A Theoretical Analysis of Public Intermediate Goods and International Trade*

Makoto Tawada**
Nagoya University

Abstract

The aim of this paper is to survey the theoretical studies of international trade with public intermediate goods by focusing my works relating this topic and to discuss future problems in this field.

Key Words: public intermediate goods; production possibilities; international trade theorems, gains from trade

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1 Introduction

The aim of this paper is to survey the theoretical studies of international trade with public intermediate goods by focusing my works relating this topic and to discuss future problems in this field.

Historically, in the theoretical studies of international trade, since after Samuelson elaborated the Heckscher-Ohlin trade model and proposed trade theorems with rigorous proofs based on the Heckscher-Ohlin trade model, many trade theorists have been attracted to this model and tried to make extensions. There are two ways of the extensions. One is to generalize the numbers of goods, factors and countries and to allow non-constant returns to scale technologies. The other is to accommodate some additional factors like intermediate goods, non-traded goods, natural resources, external economies and so on.

As a latter type of extensions, to accommodate public intermediate goods into the Heckscher-Ohlin trade model is one of important issues and many studies appeared to re-examine the traditional trade theorems like Stolper-Samuelson, Rybczynski, and factor price equalization theorems. There are some reasons why this topic is important. The existence of the public intermediate goods affects the private production in the way of scale effect even though every technology is constant returns to scale. This is because public intermediate goods can be used

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**Corresponding author: Graduate School of Economics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, 464-8601, Japan, E-mail: mtawada@soec.nagoya-u.ac.jp
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jointly to all industries under the assumption that the use of public goods does not bring forth of congestion. Thus, in this case, the production frontier may be non-concave. Another reason is related to the supply of the public goods. Usually private sectors cannot supply these goods and thus the government should play the role. Then, we have to set the rule to supply them by the government. Finally we have to distinguish the types of public goods. If a public good is of pure type, there is no congestion no matter who and how many people make use of it. But the good is of impure type, there may be congestion depending on who and how many people make use it. When we tackle this topic, we have to take these aspects into our consideration at least.

The following section is devoted to a review of existing studies relating to the analysis of the Heckscher-Ohlin model with public intermediate goods. Then, my recent study on this topic is briefly introduced in Section 3. Finally some topics worth investigating in future are discussed in the last section.

2 Review on the Heckscher-Ohlin Analysis with Public Intermediate Goods

As I pointed out in the introduction, we have to characterize the properties of public intermediate goods when we build a Heckscher-Ohlin model accommodating public intermediate goods. Meade(1952) first distinguished two types of public intermediate goods. The first type is called as unpaid factors of production and the second as creation of atmosphere. In the use of the public intermediate goods of the first type, there is congestion within an industry but no congestion among industries. Thus the public good of this type can be regarded as a semi-public good. The use of the second type of public goods generates no congestion even within any industries. Thus this type can be considered as pure public goods. Following Meade's clarification, Negishi(1973) formulated the first type such that the industry production function is linearly homogeneous with respect to private inputs excluding the public inputs, whereas the second type such that the industry production function is linearly homogeneous with respect to all inputs including public inputs. Between these two types, the obtained results are quite

Figure 1

Commodity 2

Production Equilibrium

Commodity 1

Production Frontier

Figure 1
different.

First we treat the case of unpaid factors of production. One important feature of this case is that the production possibility frontier is strictly concave to the origin whatever the numbers of goods, primary factors and public inputs. To derive this result, the public inputs are assumed to be produced by the use of primary factors under the constant returns to scale technologies. (See Tawada (1980).) In this economy, suppose that the government supply the public intermediate goods by the Lindahl pricing rule in order to satisfy the Samuelson-Kaizuka efficient condition. Then the equilibrium production point is in the production frontier and the price line is tangent to the frontier on that point if the all private sectors are subject to perfect competition. Figure 1 shows the production possibility frontier to be concave to the origin. Let $P$ be the output price ratio of commodity 1 to commodity 2. Then point A in the figure is an equilibrium production point where the price line is tangent to the frontier.

