Recognition Accuracy Improvement of Obfuscated QR Code by Using Reliability Information and GMD Decoding

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\textbf{Abstract:} In recent years, QR code is used in various situations such as in medical prescription and the boarding procedure at the airport in addition to conventional applications. With expanding application, there are some QR codes that cannot be read by the conventional method. In this paper, we refer to this kind of QR code as obfuscated QR code. Considering the situation that the QR code is more common, improving the encoding method is not realistic. Hence, in reading obfuscated QR code, improving only the decoding method is required. In general, Euclidean decoding is used in the decoding method of QR code. On the other hand, Generalized Minimum Distance (GMD) decoding has been proposed. GMD decoding is a method of approximately performing maximum likelihood decoding by using information of the likelihood of each symbol called reliability. However, a method for calculating the reliability information of each symbol of the two-dimensional code has not been proposed. In this paper, we propose a method of calculating reliability information using the graphical features of QR code. Then, we show the proposed method is more useful in recognition accuracy of an obfuscated QR code than the conventional method using Euclidean decoding.

\textbf{Keywords:} Reed-Solomon codes, Euclidean decoding, GMD decoding, QR code

1. Introduction

Two-dimensional code typified by a QR code\cite{1,2} is used in the boarding procedure at the airport and prescription at medical treatment, in addition to traditional applications such as in inventory management and exchanging contacts. In particular, the recognition accuracy of QR code is excellent, and QR code is widely recognized. In recent years, movements to improve the convenience of services by using QR code, when paying or registering are increasing. The recognition accuracy of the QR code is guaranteed by the high position detection accuracy and the high error correction capability. However, due to the expanding applications of the QR code, there are some cases where reading fails in the conventional decoding method. For example, blurred noise tends to be added to the QR code printed on the prescription due to the influence of the printing environment, and there are many cases where reading fails in the conventional decoding method. Errors exceeding the correcting capability, generated in the encoding region\cite{1,2} of the QR code, and failure in detection using geometric features of QR code, are the main causes of recognition failure in the conventional decoding method. In this paper, QR code which cannot realize high recognition accuracy in the conventional decoding method is expressed as obfuscated QR code.

By changing the standard of the encoding method of QR code, a method that improves the error correction capability has been proposed in Ref.\cite{3}. However, some applications have been expanded based on the high awareness of QR code, it is important to be able to read the previously printed QR code. In other words, without changing the encoding method of QR code, methods of improving the recognition accuracy by changing only the decoding method is desirable. Euclidean decoding\cite{4} is widely used in the conventional decoding method in the decoding processing of the QR code. On the other hand, Generalized Minimum Distance (GMD) decoding has been proposed in Ref.\cite{5}. GMD decoding is a method of approximately performing maximum likelihood decoding by using information of the likelihood of each symbol called reliability and repeating the erasure correction\cite{6} while changing the erasure location. In the GMD decoding method, it is difficult to improve the decoding accuracy unless the reliability information can be acquired with high accuracy. The symbol\cite{1,2} in the QR code is composed of 8 modules which are processing units for performing encoding and decoding processing. The module\cite{1,2} is the minimum unit constituting the QR code, and consists of light and dark rectangles. In order to use the GMD decoding in the decoding processing of the QR code, reliability information of each symbol is necessary. However, a method for calculating the reliability information of each symbol of the two-dimensional code has not been proposed. In this paper, we propose a method to calculate the reliability information of each symbol from the tendency of the errors that occur in the QR code. By using the proposed method and GMD decoding, we show that it is possible to realize high recognition accuracy compared with the conventional method in reading an obfuscated QR code.

The paper is organized as follows. Section 2 explains about the QR code and GMD decoding. Section 3 proposes a method that uses reliability information and GMD decoding. Section 4 evaluates the proposed method and Section 5 concludes the paper.
2. QR Code and GMD Decoding

The procedure for generating the QR code is described in Section 2.1, and the procedure for reading the QR code is described in Section 2.2. In addition, the algorithm of GMD decoding used in this paper is described in Section 2.3.

2.1 Generation of QR Code

The QR code is generated by arranging modules composed of light and dark in a lattice pattern. The QR code is divided into a functional pattern and an encoding region. A functional pattern is composed of a finder pattern and a timing pattern. An encoding region is composed of a data code word, an error correction code word, and format information. Up to 40 versions exist in the QR code depending on the data capacity to store and the error correction level. For example, in version 1, it consists of 21 modules. To store data in the QR code, shortened Reed-Solomon (RS) codes in the finite field \( \mathbb{F}_2^8 \) is used to correct a certain amount of errors. Four levels of error correction levels are prepared: L (7%), M (15%), Q (25%), and H (30%). For example, in error correction level M, it can be corrected in the case where an error occurs of 15% or less of the entire encoding region. In this paper, QR code with error correction level L in version 1 is expressed as 1-L type.

