed to developing the exchange rate theory¹. However, these models have overlooked some characteristic features of a monetary economy observed in the present world.

The first significant feature overlooked in many existing international portfolio models is that nowadays both monies and bonds issued in many countries are internationally traded and thus investors can freely hold all of them in their portfolios. Therefore, it seems very curious to assume that international investors hold only either monies or bonds in their portfolios.

In practice, many investors in the main countries such as the G. 7 that have given the great

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effects to the world economy participate in the international asset markets and hold both monies and bonds issued in these countries. Since the exchange rate and its expectations affect the substitutive relationships among monies and bonds it is necessary for any models examining the exchange rate to consider monies and bonds issued in different countries².

The second essential feature of a monetary economy overlooked in almost of all existing models is that investors in a monetary economy usually evaluate their wealth expressed in terms of the money of the country where they permanently live. For example, Japanese evaluate their wealth measured in yen, Americans their wealth expressed in dollars, Germans their wealth assessed in marks and so on. Though many existing models have ignored this feature, this is crucial from both practical and theoretical points of view.

As has been indicated by J. Flemming, S. Turnovsky and M. Kemp (1977), a choice of numeraire must be careful in a model considering uncertainty. This relates to the Siegel's paradox that some results of models including random factors depend on the choice of numeraire and become undeterminable. One way to solve this difficulty is to take account of some actual behaviors of economic agents in the real world. Thus, considering the feature of a monetary economy that international investors evaluate their wealth in terms of money in their country has essential meanings.

The last significant feature of a monetary economy overlooked in lots of existing models is that the adjustment speed of the financial markets is much faster than that of the real markets. Adjustments in real decisions such as production and international goods trade require much longer time than those in financial decisions do. Moreover, data on financial variables such as the exchange rates and the asset prices are available more frequently than those on real variables such as outputs and trade imbalances. J. Frenkel (1988) has also indicated :

We do not expect any more flexible exchange rates to eliminate current account imbalances. Data on exchange rates and interest rates are available in a much greater frequency than data on national income or on the current account.

The difference between adjustment speeds in the financial markets and in the good markets are reflected in behaviors of international trades of assets and goods. That is, while the financial asset markets attain equilibrium at each instant, the current account imbalances may last for a considerably long time unless they have a crucial effect on the world economy³. Thus, in such circumstances the asset prices including the exchange rates are determined so as to equilibrate the international asset markets at each instant given the real variables such as incomes, current account imbalances, the good prices and so on which are merely adjusted in the long run.

In what follows, I establish a more generalized model of the exchange rate determination which explicitly considers the essential features of a monetary economy mentioned above. The model also modifies the assumptions of purchasing power parity (PPP)⁴, interest rate parity (IRP) and identical investors⁵. However, I do not adopt the cash-in-advance assumption proposed by R.

Lucas Jr. (1982) in this paper.

Nowadays residents of many countries can buy not only domestic goods but also imported goods with their domestic money in their country. Moreover, whenever they need foreign monies, they can easily get them through exchanging their domestic money in the international money (or exchange) market. They need not prepare in advance any foreign monies even in buying foreign goods within their country. Hence, the cash-in-advance assumption seems to be rather curious from an actual point of view.

In addition to the above point, the cash-in-advance assumption is also restrictive from a theoretical point of view. Though the cash-in-advance models have included both monies and bonds issued in different countries, they have excluded any money holdings stemming from speculative motives. This seems to be a serious flaw in discussing the exchange rate determination.

This paper is organized as follows. Section II presents a basic model of optimal consumption-portfolios of international investors. For investigating effectively the different roles of monies and bonds I establish a two-country, two-good, two-money, two-bond model where both monies and bonds are issued in two countries, respectively.

In Section III, I examine the effects of changes in investors' expectations on their optimal portfolio rules and modify some results which are derived in the models considering only either monies or bonds. I demonstrate that it is essentially significant to consider that investors evaluate their wealth measured in their domestic money. Furthermore, I show that monies and bonds supplied in different countries are not perfectly gross substitutive for each other respectively and that both PPP and IRP do not generally hold.

Section IV examines the exchange rate and the bond prices which equilibrate the international asset markets⁶. In addition to money holdings of investors this section also considers the firms' money holdings stemming from imbalances of goods trade as a part of international money holdings. Moreover, it takes into consideration the difference between adjustment speeds of the asset markets and of the good markets. Then it analyzes the effects of changes in amounts of trade imbalances, money supplies and bond supplies in two countries on the exchange rate. As a result, it is shown that while the signs of effects of fairly large changes in these amounts on the exchange rate are definitely judged respectively, those of marginal changes in these amounts on the exchange rate become ambiguous as the volume of international asset trade grows significantly. Finally section V presents some concluding remarks and discusses some directions for extending the model.

1. The Basic Model and Assumptions

In this section I first establish a generalized model of consumption-portfolio choices of individuals who participate in both the international goods and asset markets, and then I discuss

some features of their optimal consumption-portfolio rules.

As its analytical methodology of individuals' international portfolios this paper adopts the same continuous-time framework as that exploited by R. Merton (1975) and extended by D. Breeden (1979), J. Macedo (1983), R. Stulz (1984), A. Fraga (1986) et al. However, it extends their models so as to include two goods, two monies and two bonds supplied by home and foreign countries. Therefore, the goods prices, the bond prices and the exchange rate are explicitly introduced into the model.

It is here assumed that the goods prices, the bond prices and the exchange rate are all uncertain to individuals. Furthermore, it is also supposed that the expectations of these uncertain variables and the utility functions of individuals in the home country are both different from those of residents in the foreign country.

A. Optimal Consumption-Portfolio Rule of the Home Investor

In this subsection, consider an average individual in the home country consuming two goods produced in two countries and holding a variable portfolio of two monies and two bonds issued in two countries. He takes all the market prices, P_i , P_b , P_b^* and E as given, where P_i , P_b , P_b^* and E are the i -th good price ($i = 1, 2$), the home bond price, the foreign bond price and the foreign money price expressed in terms of home money, respectively.

It is assumed that these prices are expected by the Itô type of continuous stochastic processes, i.e.,

$$\begin{aligned} dP_i/P_i &= p_i (P_i, P_b, P_b^*, E) dt + \sigma_{pi} (P_i, P_b, P_b^*, E) dz_{pi}, \quad i = 1, 2, \\ dP_b/P_b &= p_b (P_i, P_b, P_b^*, E) dt + \sigma_{pb} (P_i, P_b, P_b^*, E) dz_{pb}, \\ dP_b^*/P_b^* &= p_b^* (P_i, P_b, P_b^*, E) dt + \sigma_{pb^*} (P_i, P_b, P_b^*, E) dz_{pb^*}, \\ dE/E &= e (P_i, P_b, P_b^*, E) dt + \sigma_e (P_i, P_b, P_b^*, E) dz_e, \end{aligned} \quad (1)$$

where p_i ($i = 1, 2$), p_b , p_b^* and e are the instantaneous conditional expected mean proportional changes per unit of time of P_i ($i = 1, 2$), P_b , P_b^* and E , σ_{pi} , σ_{pb} , σ_{pb^*} and σ_e are the instantaneous conditional standard deviations of P_i ($i = 1, 2$), P_b , P_b^* and E , and dz_{pi} ($i = 1, 2$), dz_{pb} , dz_{pb^*} and dz_e are the Wiener processes, respectively. It is also assumed that the instantaneous nominal interest rates of home and foreign monies are both zero.