Bearing these facts into our mind, we accommodate a public intermediate good of semi-public type into the Heckscher-Ohlin two commodity and two primary factor model. Then, under the factor intensity condition that the capital-labor ratio of the public intermediate good production industry lies in between the ratios of the two private industries, the weak Rybczyinski theorem and the weak Stolper-Samuelson theorem hold. Here, the 'weak' means the relative response to a relative change. For example, the weak Rybczyinski theorem implies that an increase in a factor endowment creates an increase in the output ratio of the industry where the increasing factor is intensively used. Thus, the Heckscher-Ohlin Theorem can be also shown to carry over. As for the factor price equalization theorem, it also holds in a weak sense, implying that the ratio of the factor prices tends to be equalized between countries by the opening of trade. The detailed and precise proofs of these theorems are demonstrated in Tawada and Okamoto (1983). Although they did not discuss about the gains from trade, the gains from trade is obviously assured by the concave property of the production frontier and the fact that the price line is tangent to the frontier at the production equilibrium point.

Figure 2

![Production Frontier and Equilibrium Point](image-url)
Now we concentrate on the case of pure public intermediate goods. This case was treated first by Manning and McMillan (1979) in a model of Ricardian type where only one primary factor exists. They made it clear that the production frontier is strictly convex to the origin in general. The shape of the production frontier is shown as in Figure 2. The reason why the frontier is convex to the origin is that the public input plays a role similar to the production externalities and so the private production functions including the public input can enjoy increasing returns to scale as a whole. Actually Ethier (1981) presented a convex production frontier in the Ricardian type of economy where Marshallian external economies is working in one industry. In a Ricardian small country economy with a convex production frontier, the essential discussion of the gains from trade follows the usual Ricardian analysis. Thus, after opening trade, specialization to one industry occurs and the trade gains are assured. These are shown in Manning and McMillan (1979).

Then, our interest is naturally directed to the Heckscher-Ohlin framework from that of Ricardian. Similar to the effect of external economies analyzed by Herberg and Kemp (1969), pure public inputs distort the production possibility frontier in a complicated manner once we allow two primary factors to exist. Thus, the frontier usually has concave as well as convex portions and there are possibly multiple equilibria, as is illustrated in Figure 3. In the figure, both point A and B are production equilibria for relative price $P$. Then, we cannot expect the validity of the familiar trade theorems. So our next step is to seek a condition for the production frontier to be concave to the origin. Tawada and Abe (1981) proposed the production functions of two industries as

$$X_i = R^a F^i(V_1^i, V_2^i), \quad \text{for } i = 1, 2,$$

where $X_i$ is the output of industry $i$, $V_j^i$ is the input of factor $j$ in industry $i$, $R$ is the public input, $a$ is a parameter satisfying $0 < a < 1$ and $F^i(V_1^i, V_2^i)$ is a linearly homogeneous and quasi-concave function. Notice that the public input falls into both production functions in the same degree. Under this special production functions, the production frontier becomes concave to the origin as in Figure 1.

In this economy, Tawada and Abe demonstrated that the weak Stolper-Samuelson, the weak Rybczynski, the
factor price equalization, and the Heckscher-Ohlin theorems carry over without any restriction on the factor intensity of the public intermediate good production. In this case again, trade is obviously gainful, though they did not explicitly mention it.

Okamoto (1985) and Ishizawa (1988 and 1991) made use of the stability condition of a dynamic adjustment process in order to confine their discussion to a concave portion of the production frontier. They employed different dynamic processes with each other, but reached similar conclusions on trade theorems. Relying on their analyses, we can speculate in a general case that, even if there are multiple equilibria, a stable equilibrium should be somewhere in the concave portion of the production frontier. Thus, trade is gainful as long as the prices do not change greatly after opening trade. Further extensions of the analysis were made by Altenburg (1987 and 1992) and Terasaki (1990), but the analysis of the trade theorems becomes complicated without any additional assumptions or conditions.