We describe a generation procedure by taking a case of generating a 1-M type QR code as an example. BCH codes is used in format information. And shortened (26, 16, 4) RS code is used in the data code word and the error correction code word of the 1-M type QR code. The shortened (26, 16, 4) RS code is used for generating a data code word of 16 symbols and an error correction code word of 10 symbols. When arranging the data code word and the error correction code word on the QR code, an XOR operation with the determined mask pattern is performed so as not to bias the proportion of light and dark. The configuration of the 1-M type QR code is shown in Fig. 1. Each rectangle in the data code word and the error correction code word of Fig. 1 indicates one symbol, symbols corresponding to D1 to D16 are the data code words, and symbols corresponding to E1 to E10 are the error correction code words. The symbol in the QR code is composed of 8 modules where each of the modules expresses one part of light or dark image. Furthermore, the module is read by imaging device which has higher resolution than the module size for detection of QR code. As the example of the imaging device is discussed as a size of pixel, namely, it means “symbol size > module size > pixel size.” The Construction of symbols, modules, and pixels in 1-M type QR code is shown in Fig. 2.

2.2 Decoding of QR Code

In this section, we describe the procedure to read stored data from the image containing the QR code. First, an operation of detecting a region including the QR code from an image is performed. When detecting a region including the QR code, the geometric features of the QR code are used. This geometric feature is a finder pattern, and its features are shown in Fig. 3. In Fig. 3, there is the feature that the ratio of the light-dark module width is always 1 : 1 : 3 : 1 : 1. When the scanning line passes through the center of the finder pattern, even if the QR code is inclined in the image, it can be read because this ratio is kept from any direction. In the decoding processing of the encoding region, the mask applied to the encoding region is canceled, and the data code word and the error correction code word are restored. Then, decoding processing of RS code is performed to obtain stored data. Euclidean decoding is widely used for the decoding processing of the RS code. The above reading flow is shown in Fig. 4.

2.3 An Algorithm of the GMD Decoding

In erasure correction [6], when the number of erasures is \( \epsilon \) and the number of errors is \( e \), errors including erasure can be corrected within a range that satisfies the following equation with minimum distance \( d \).

\[
2e + \epsilon \leq d - 1
\]
In GMD decoding, correction is performed while changing the number of erasures. First, \( d - 1 \) pieces of received symbols are handled as erasures from the smaller reliability, and correction is attempted on the assumption that there is no other error. If correction fails, \( d - 3 \) pieces of received symbols are handled as erasures from the smaller reliability, and correction is attempted on the assumption that there is one error. After that, every time the correction fails, the correction is repeated while decreasing the number of erasures. Finally, if it cannot be corrected when the number of erasures is 0 and the number of errors is \( [(d - 1)/2] \), decryption fails. In this way, in the GMD decoding, correction is attempted at maximum \( [(d - 1)/2] \) times. Also, suppose that the correction is successful when Eq. (2) is satisfied with the sum of the reliability information of all symbols being \( \theta \) and the length of the code word being \( n \).

\[
\theta - 2 \sum_{\text{er}} \theta_{\text{er}} > n - d \tag{2}
\]

Kamiya proposed a scheme to reduce the computational complexity of GMD decoding, by relating the GMD decoding problem to the multisequence shift register synthesis problem [10]. In the experiment of Section 4, we used Kamiya’s method as an algorithm of the GMD decoding.

**Algorithm 1: Kamiya’s Method**

1. \( k \leftarrow 0 \), \( L_0 \leftarrow 0 \),
   \[ P_0(z) \leftarrow 1, \ R_0(z) \leftarrow T(z), \ U_0(z) \leftarrow 0, \ V_0(z) \leftarrow z^{2l} \]

2. \[ Q_k(z) \leftarrow \frac{R_k(z)}{1 - x_k z}, \ y_k \leftarrow R_k(z) - (1 - x_k z)Q_k(z) \]
   \[ W_k(z) \leftarrow \frac{V_k(z)}{1 - x_k z}, \ v_k \leftarrow V_k(z) - (1 - x_k z)W_k(z) \]