Under the conditions given by (1), ignoring bequests and the discounting of future utility, the home investor maximizes his expected utility from time 0 to time T defined as⁷

$$U = E_0 \int_0^T (1/\gamma) \{x_1(t)^{\alpha_1} x_2(t)^{\alpha_2}\}^\gamma dt = E_0 \int_0^T \tilde{u}(t) dt \quad (2)$$

where E_0 denotes expectation conditional upon the information available at time 0, $x_i(t)$ is the amount of good i consumed at time t , α_i is the share of goods i in nominal consumption expenditure measured by home money ($i = 1, 2$), $\alpha_1 + \alpha_2 = 1$, and $\tilde{u}$ is the instantaneous utility function with constant relative risk aversion $(1 - \gamma)$, that is,

$$\tilde{u} = (1/\gamma) \prod x_i^{\alpha_i \gamma}, \quad i = 1, 2. \quad (3)$$

When the average home investor has a Cobb-Douglas type of instantaneous utility function

given by (3), his real consumption expenditure X_c deflated by the purchasing power of home money can be written as

$$X_c = QE_N, \quad (4)$$

where $E_N = (P_1 x_1 + P_2 x_2)$ is the investor's nominal consumption expenditure expressed in terms of home money, and $Q = 1/P_1^{a_1} P_2^{a_2}$ is the purchasing power of home money over two goods.

Similarly, the home investor's real wealth W is defined as the purchasing power of his nominal wealth consisting of two monies and two bonds issued in two countries,

$$W = Q \{M_h + P_b B_h + E(M_h^* + P_b^* B_h^*)\}, \quad (5)$$

where M_h and M_h^* are the amounts of home and foreign monies held by the home investor, respectively, and B_h and B_h^* are the quantities of home and foreign bonds held by the home investor, respectively.

Since the consumption of two goods and the new portfolio of monies-bonds at each instant are simultaneously chosen, the investor's instantaneous real consumption of two goods is equal to the sum of real income from reshuffling the portfolio and instantaneous flow dY of real noncapital income (which is assumed as nonstochastic in this paper),

$$\begin{aligned} dY - X_c dt = & dQ dM_h + d(QP_b) dB_h + d(QE) dM_h^* + d(QEP_b^*) dB_h^* \\ & + Q dM_h + (QP_b) dB_h + (QE) dM_h^* + (QEP_b^*) dB_h^*. \end{aligned} \quad (6)$$

Thus, taking into consideration the $\hat{I}\hat{o}$ Lemma, the Wiener proceses defined in (1), $m_h + b_h + m_h^* + b_h^* = 1$ and the investor's consumption pattern, one gets

$$\begin{aligned} dW/W = & [(p_b - \alpha_1 \sigma_{1b} - \alpha_2 \sigma_{2b}) b_h + (e - \alpha_1 \sigma_{e1} - \alpha_2 \sigma_{e2}) m_h^* + \{e + p_b^* \\ & + \sigma_{eb}^* - \alpha_2 (\sigma_{e2} + \sigma_{2b}^*) - \alpha_1 (\sigma_{1b}^* + \sigma_{e1})\} b_h^* + \alpha_1 \alpha_2 \sigma_{12} - \alpha_1 p_1 \\ & - \alpha_2 p_2 + \alpha_1 (1 + \alpha_1) \sigma_{11}/2 + \alpha_2 (1 + \alpha_2) \sigma_{22}/2 + (dY - X_c)/W] dt \\ & + \sigma_b dz_b b_h + \sigma_e dz_e (m_h^* + b_h^*) + \sigma_b^* dz_b^* b_h^* - \alpha_1 \sigma_1 dz_1 - \alpha_2 \sigma_2 dz_2 \\ = & [Ab_h + Bm_h^* + Cb_h^* + D + (dY - X_c)/W] dt + \sigma_b dz_b b_h + \sigma_e dz_e (m_h^* \\ & + b_h^*) + \sigma_b^* dz_b^* b_h^* - \alpha_1 \sigma_1 dz_1 - \alpha_2 \sigma_2 dz_2, \end{aligned} \quad (7)$$

where $m_h = QM_h/W$, $b_h = QP_b B_h/W$, $m_h^* = QEM_h^*/W$ and $b_h^* = QEP_b^* B_h^*/W$, respectively, σ_{ij} is the covariance between stochastic variables i and j ($i, j = P_1, P_2, P_b, P_b^*$ and E) and A, B, C and D are abbreviations of the corresponding terms in the former equation, respectively, but p_s in the subscripts of σ_{ij} in (7) are all omitted, for example, like $\sigma_{p_1 e} = \sigma_{1e}$, $\sigma_{p_b p_b^*} = \sigma_{bb^*}$, $\sigma_{p_1 p_2} = \sigma_{12}$, and so on. The home investor seeks the optimal consumption and portfolio rule which maximizes his expected utility function given by (2) subject to his continuous-time stochastic budget constraint defined as (7)⁸.

From (A. 4) in Appendix the optimal portfolio choices of the home investor with respect to b_h, b_h^*, m_h^* and m_h are obtained, respectively, by

$$\begin{aligned} b_h = & \left(\sum_{i=1}^3 Z_i \Delta_{i1} \right) / \Delta, & b_h^* = & \left(\sum_{i=1}^3 Z_i \Delta_{i2} \right) / \Delta, & m_h^* = & \left(\sum_{i=1}^3 Z_i \Delta_{i3} \right) / \Delta, \\ m_h = & 1 - b_h - b_h^* - m_h^*, \end{aligned} \quad (8)$$

where $Z_1 = \alpha_1 \sigma_{1b} + \alpha_2 \sigma_{2b} - A (J_w/WJ_{ww}) - \sum (KJ_{wk}/WJ_{ww}) \sigma_{bK}$, $Z_2 = \alpha_1 \sigma_{1b}^* + \alpha_2 \sigma_{2b}^* - (C - B)(J_w/WJ_{ww}) - \sum (KJ_{wk}/WJ_{ww}) \sigma_{b^*K}$, $Z_3 = \alpha_1 \sigma_{1e} + \alpha_2 \sigma_{2e} - B (J_w/WJ_{ww}) - \sum (KJ_{wk}/WJ_{ww}) \sigma_{eK}$, Δ is the determinant of the variance-covariance matrix in (A. 4) which is expressed as

$$\Delta = \begin{vmatrix} \sigma_{bb} & \sigma_{b^*b} & \sigma_{eb} \\ \sigma_{bb}^* & \sigma_{b^*b}^* & \sigma_{eb}^* \\ \sigma_{be} & \sigma_{b^*e} & \sigma_{ee} \end{vmatrix}$$

and Δ_{ij} ($i, j = 1, 2, 3$) are the cofactors of the element in the i th row and j th column of Δ .

Since the relative risk tolerance of the home investor is defined as $T_R = -J_w/WJ_{ww}$ and the elasticities of the Bellman function with respect to $K (= P_1, P_2, P_b, P_b^*, E)$ are expressed as $\mathcal{U}_{WK} = (K/J_w)(\partial J_w/\partial K)$, Z_i ($i = 1, 2, 3$) are rewritten, respectively, as

$$\begin{aligned} Z_1 &= (\alpha_1 \sigma_{1b} + \alpha_2 \sigma_{2b}) (1 - T_R) + (p_b + \sum_K \mathcal{U}_{WK} \sigma_{Kb}) T_R, \\ Z_2 &= (\alpha_1 \sigma_{1b}^* + \alpha_2 \sigma_{2b}^*) (1 - T_R) + (p_b^* + \sigma_{eb}^* + \sum_K \mathcal{U}_{WK} \sigma_{Kb}^*) T_R, \\ Z_3 &= (\alpha_1 \sigma_{1e} + \alpha_2 \sigma_{2e}) (1 - T_R) + (e + \sum_K \mathcal{U}_{WK} \sigma_{eK}) T_R. \end{aligned} \quad (9)$$

Hence it is immediately obvious from (1), (8) and (9) that the optimal portfolio rule of the home investor generally depends on his relative risk tolerance, his consumption pattern, the state variables and his expectations of these state variables. Though it is very interesting to analyze the investor's portfolio rules and to make clear their characteristic features, the more detailed examinations about the home investor's portfolio rule given by (1), (8) and (9) will be presented in section III, together with those of the foreign investor's portfolio rule derived in the next subsection.