3 Two-Country analysis with a public intermediate good

The analysis of international trade with public intermediate goods was confined to a small country case so far. Suga and Tawada (2007) tackled with a two-country analysis recently and derived an interesting result on the trade gains. They extended Manning and McMillan's small country model (1979) to a two-country framework. Then, following Ethier's manner (1982), they adopted the Marshallian dynamic adjustment process in order to determine which equilibrium is actually attained among multiple equilibria.

They assume that the difference between two countries is placed only on the amount of labor endowment. The public intermediate good serves both final good industries such a way that the production functions can be formulated as

\[ X_i = A_i(R)L_i, \quad \text{for } i = 1,2, \]

where \( L_i \) is labor input in industry \( i \) and \( A_i \) is a function satisfying \( A'' > 0 \) and \( A'''' < 0 \). Then, the pattern of trade is shown to be that a country endowed with a larger amount of labor exports the commodity having a larger elasticity of marginal labor productivity with respect to the public input. Here we should notice that specialization is not necessary by trade in a two-country model. Depending on the world demand, one country possibly produces both commodities, which creates a source of an interesting result on trade gains.

Now, we move to the analysis of trade gains. Remind that the production frontier is convex to the origin and the efficient production must be in the frontier. Any country whose indifference curves never touch the commodity axes must produce both commodities. So, at the autarkic equilibrium, both commodities must be produced. This, together with the Lindahl pricing rule, the efficient production occurs at the interior point of the frontier where the autarkic price line is tangent. Consider the country producing both commodities even after opening trade. The production equilibrium of this incompletely specializing country is at the point in the frontier where the international equilibrium price line is tangent. Figure 4 illustrates the autarkic and international equilibrium production and consumption points. Point A is the point of both production and consumption at the
autarky. Point C and T are the trading equilibrium points of consumption and production, respectively. The curves of WW and W'W' display the autarkic and trading equilibrium welfare indifference curves, respectively. It is clear from the figure that the curve W'W' situates below that of WW. Therefore, the incompletely specializing country must lose from trade. Different from a small country case where Manning and McMillan asserted that any country must specialize by trade and have gains from trade, an incompletely specializing country possibly emerges and that country necessarily loses from trade in the two-country economy.

![Figure 4](image)

4 Future topics of international trade with public intermediate goods

The result of the loss from trade is heavily dependent on the Lindahl Pricing rule to attain an efficient production. Thus, Shimomura (2007) gave up this assumption and assumed that each government supplies public goods strategically in order to maximize its country’s welfare. Then, two governments play a Nash game and the trading Nash equilibrium may be inefficient in the sense that the production equilibrium is located under the production frontier. Nevertheless, Shimomura (2007) proved that every country has a gain from trade in a fairly general framework where the numbers of country, good, factor, and public good are arbitrary and the public goods can serve as consumption as well as intermediate goods. However, if the public goods are international, meaning that they spill overseas, then the gains from trade theorem fails to hold. We need some additional assumptions for the validity of the gains from trade theorem. Therefore, one future topic is to investigate international trade with international public goods.

In the present survey, we have not discussed a dynamic aspect. McMillan (1978) actually treated a dynamic international trade model of Ricardian type with public capital by the optimal control theory. But their analysis is confined in a small country case. Hence the study of the dynamic two-country model is still left as a future topic.

The overlapping generations models are another type of dynamic treatment and sometimes provide interesting
aspects. Galor (1992) presented a basic model suitable to accommodate an international trade aspect and then Galor (1994) tackled the analysis of international trade in a small country Heckscher-Ohlin case. Based on Galor's analysis, Mountford (1999) studied a two-country Heckscher-Ohlin economy. Thus, one extension is to accommodate public intermediate goods into their models. Recently Shinozaki, Tawada and Yanagihara (2008) have studied international trade with a public good in an overlapping generations model and obtained a different result concerning the pattern of capital accumulation. They impose, however a rather special assumption that a public intermediate good is not accumulated. Thus we need to consider some other different trade models of overlapping generations with public intermediate goods. Moreover, a case of semi-public intermediate goods may be of another interest.

References

overlapping generations model", mimeographed, Nagoya University