B. Optimal Consumption-Portfolio Rule of the Foreign Investor

Suppose an average foreign investor consuming two goods produced in two countries and holding a variable portfolio of two monies and two bonds issued in two countries. Thus, he also faces five kinds of the market prices, i.e., the consumption-good prices, P_1^* and P_2^* , the bond prices, P_b and P_b^* , and the exchange rate, E , where P_i^* is the price of a consumption good i in terms of foreign money ($i = 1, 2$) and the other prices are the same as those used in the previous subsection. He also takes all these prices as uncertain and given.

It is also assumed that the market prices are all expected by the Itô type of continuous stochastic processes which are different from those of the home investor,

$$\begin{aligned} dP_i^*/P_i^* &= p_{if}(P_i^*, P_b, P_b^*, E) dt + \sigma_{pif}(P_i^*, P_b, P_b^*, E) dz_{pif}, \quad i = 1, 2, \\ dP_b/P_b &= p_{bf}(P_i^*, P_b, P_b^*, E) dt + \sigma_{pbf}(P_i^*, P_b, P_b^*, E) dz_{pbf}, \\ dP_b^*/P_b^* &= p_{b^*f}(P_i^*, P_b, P_b^*, E) dt + \sigma_{pb^*f}(P_i^*, P_b, P_b^*, E) dz_{pb^*f}, \\ dE/E &= e_f(P_i^*, P_b, P_b^*, E) dt + \sigma_{ef}(P_i^*, P_b, P_b^*, E) dz_{ef}. \end{aligned} \quad (10)$$

In (10) and hereafter notations including f in subscripts denote those of the foreign investor.

The foreign investor maximizes his expected utility from time 0 to time T defined as

$$U_f = E_{0f} \int_0^T (1/\mu) \{x_{1f}(t)^{\beta_1} x_{2f}(t)^{\beta_2}\}^\mu dt = E_{0f} \int_0^T \tilde{u}_f(t) dt, \quad (11)$$

where β_i is the share of good i ($i = 1, 2$) in nominal consumption expenditure measured by foreign money, $\beta_1 + \beta_2 = 1$ holds by definition, and $(1 - \mu)$ is the relative risk aversion of the foreign investor which is constant in our model.

According to the same way as just adopted in the previous subsection the foreign investor's real consumption expenditure X_{cf} deflated by the purchasing power of foreign money is given by

$$X_{cf} = Q^* E_{Nf}, \quad (12)$$

where $E_{Nf} = (P_1^* x_{1f} + P_2^* x_{2f})$ is the foreign investor's nominal consumption expenditure expressed in terms of foreign money, and $Q^* = 1/P_1^{*\beta_1} P_2^{*\beta_2}$ is the purchasing power of foreign money over two goods in the foreign country.

Taking into consideration that the foreign investor also holds two kinds of monies and two kinds of bonds issued in the two countries and that he usually evaluates his own wealth measured in terms of foreign money, his real wealth W_f deflated by the purchasing power of foreign money is defined as

$$W_f = Q^* \{(M_f + P_b B_f)/E + (M_f^* + P_b^* B_f^*)\}, \quad (13)$$

where M_f and M_f^* are the amounts of home and foreign monies demanded by the foreign investor and B_f and B_f^* are the quantities of home and foreign bonds held by him, respectively.

If PPP holds, that is, $E = Q^*/Q$ is true, then (13) is rewritten as

$$W_f = Q \{(M_f + P_b B_f) + E (M_f^* + P_b^* B_f^*)\}. \quad (13)'$$

In this case, replacing the subscript f in (13)' with h , (13)' equals (5). But this is not the case in the present paper since PPP is not supposed. Moreover, if the law of one price, i.e., $P_i = EP_i^*$ ($i = 1, 2$) holds, then $Q^* = P_1^{*\beta_1} P_2^{*\beta_2} = EP_1^{\beta_1} P_2^{\beta_2}$ holds. Therefore, denoting $\tilde{Q} = P_1^{\beta_1} P_2^{\beta_2}$, (13) is rewritten as

$$W_f = \tilde{Q} \{(M_f + P_b B_f) + E (M_f^* + P_b^* B_f^*)\}. \quad (13)''$$

Hence, once again, replacing the subscript f and $\tilde{Q}$ in (13)'' with h and Q , respectively, (13)'' is equal to (5). However, it is easily shown from (13)' and (13)'' that the law of one price does not generally imply PPP except for the case where the consumption patterns of the home and foreign investors are the same, that is, $\alpha_i = \beta_i$ ($i = 1, 2$) holds. In this paper it is not supposed that the law of one price always holds.

Now, adopting the new notation, $\tilde{E} = 1/E$, (13) becomes

$$W_f = Q^* \{(M_f^* + P_b^* B_f^*) + \tilde{E} (M_f + P_b B_f)\}. \quad (13)'''$$

Clearly, since (13)''' includes (13)' and (13)'' as special cases, the arguments based on (13)''' would generalize those based on the PPP assumption and/or on the assumption of the law of one price.

Comparing (13)''' with (5), one would find once again that the definition of the real wealth of the foreign investor is quite symmetrical with that of the home investor when one can recognize that M_h , B_h , M_f , B_f , Q and E in (5) are corresponding to M_f^* , B_f^* , M_h^* , B_h^* , Q^* and $\tilde{E}$ in (13)''', respectively. In this case, from (10) and the Itô Lemma one gets

$$d\tilde{E}/\tilde{E} = -dE/E + (dE/E)^2 = (\sigma_{eef} - e_f) dt - \sigma_{ef} dz_{ef}. \quad (14)$$

Furthermore, it is also shown, comparing (10) and (11) with (1) and (2), respectively, that both the expectation pattern and the utility function of the foreign investor are similar to those of the home investor. Therefore, it is quite obvious that the foreign investor's optimal portfolio rule is derived in the same way as that used in deriving the home investor's optimal portfolio rule in the previous subsection, that is, the foreign investor's optimal portfolio choices are given by

$$\begin{aligned} b_f^* &= (\sum_{i=1} Z_{if} \Delta_{i1f}) / \Delta_f, & b_f &= (\sum_{i=1} Z_{if} \Delta_{i2f}) / \Delta_f, & m_f &= (\sum_{i=1} Z_{if} \Delta_{i3f}) / \Delta_f, \\ m_f^* &= 1 - b_f^* - b_f - m_f, \end{aligned} \quad (15)$$

where $m_f^* = Q^* M_f^* / W_f$, $b_f^* = Q^* P_b^* B_f^* / W_f$, $m_f = Q^* \tilde{E} M_f / W_f$, $b_f = Q^* \tilde{E} P_b B_f / W_f$,

$$\Delta_f = \begin{vmatrix} \sigma_{b^*b^*f} & \sigma_{bb^*f} & \sigma_{eb^*f} \\ \sigma_{b^*bf} & \sigma_{bbf} & \sigma_{ebf} \\ \sigma_{b^*ef} & \sigma_{bef} & \sigma_{eef} \end{vmatrix},$$

$$\begin{aligned} Z_{1f} &= (\beta_1 \sigma_{1b^*f} + \beta_2 \sigma_{2b^*f})(1 - T_{Rf}) + (p_b^* + \sum_K \chi_{WKf} \tilde{\sigma}_{Kb}^*) T_{Rf}, \\ Z_{2f} &= (\beta_1 \sigma_{1bf} + \beta_2 \sigma_{2bf})(1 - T_{Rf}) + (p_b - \sigma_{ebf} + \sum_K \chi_{WKf} \tilde{\sigma}_{Kb}) T_{Rf}, \\ Z_{3f} &= -(\beta_1 \sigma_{1ef} + \beta_2 \sigma_{2ef})(1 - T_{Rf}) + (\sigma_{eef} - e_f + \sum_K \chi_{WKf} \tilde{\sigma}_{eK}) T_{Rf}, \\ (\because \tilde{\sigma}_{eK} &= -\sigma_{eKf} \text{ for } K \neq e, \tilde{\sigma}_{ee} = \sigma_{ee}, \tilde{\sigma}_{Kb}^* = \sigma_{Kb}^* f, \tilde{\sigma}_{Kb} = \sigma_{Kbf}). \end{aligned} \quad (16)$$

It is shown from (8) and (15) that the optimal portfolio choices of the foreign investor are quite different from those of the home investor though their derivations are similar to each other. This is one of the features stemmed from the fact that international investors evaluate their wealth by their domestic money. The other features stemming from different money choices are discussed in the next section.

2. Risk Aversion, Expectations and the Optimal Portfolio Rules

In this section, to save space I mainly examine the effects of the home investor's risk tolerance and his expectations of the state variables on his optimal portfolio choices. However, I also refer to the features of optimal portfolio choices of the foreign investor in discussing the differences between the optimal portfolio choices of the home and the foreign investors.

If the risk tolerance, T_R , of the home investor is zero, his optimal portfolio choices of b_h , b_h^* , m_h^* and m_h are, from (8) and (9), given by

$$\begin{aligned} b_h &= (\sum Z_i \Delta_{i1}) / \Delta, \\ b_h^* &= (\sum Z_i \Delta_{i2}) / \Delta, \\ m_h^* &= (\sum Z_i \Delta_{i3}) / \Delta, \text{ and} \\ m_h &= 1 - b_h - b_h^* - m_h^*, \quad i = 1, 2, 3, \end{aligned} \quad (17)$$

where $Z_1 = (\alpha_1 \sigma_{1b} + \alpha_2 \sigma_{2b})$, $Z_2 = (\alpha_1 \sigma_{1b^*} + \alpha_2 \sigma_{2b^*})$ and $Z_3 = (\alpha_1 \sigma_{1e} + \alpha_2 \sigma_{2e})$. Therefore, it is clear in the case of $T_R = 0$ that the home investor's optimal portfolio rule is equivalent to the minimum variance portfolio rule which minimizes the instantaneous conditional variance of dW/W and is independent of the state variables, P_1 , P_2 , P_b , P_b^* and E as well as their expected mean

proportional changes per unit time, p_1 , p_2 , p_b , p_b^* and e .

On the other hand, if T_R is unity, the home investor's portfolio rule is reduced, from (8) and (9), as

$$\begin{aligned} b_h &= (\sum Z_i \Delta_{i1}) / \Delta, \\ b_h^* &= (\sum Z_i \Delta_{i2}) / \Delta, \\ m_h^* &= (\sum Z_i \Delta_{i3}) / \Delta, \text{ and} \\ m_h &= 1 - b_h - b_h^* - m_h^*, \quad i = 1, 2, 3, \end{aligned} \quad (18)$$

where $Z_1 = (p_b + \sum \psi_{WK} \sigma_{Kb})$, $Z_2 = (p_b^* + \sigma_{eb}^* + \sum \psi_{WK} \sigma_{Kb}^*)$ and $Z_3 = (e + \sum \psi_{WK} \sigma_{eK})$. Hence, it is shown in this case that the optimal portfolio rule of the home investor is equal to the logarithmic portfolio rule and that his portfolio choices are insensitive to both his consumption pattern, α_i ($i = 1, 2$) and the expected mean proportional changes per unit time of P_i , i.e., p_1 and p_2 .

The above arguments demonstrate that the home investor's portfolio rule is generally expressed as a weighted average of the minimum variance portfolio rule and the logarithmic portfolio rule as W. Bransson and D. Henderson (1985) have indicated by using a special one of our model. Therefore, as the investor's risk tolerance approaches unity (zero), his portfolio rule also does the logarithmic portfolio rule (the minimum variance portfolio rule).

Since the home investor's portfolio rule depends on all of his consumption pattern, his risk tolerance, his conditional instantaneous expectations of the state variables except for p_1 and p_2 , and the current state variables, if the bonds issued in two countries are perfectly gross substitutive for each other, $\partial b_h / \partial p_b = - \partial b_h^* / \partial p_b$ and/or $\partial b_h / \partial p_b^* = - \partial b_h^* / \partial p_b^*$ must be true. So I now examine these relations.

Differentiating respectively b_h and b_h^* in (8) with respect to p_b and p_b^* , and taking the Cauchy-Schwarz inequality into consideration, one gets

$$\begin{aligned} \partial b_h / \partial p_b &= \Delta_{11} T_R / \Delta > 0, \\ \partial b_h^* / \partial p_b^* &= \Delta_{22} T_R / \Delta > 0, \\ \partial b_h^* / \partial p_b &= \Delta_{12} T_R / \Delta > (<) 0 \text{ as } \Delta_{12} > (<) 0, \\ \partial b_h / \partial p_b^* &= \Delta_{21} T_R / \Delta > (<) 0 \text{ as } \Delta_{21} > (<) 0. \end{aligned} \quad (19)$$

Thus, only the signs of $\partial b_h / \partial p_b$ and $\partial b_h^* / \partial p_b^*$ are determined as positive since Δ , Δ_{11} and Δ_{22} are positive, which implies that an increase in the instantaneous conditional expected mean proportional change per unit time of the home(foreign) bond price raises home(foreign) bond holdings of the home investor, and *vice versa*. However, the signs of $\partial b_h^* / \partial p_b$ and $\partial b_h / \partial p_b^*$ are both ambiguous due to the equivocation of the signs of Δ_{12} and Δ_{21} . Moreover, it is shown that the effects of a change in p_b (p_b^*) on b_h and b_h^* are different from each other since $\Delta_{11} \neq \Delta_{12}$ ($\Delta_{22} \neq \Delta_{21}$) holds in general. Thus, the home and foreign bonds are not perfectly gross substitutive for each other except for a special case of $T_R = 0$ where both b_h and b_h^* are insensitive to p_b (p_b^*).

It is also shown from (8) that monies issued in two countries are not also perfectly gross substitutive for each other. Differentiating m_h^* and m_h in (8) with respect to e and taking account of the Cauchy-Schwarz inequality one obtains

$$\begin{aligned}\partial m_h^*/\partial e &= \Delta_{33} T_R / \Delta > 0, \\ \partial m_h/\partial e &= -(\Delta_{31} + \Delta_{32} + \Delta_{33}) T_R / \Delta.\end{aligned}\quad (20)$$

Obviously an increase in the expected mean proportional change per unit time of the exchange rate increases the foreign money holdings of the home investor and *vice versa*. However the sign of the second equation of (20) is generally ambiguous and thus the absolute value of the right side of $\partial m_h^*/\partial e$ is different from that of $\partial m_h/\partial e$ except for a special case of $T_R = 0$ where both m_h^* and m_h are insensitive to e .

In the present model considering uncertainty of the good prices, the bond prices and the exchange rate, the risk premium in international portfolios is defined as the expected return differential. Therefore, the risk premium of the home investor is given by

$$\begin{aligned}p_b - p_b^* - e \\ &= \alpha_1 (1 + R_R)(\sigma_{1b} - \sigma_{1b}^* - \sigma_{1e}) + \alpha_2 (1 + R_R)(\sigma_{2b} - \sigma_{2b}^* - \sigma_{2e}) \\ &\quad - R_R \{b_h(\sigma_{bb} - \sigma_{bb}^* - \sigma_{be}) + b_h^*(\sigma_{bb}^* - \sigma_b^* b^* - \sigma_{eb}^*) \\ &\quad + (m_h^* + b_h^*)(\sigma_{eb} - \sigma_{eb}^* - \sigma_{ee})\} + \sum_K U_{KW}(\sigma_{bK} - \sigma_b^* K - \sigma_{eK}),\end{aligned}\quad (21)$$

where $R_R (= 1/T_R)$ is the relative risk aversion of the home investor. On the other hand, the risk premium of the foreign investor is given by

$$\begin{aligned}p_b^* f - p_{bf} + e_f \\ &= \beta_1 (1 + R_{Rf})(\sigma_1^* b^* f - \sigma_1^* b_f + \sigma_1^* e_f) + \beta_2 (1 + R_{Rf})(\sigma_2^* b^* f - \sigma_2^* b_f \\ &\quad + \sigma_2^* e_f) - R_{Rf} \{b_f^*(\sigma_b^* b^* f - \sigma_b^* b_f + \sigma_b^* e_f) + b_f(\sigma_b^* b_f - \sigma_{bbf} + \sigma_{eb}^* f) \\ &\quad + (m_f + b_f)(\sigma_{ebf} - \sigma_{eb}^* f - \sigma_{eef})\} + \sigma_{eef} + \sum_K U_{KWf}(\sigma_b^* Kf - \tilde{\sigma}_b^* Kf - \sigma_{eKf}),\end{aligned}\quad (22)$$

where $R_{Rf} (= 1/T_{Rf})$ is the relative risk aversion of the foreign investor.

If the bonds issued in two countries are perfectly gross substitutive for each other and if the international asset markets are perfectly competitive, then both the right sides of (21) and (22) must be zero, and thus the more generalized interest rate parity including monies and bonds issued in two countries would be true. However, there is no guarantee that both the right sides of (21) and (22) are always zero. Hence, interest rate parity, i.e., IRP, would not generally hold. It is also shown from (21) and (22) that even if the expectations, the consumption patterns and the risk aversions are all the same among the home and foreign investors, their risk premiums are not equal to each other. This difference originally stems from the fact that investors usually evaluate their wealth in terms of their domestic money.

The fact that the risk premiums of the home and foreign investors are different each other implies, in turn, that the equilibrium asset prices regarded by them as the international asset market clearing prices are not always equal to each other. In such circumstances, even if the home investor supposes that international asset markets are in equilibrium, the foreign investor

does not think so, and *vice versa*. Hence, at least one of the home and foreign investors always adjusts his portfolio and the international asset markets including the exchange market tend to be volatile.

In any models where only monies are considered as international assets (say the monies models, hereafter), since investors could not hold any bonds in their portfolios, $b_h = b_h^* = b_f = b_f^* = 0$, $m_h + m_h^* = 1$ and $m_f + m_f^* = 1$ hold. Thus, one can easily get $\partial m_h / \partial e = -\partial m_h^* / \partial e < 0$, $\partial m_f^* / \partial e = -\partial m_f / \partial e > 0$, $\partial m_h / \partial \sigma_e = -\partial m_h^* / \partial \sigma_e$ and $\partial m_f / \partial \sigma_e = -\partial m_f^* / \partial \sigma_e$, which implies that in the monies models the home and foreign monies are always perfectly gross substitutive for each other. However, these are contrary to the results in the present model including both monies and bonds. Hence, it is shown that the monies models are very restrictive for analyzing the optimal portfolio rules of international investors and exchange rate determination⁹.

In examining the effects of changes in the foreign investor's expectations of the prices and the exchange rate on his portfolio rule, it is necessary to recall that this paper presumes neither PPP nor the law of one price. If PPP is adopted, the real wealth of the foreign investor is given by (13)' and it is the same as that of the home investor given by (5) except that h and f in the subscripts of the parameters are only the difference. And, if the law of one price is assumed, the real wealth of the foreign investor is defined as (13)'' and it is essentially the same as that of the home investor defined by (5) except that, in addition to h and f in the subscripts of the parameters, investor's consumption expenditure shares α_i and β_i ($i = 1, 2$) are different. Therefore, all the signs of the effects of changes in the foreign investor's expectations of the prices and the exchange rate on his portfolio rule are the same as those of the effects of changes in the home investor's expectations of the prices and the exchange rate on his portfolio rule, respectively. These propositions, however, do not hold in this paper which adopts neither PPP nor the law of one price.

3. Trade Imbalances, Monies, Bonds and the Exchange Rate

This section first derives the market exchange rate from the equilibrium conditions of international asset markets and then investigates the effects of changes in trade imbalances, monetary and fiscal policies on this rate.

A. International Financial Markets and the Exchange Rate

Since the financial markets of monies and bonds issued in home and foreign countries are all internationally open and perfectly competitive, individuals in two countries can equally participate in these markets and take the prices observed in these markets as common ones. Moreover, the firms in two countries also take part in the international asset markets.

It is here assumed, without losing the generality, that while the firms hold monies on transaction motives, they do not demand them on speculative motives. It is also supposed, for

simplicity, that the total amount of home money TA (foreign money TAF) held by the firms at each instant is equal to the instantaneous trade surplus of home country ES (or the instantaneous trade deficit of foreign country-S), where S is measured in terms of foreign money. This reflects the actual observation that the home firms facing trade surplus paid with foreign money supply them in order to get home money, and vice versa. Thus, the trade surplus of home country results in increases of demand for home money and of supply of foreign money. Then,

(23)

hold as demands for monies induced by trade imbalances between two countries.

In this model, the real variables such as the trade imbalances S and the good prices P_i ($i=$

1, 2) are all given to the international asset markets at each instant since both the goods

markets and the trade imbalances are merely adjusted so as to attain the long-run equilibrium,

that is, $\dot{S} = 0$ and $\dot{P}_i = 0$ hold only in long run, where X_i and D_i are

instantaneous aggregate supply of and demand for a goods i in the world ($i=1, 2$).

On the other hand, since the asset markets attain equilibrium at each instant, the

instantaneous equilibrium conditions for the markets of two monies and two bonds are

respectively given by

(24)

where M_h , M_f^* , B_h , B_f^* , n_h and n_f denote the amounts of monies supplied by the home and foreign monetary authorities, the numbers of bonds issued by the home and foreign governments and the numbers of individuals in the two countries, respectively, and M_h , M_f^* , B_h , B_f^* , n_h and n_f are all supposed to be positive.

The equilibrium conditions of home and foreign money markets, i.e., (24-i) and (24-ii), are expressed by their monies, respectively. It should be also noted that even if some bonds issued by private sectors are included in bond holdings of investors, they do not appear in the equilibrium conditions (24-iii) and (24-iv) of the international bond markets since these private bonds cancel out each other among themselves in equilibrium. Furthermore, M_h , M_f^* , B_h , B_f^* , n_h and n_f are all regarded as parameters since the former four are political variables of the home and foreign governments, and the latter two are noneconomic ones which are not determined in the asset markets.

Substituting $m_h = Q M_h / W$, $b_h = Q P_b B_h / W$, $m_h^* = Q E M_h^* / W$, $b_h^* = Q E P_b^* B_h^* / W$, $m_f = Q^* E M_f / W_f$, $b_f = Q^* E P_b B_f / W_f$, $m_f^* = Q^* M_f^* / W_f$, $b_f^* = Q^* P_b^* B_f^* / W_f$ (15) and (23) into (24), and arranging the results, the equilibrium conditions (24) of the international asset markets are rewritten as

$$\begin{aligned}
(i) \quad M_h &= m_h W_M + m_f E W_{Mf} + ES, \\
(ii) \quad E M_f^* &= m_h^* W_M + m_f^* E W_{Mf} - ES, \\
(iii) \quad P_b B_h &= b_h W_M + b_f E W_{Mf}, \\
(iv) \quad E P_b^* B_f^* &= b_h^* W_M + b_f^* E W_{Mf},
\end{aligned} \tag{24}'$$

where $W_M = n_h \{(M_h + P_b B_h) + E(M_h^* + P_b^* B_h^*)\} = n_h W/Q$ and $W_{Mf} = n_f \{(M_f + P_b B_f) \tilde{E} + (M_f^* + P_b^* B_f^*)\} = n_f W_f/Q^*$ are the total wealth held by individuals of the home and foreign countries, respectively. Therefore, (24)' indicates that the equilibrium asset prices depend on the goods prices, the investors' portfolio rules in two countries, the aggregate nominal wealth of individuals in two countries, the total amounts of monies and government bonds supplied by two countries and the trade imbalances between two countries.

Since $m_h + m_h^* + b_h + b_h^* = 1$, $m_f + m_f^* + b_f + b_f^* = 1$ and (24)' combine to give $M_h + E M_f^* + P_b B_h + E P_b^* B_f^* = W_M + E W_{Mf}$ as the world financial budget constraint, it is clear that though (24)' consists of four equations including the good prices, the bond prices and the exchange rate as its variables, any three equations of (24)' solve the exchange rate E and the two bond prices P_b and P_b^* which attain the asset market equilibrium if the trade imbalance S and the goods prices P_i and P_i^* ($i = 1, 2$) are determined in the real markets. Thus, considering (8), (15) and (24)', the bond prices P_b and P_b^* and the exchange rate E at the equilibrium of international asset markets are given by

$$\begin{aligned}
(i) \quad P_b &= P_b(\alpha_1, \beta_1, p_g, p_{gf}, p_b, p_b^*, e, p_{bf}, p_b^* f, e_f, \sigma_{ij}, \sigma_{ijf}, P_g, P_g^*, \\
&\quad T_R, T_{Rf}, M_h, M_f^*, B_h, B_f^*, S, n, n_f), \\
(ii) \quad P_b^* &= P_b^*(\alpha_1, \beta_1, p_g, p_{gf}, p_b, p_b^*, e, p_{bf}, p_b^* f, e_f, \sigma_{ij}, \sigma_{ijf}, P_g, \\
&\quad P_g^*, T_R, T_{Rf}, M_h, M_f^*, B_h, B_f^*, S, n, n_f), \\
(iii) \quad E &= E(\alpha_1, \beta_1, p_g, p_{gf}, p_b, p_b^*, e, p_{bf}, p_b^* f, e_f, \sigma_{ij}, \sigma_{ijf}, P_g, P_g^*, \\
&\quad T_R, T_{Rf}, M_h, M_f^*, B_h, B_f^*, S, n, n_f), \\
&\quad (g = 1, 2 \text{ and } i, j = 1, 2, b, b^*, e).
\end{aligned} \tag{25}$$

It is shown from (25) that the exchange rate and the bond prices are the functions of all individual' consumption patterns, their expectations of the asset and goods prices, their degrees of risk tolerance, the goods prices, the amounts of monies and government bonds supplied in two countries, the amount of trade imbalances, and the number of individuals in two countries.

The feature that the exchange rate function includes the goods prices and the trade imbalances as its arguments among others stems from the dual equilibrium hypothesis adopted in this model. Indeed, if the international asset markets, the goods markets and the trade balances attain the simultaneous equilibrium, all the asset prices, the goods prices and the trade balances are determined simultaneously. Thus, adopting the usual equilibrium hypothesis which assumes the simultaneous equilibrium of the international asset markets, the goods markets and the trade balances, one cannot assert that the exchange rate depends on the levels of goods prices and the amount of trade imbalances.

B. Trade Surplus, Monetary and Fiscal Policies and the Exchange Rate

I here examine the effects of the amounts of monies and bonds issued by the home and foreign governments and the trade imbalances on the exchange rate. So, for simplicity, it is additionally assumed that all the investors' expectations of the goods prices and the asset prices follow the "geometric Brownian motion" which has often been adopted in many papers. In such a case the wealth is an only state variable and thus all Ψ_{WK} ($K = P_i, P_b, P_b^*, E$) become zero.

Before proceeding to the derivation of the exchange rate, I consider an interesting financial feature of the local asset preferences in the sense that the home(foreign) investors hold larger proportions of home(foreign) assets in their wealth than the foreign(home) investors do, respectively. This feature is taken as very plausible since the home(foreign) investors have tendencies to suppose that home(foreign) assets are much safer than foreign(home) assets. In this paper with four assets, the local asset preference implies $m_h > m_f$, $m_f^* > m_h^*$, $b_h > b_f$ and $b_f^* > b_h^*$. Then, one easily obtains

$$(m_h^* b_f - m_f^* b_h) < 0, \quad \text{and} \quad (m_f^* m_h - m_h^* m_f) > 0. \quad (26)$$

In the following, I shall adopt (26) in analyzing the effects of large and marginal changes in the trade imbalances, the monetary and fiscal policies of two countries on the exchange rate and so on. Moreover, it is also supposed that the home country faces trade surplus like Japan.

B.I Large Changes in Amounts of Trade Imbalances and Monies

Solving W_M and W_{Mf} from (24. i)' and (24. ii)' and taking account of $0 < W_M$, $0 < W_{Mf}$ and (26), one gets

$$\rho < E < \delta \quad (27)$$

where

$$\rho = m_h^* M_h / \{m_h M_f^* + (m_h + m_h^*) S\},$$

$$\delta = m_f^* M_h / \{m_f M_f^* + (m_f + m_f^*) S\}.$$

Obviously, the exchange rate is generally bounded both from the above and from the below, and both these limits increase as the supply of home money rises and as the supply of foreign money and/or the trade surplus of the home country decrease, and *vice versa*. Hence, while a considerably large increase(decrease) in the trade surplus of home country leads to the appreciation(depreciation) of home money, a considerably huge increase in the supply of home (foreign) money results in the depreciation(appreciation) of home money, and *vice versa*.

In this context, a considerably large change in trade imbalance and/or money supply implies that it is large enough to make the upper(lower) limit of the exchange rate lower(higher) than the current level. In practice it would be shown that the Japanese trade surplus in the recent years has been large enough to make the upper limit, δ , lower than the current exchange rate E (the price of a dollar in terms of yen), and yen has been appreciating against dollar as its trend. Thus, it is proposed that any market interventions by the Japanese and/or U.S. monetary authorities aiming to control the drastic appreciation of yen must be large enough to stop the

shift of this upper limit or to return such a shift to the opposite direction.

B.II Marginal Changes in Amounts of Trade Imbalance, Monies and Bonds

In order to analyze the effects of marginal changes in the amounts of trade imbalances, monies and bonds controlled by the home and foreign governments on the exchange rate, I derive the equation which expresses a relationship among the exchange rate, the trade imbalances and the amounts of monies and bonds supplied by two countries. From (24-i)', (24-ii)' and (24-iii)' we obtain

$$E = \frac{(m_h^* b_f - m_f^* b_h) M_h + (m_f^* m_h - m_f m_h^*) P_b B_h}{(m_h b_f - b_h m_f) M_f^* + (m_h b_f - b_h m_f + m_h^* b_f - m_f^* b_h) S}. \quad (28)$$

This demonstrates that the exchange rate depends on the portfolio rules of individuals in two countries, the amounts of monies and bonds supplied by two countries and the trade imbalances between two countries. If the trade account is balanced at each instant, then the exchange rate E does not depend on S .

As a monetary policy engaged by the monetary authorities let us consider the open market operation. In this case any increase(decrease) in the amount of home money operated by the home monetary authorities is accompanied by the same volume of decrease(increase) in the amount of home bonds, and thus one has $\partial P_b B_h / \partial M_h = -1$. Hence, the effect of a change in the amount of home money controlled by the home monetary authorities on the exchange rate is given by

$$\frac{\partial E}{\partial M_h} = \frac{(m_h^* b_f - m_f^* b_h) - (m_f^* m_h - m_f m_h^*)}{(m_h b_f - b_h m_f) M_f^* + (m_h b_f - b_h m_f + m_h^* b_f - m_f^* b_h) S}. \quad (29)$$

Since $(m_h^* b_f - m_f^* b_h)$ and $(m_f^* m_h - m_f m_h^*)$ are negative and positive from (26) respectively, the right side of (29) is positive if $(m_h b_f - b_h m_f)$ is negative. But, if $(m_h b_f - b_h m_f)$ is positive, the sign of (29) is ambiguous.

From (24) one gets $\text{sign}(m_h b_f - b_h m_f) = \text{sign}(b_f/b_h - m_f/m_h) = \text{sign}\{B_h/n_h B_h - (M_h - T_A)/n_h M_h\}$. It follows that $(m_h b_f - b_h m_f)$ is negative(positive) when the ratio between quantities of bonds demanded and supplied in home country, $B_h/n_h B_h$, is larger(smaller) than the ratio between net values of home money held and issued in home country, $(M_h - T_A)/n_h M_h$. In what follows, when $n_h B_h/B_h > (<) n_h M_h/(M_h - T_A)$ holds, I call the home country a relatively bond(money) preferring country. Then, one can propose:

$$(m_h b_f - b_h m_f) < (>) 0 \quad \text{as the home country is a relatively} \\ \text{bond(money) preferring country.} \quad (30)$$

In practice, whether the home country is a relatively bond preferring country or a relatively money preferring country depends only on the portfolio rules of individuals in the home country.

Now, considering (29) and (30), it is shown that if the home country is a relatively bond preferring country a marginal expanding open market operation by the home monetary authorities depreciates home money against foreign money, and *vice versa*. However, if the home country is

a relatively money preferring country the sign of (29) is ambiguous.

On the other hand, in the case where the international trades of assets are sufficiently small, that is, $m_h^* \doteq 0$, $b_h^* \doteq 0$, $m_f \doteq 0$ and $b_f \doteq 0$ hold, (29) is rewritten as

$$\frac{\partial E}{\partial M_h} \doteq \frac{b_h + m_h}{b_h S} > 0. \quad (31)$$

It follows that if the international assets trades are sufficiently small the marginal expanding open market operation by the home monetary authorities always raises the exchange rate E , and *vice versa*.

Now (29) and (31) combine to show that as international trades of assets become significant, there emerges a possibility that the effect of a marginal change in open market operation by the home monetary authorities on the exchange rate is negative. Thus, in such circumstances the co-existence of the appreciation(depreciation) and the expansion(reduction) of home money through the open market operation is observed¹⁰.

The effect of a change in the amount of foreign money M_f^* supplied by the foreign monetary authorities on the exchange rate E is given by

$$\frac{\partial E}{\partial M_f^*} = \frac{(m_f^* m_h - m_f m_h^*) \partial P_b B_h / \partial M_f^* - E (m_h b_f - b_h m_f)}{(m_h b_f - b_h m_f) M_f^* + (m_h b_f - b_h m_f + m_h^* b_f - m_f^* b_h) S}. \quad (32)$$

Clearly $(\partial P_b / \partial M_f^*)$ is positive since an increase(a decrease) in M_f^* increases(decreases) demands for home bonds and, thus, raises(declines) the home bond price. Thus (26), (30) and (32) show that, if the home country is a relatively bond preferring country, a marginal increase in the amount of money supplied by the foreign monetary authorities appreciates home money against foreign money and *vice versa*. However, if the home country is a relatively money preferring country, the effect of a marginal change in the supply of foreign money on the exchange rate is ambiguous and there is a possibility that the exchange rate and the supply of foreign money move in the same directions.

When the international asset trades are sufficiently small, (32) reduces to

$$\frac{\partial E}{\partial M_f^*} \doteq \frac{m_f^* m_h \partial P_b B_h / \partial M_f^*}{(m_h^* b_f - m_f^* b_h) S}. \quad (33)$$

Then, since $\partial P_b B_h / \partial M_f^* > 0$ and $(m_h^* b_f - m_f^* b_h) S > 0$ hold for the home country from the above arguments and (26), the right side of (33) is always negative. Therefore, if the international asset trades are sufficiently small a marginal increase in the foreign money supply always appreciates home money against foreign money, and *vice versa*.

As a fiscal policy, consider a pure bond policy that a change in the amount of home government bonds by the fiscal authorities is not accompanied by any change in the supply of home money by the monetary authorities. This kind of bond policy is often observed in such the case where the fiscal authorities independently vary the existing amount of bonds in order to pursue its own political purpose without any cooperation of the monetary authorities. In this

case, since $\partial M_h / \partial (P_b B_h) = 0$ holds, the effect of a change in the amount of government bonds on the exchange rate is given by

$$\frac{\partial E}{\partial P_b B_h} = \frac{(m_f^* m_h - m_f m_h^*)}{(m_h b_f - b_h m_f) M_f^* + (m_h b_f - b_h m_f + m_h^* b_f - m_f^* b_h) S}. \quad (34)$$

From (26), (30) and (34) it is demonstrated that if the home country is a relatively bond preferring country, a marginal increase in the volume of home bonds issued by the home government appreciates home money, and *vice versa*. On the other hand, the sign of this effect is ambiguous in a case where the home country is a relatively money preferring country and there is a possibility that a marginal increase (decrease) in the volume of home bonds issued by the home government depreciates (appreciates) home money¹¹.

If the international trades of assets are sufficiently small, the effect of a change in the amount of government bonds on the exchange rate is given by

$$\partial E / \partial P_b B_h \doteq 0. \quad (35)$$

Hence, in circumstances where the international asset trades are sufficiently small the exchange rate is independent of a marginal change in the volume of the bonds issued by the home government and does not depend on whether the home country is a relatively bond preferring country or a relatively money preferring country.

Finally, the effect of a marginal change in trade surplus of the home country on the exchange rate is given by

$$\frac{\partial E}{\partial S} = \frac{(m_f^* m_h - m_f m_h^*) \partial P_b B_h / \partial S - E (m_h b_f - b_h m_f + m_h^* b_f - m_f^* b_h)}{(m_h b_f - b_h m_f) M_f^* + (m_h b_f - b_h m_f + m_h^* b_f - m_f^* b_h) S}. \quad (36)$$

Considering that $\partial P_b / \partial S$ is regarded as positive since an increase (a decrease) in S increases (decreases) demands for home bonds and raises (declines) the home bond price, (26) and (30) ensure that the right side of (36) is negative in a case of $(m_h b_f - b_h m_f) < 0$. Consequently, a marginal increase (decrease) in trade surplus of the home country appreciates (depreciates) home money when the home country is a relatively bond preferring country. On the contrary, when the home country is a relatively money preferring country the effect of a marginal change in trade surplus of the home country on the exchange rate is ambiguous, and thus there is a possibility that a marginal increase (decrease) in trade surplus of the home country depreciates (appreciates) home money against foreign money.

If the international trades of assets are sufficiently small, (36) becomes

$$\partial E / \partial S \doteq - (m_h \partial P_b B_h / \partial S + E b_h) / b_h S, \quad (37)$$

which implies that a marginal increase (decrease) in trade surplus of the home country always appreciates (depreciates) home money against foreign money when the international asset trades are sufficiently small. The propositions with respect to (36) and (37) also combine to show that as the international asset trades become significant, we have a possibility that the effect of a marginal increase (decrease) in trade surplus of the home country on the exchange rate becomes

4. Concluding Remarks

This paper firstly establishes a generalized two-country, two-good, two-money, two-bond continuous-time international portfolio model considering the uncertainty with respect to the exchange rate, the goods prices and the bond prices in order to examine the portfolio rules of international investors. Then the equilibrium conditions of international asset markets are proposed to analyze the exchange rate determination and the effects of changes in trade imbalances, monetary and fiscal policies on the exchange rate. This paper does not assume purchasing power parity(PPP), interest rate parity(IRP), the law of one price and the investors' homogeneous expectations and the one-kind-asset world with only either monies or bonds.

Since the PPP assumption implicitly requires the same consumption patterns of individuals in two countries and the law of one price, PPP does not hold in general. IRP does not generally hold either since both the expectations and attitudes toward risk of home investors are not the same as those of foreign investors. It is well known that uncertainty of E is not equal to that of $1/E$. One of the significant features of a monetary economy is that international investors usually evaluate their own wealth in terms of their domestic monies.

In discussing the individuals' international portfolio rules this paper considers four kinds of assets, that is, monies and bonds supplied in the two countries. Therefore, it can investigate more actually some of interesting phenomena observed in the international asset markets. It describes the international portfolio rules including these four assets and proves that home and foreign monies as well as home and foreign bonds are not perfectly gross substitutive for each other. This implies, in turn, that the propositions on the assumption of homogeneous investors do not hold in general.

The general equilibrium analysis of international asset markets shows that the exchange rate depends on the consumption and portfolio rules of investors in two countries which, in turn, depend on their expectations and their attitudes toward risk, the amounts of monies and bonds issued in two countries and the volume of trade imbalances between two countries, among others. In particular, the main result that the exchange rate depends on the volume of trade imbalances between two countries stems from the fact that the adjustment speed of asset markets is much faster than that of trade imbalances.

Finally the paper examines the effects of changes in the amount of monies and bonds controlled by the home and foreign governments and the volume of trade imbalances between two countries on the exchange rate. It is shown that the signs of large changes in these parameters on the exchange rate are always judged definitely and that the signs of marginal changes in these parameters on the exchange rate are also determined in the case where the volumes of international asset trades are sufficiently small.

However, it is proposed that as the volume of international asset trades becomes significant the signs of marginal changes in these parameters on the exchange rate are ambiguous without the condition that the home country is a bond preferring country. Therefore, if this condition is not satisfied there is a possibility that a marginal expansion(reduction) of home money through open market operation and/or a marginal decrease(increase) in trade surplus of the home country appreciates(depreciates) home money against foreign money.

The model in this paper could be further extended in some directions. The explicit introduction of real sectors will make clearer long-run relationships between financial and real sectors. And the model will also be extended so as to investigate some welfare aspects of international financial markets.

Appendix

To solve the continuous-time consumption-portfolio problem of the home investor, the technique of stochastic dynamic programming is used. Define

$$J(W, P_1, P_2, P_b, P_b^*, E) \equiv \max_t E_t \left[\int_t^T \tilde{u}(\tau) d\tau \right], \quad (A.1)$$

where E_t is the conditional expectation operator, conditional on $W(t) = W$, $P_1(t) = P_1$, $P_2(t) = P_2$, $P_b(t) = P_b$, $P_b^*(t) = P_b^*$ and $E(t) = E$. Since $\tilde{u}$ is strictly concave in x_i , there exist, by Bellman's theorem, optimal controls, x_i , m_h , b_h , m_h^* and b_h^* which maximize F defined such as

$$F(x_1, x_2, m_h, b_h, m_h^*, b_h^*, W, P_1, P_2, P_b, P_b^*, E) = \tilde{u} + \Theta(J), \quad (A.2)$$

where $\Theta(J)$ denotes the Dynkin operator.

Denoting as subscripts the partial derivatives of the Bellman function J with respect to its arguments, and considering $m_h + b_h + m_h^* + b_h^* = 1$, (1) and (A.1), one obtains its average expected change per unit of time as

$$\begin{aligned} \Theta(J) = & J_W[(A b_h + B m_h^* + C b_h^* + D)W - X_c] + \sum_i J_{P_i} P_i P_i + J_{P_b} P_b P_b \\ & + J_{P_b^*} P_b^* P_b^* + J_E E E + \frac{1}{2} J_{WW} [\{ \alpha_1 \sigma_1 + \alpha_2 \sigma_2 + \sigma_b b_h + \sigma_b^* b_h^* \\ & + \sigma_e (m_h^* + b_h^*) \}^2 W^2] + \sum_K J_{WK} [\{ -\alpha_1 \sigma_{1K} - \alpha_2 \sigma_{2K} + \sigma_{bK} b_h + \sigma_{b^*K} b_h^* \\ & + \sigma_{eK} (m_h^* + b_h^*) \} WK] + \frac{1}{2} \sum_H \sum_K J_{HK} \sigma_{HK} HK, \end{aligned} \quad (A.3)$$

where $i = 1, 2$, and K (and H) = P_1, P_2, P_b, P_b^*, E are used, for notational simplicity. Hence, substituting (A.3) into (A.2), the first-order conditions are given in the usual fashion by differentiating F with respect to x_i , b_h , b_h^* and m_h^* and by equating the results to zero, i.e.,

$$\gamma u_{x_i} / x_i - J_W P_i Q = 0, \quad i = 1, 2, \quad (A.4.i)$$

$$\begin{aligned} J_W A W + J_{WW} [\{ -\alpha_1 \sigma_1 b - \alpha_2 \sigma_2 b + \sigma_{bb} b_h + \sigma_{b^*b} b_h^* \\ + \sigma_{eb} (m_h^* + b_h^*) \} W^2] + \sum_K J_{WK} \sigma_{Kb} WK = 0, \end{aligned} \quad (A.4.ii)$$

$$\begin{aligned} J_W C W + J_{WW} [\{ \sigma_b^* + \sigma_e \} \{ -\alpha_1 \sigma_1 - \alpha_2 \sigma_2 + \sigma_b b_h \\ + \sigma_b^* b_h^* + \sigma_e (m_h^* + b_h^*) \} W^2] + \sum_K J_{WK} (\sigma_{b^*K} + \sigma_{eK}) WK = 0, \end{aligned} \quad (A.4.iii)$$

$$\begin{aligned} J_W B W + J_{WW} [\{ -\alpha_1 \sigma_1 e - \alpha_2 \sigma_2 e + \sigma_{be} b_h + \sigma_{b^*e} b_h^* \\ + \sigma_{ee} (m_h^* + b_h^*) \} W^2] + \sum_K J_{WK} \sigma_{eK} WK = 0, \end{aligned} \quad (A.4.iv)$$

$$K = P_1, P_2, P_b, P_b^*, E.$$

Thus, we obtain (8) in section II from these equations of (A.4).

Notes

- (1) See D. Breeden (1979), D. Caves and E. Feige (1980), R. Clarida (1990), A. Fraga (1986), J. B. Macedo (1983), M. Mussa (1988), P. Neary (1988), A. C. Stockman and L. Svenson (1987), R. Stulz (1984), L. Svensson (1985) and their references, for example.
- (2) For example, W. Branson and D. Henderson (1985), R. Clarida (1990), A. Fraga (1986) and J. Macedo, J. Goldsrein and D. Meerscham (1988) have not considered monies, and R. E. Jr. Lucas (1982) and J. Macedo (1983) have not included bonds.
- (3) In fact, Japan has continued a great deal of trade surplus for about the last two decades, but the exchange rate has not adjusted so as to attain the equilibrium of trade balance. Thus, it is recognized today that the exchange rate which is supposed to equilibrate trade balance of goods is generally different from that observed in international financial markets. As regard these arguments, see Matsumoto and et al. (1989), for example.
- (4) J. Bergstrand (1991), W. Branson (1988), R. Clarida (1990), R. Driskill, M. Nelson and S. Steven (1992), J. Frankel (1988), J. Frankel and A. Kenneth (1990), J. Macedo (1983), L. Svenson (1985) have indicated that PPP has lost its validity to explain the exchange rate movement.
- (5) Many papers have adopted the assumptions of IRP and identical investors. However, R. Driskill, M. Nelson and S. Steven (1992) have presented some evidences of the investors' heterogeneous expectations, for example.
- (6) See A. Fraga (1986), R. Hodrick (1990), Y. Ishii (1993) and R. Stulz (1984), for example, as the models of the general equilibrium analysis of the exchange rate based on the international portfolio approach.
- (7) A bequest function with elasticity γ with respect to terminal wealth does not change the results. A discount factor on instantaneous utility is also easily introduced. Finally T is supposed to be very large so that we may regard it as an approximately infinite horizon without losing the essential generality of our model.
- (8) In this paper, since I concentrate on the investigations of relationships among the exchange rate, the investors' portfolio choices and the government policies in the home and foreign countries, the instantaneous flow of real noncapital income is assumed to be zero without losing the essential features of the results in this paper.
- (9) Similarly, it is demonstrated that in the bonds models where only bonds are internationally traded the home and foreign bonds are always perfectly gross substitutive for each other.
- (10) See W. Branson (1988), D. Papell (1989) and H. Yoshikawa (1990).
- (11) See W. Branson (1988), D. Papell (1989) and H. Yoshikawa (1990) again.

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