3. if \( (y_k \neq 0) \) then
   \[ L_{k+1} \leftarrow L_k \]

\[ P_{k+1}(z) \leftarrow P_k(z), \ R_{k+1}(z) \leftarrow Q_k(z), \]
\[ U_{k+1}(z) \leftarrow (1 - x_k z)U_k(z), \ V_{k+1}(z) \leftarrow V_k(z) \]

04: if \( (y_k \neq 0) \) then

(a) if \( (v_k \neq 0 \) and \( 2L_k \geq k + 1 \) then
   \[ L_{k+1} \leftarrow L_k \]
   \[ P_{k+1}(z) \leftarrow P_k(z) - \frac{y_k}{v_k} U_k(z) \]
   \[ R_{k+1}(z) \leftarrow Q_k(z) - \frac{y_k}{v_k} W_k(z) \]
   \[ U_{k+1}(z) \leftarrow (1 - x_k z)U_k(z), \ V_{k+1}(z) \leftarrow V_k(z) \]

(b) else
   \[ L_{k+1} \leftarrow L_k \]
   \[ P_{k+1}(z) \leftarrow (1 - x_k z)P_k(z), \ R_{k+1}(z) \leftarrow R_k(z) \]
   \[ U_{k+1}(z) \leftarrow U_k(z) - \frac{v_k}{y_k} P_k(z) \]
   \[ V_{k+1}(z) \leftarrow W_k(z) - \frac{v_k}{y_k} Q_k(z) \]

05: if \( k = k + 1 \), output \( L_k, P_k(z), R_k(z) \)

if \( (k < d - 1) \) then return to (02), else stop.

\( T(z) \) is a modified syndrome polynomial in the case where the number of erasures is \( d - 1 \). \( x_k \) is a symbol with high reliability at the \( k \)-th when the highest reliability information is selected as the 0-th among the \( d - 1 \) erasures. Also, in Algorithm 05, the reliability information of \( x_k \) is set to 0 before updating \( k \). When executing the GMD decoding, after finding \( L_k, P_k(z), R_k(z) \), find their each roots. Let \( P[k] \) and \( R[k] \) be the set of roots and let \( |P[k]| \) be the number of roots of \( P_k(z) \). Also, \( k = 0 \), and let \( \theta \) be the sum of the reliability information of all symbols. However, even if the reliability information of the erasure location \( x_k \) is set to 0, the value of \( \theta \) is not updated. When at least one condition of \( 2L_{k} \geq k + 1 \) or \( |P[k]| \neq L_k \) is satisfied, confirm the condition again with \( k + 2 \) as \( k \) and finish at the point where it becomes \( k > d + 1 \). If the condition is not satisfied, let \( \theta_{i} \) be the sum of the symbols at the location \( P[i] \) and the erasure used at that time, excluding \( R[k] \). If \( \theta - 2\theta_{i} > n - d \) is satisfied, it is determined that the GMD decoding is successful. The error location is an element of \( P[k] \).

Let \( \Psi(z) \) be the erasure location polynomial, the number of errors \( Y_j \) is obtained by the following equation.

\[
Y_j = \frac{\alpha^{-j(d-2)}}{U_{j}(\alpha^{-j})\Psi(\alpha^{-j})P_{j}(\alpha^{-j})} \tag{3}
\]

When \( \alpha^{-j} \) is used as an erasure, the following equation is used.

\[
Y_j = \frac{R_{j}(\alpha^{-j})}{\alpha^{-j}\Psi(\alpha^{-j})P_{j}(\alpha^{-j})} \tag{4}
\]

### 3. Calculating the Reliability Information of Obfuscated QR Code

In this section, we propose a method to calculate reliability information of symbols in an obfuscated QR code used in GMD decoding. Section 3.1 defines the obfuscated QR code. Section 3.2 describes the method of calculating the reliability information of the symbol.
3.1 Definition of Obfuscated QR Code

There are cases where blurred noise is added to the QR code at the time of printing or use, and there are cases where some areas of the QR code are missing. Figure 5 shows an example of an obfuscated QR code added with blurred noise over the entire encoding region, and an obfuscated QR code missing the lower right part of the QR code. In this paper, the QR code added with the blurred noise over the entire encoding region and the QR code shadowing by the circular shape are defined as the obfuscated QR code. And, we deal with the image of printed obfuscated QR code, and the constituent unit of the RGB value in the image is expressed as pixel.

3.2 Calculating the Reliability Information of Obfuscated QR Code

We propose a method to read an obfuscated QR code using GMD decoding in this section. As described in Section 2.1, the shortened RS code is used when generating the QR code, and the GMD decoding can be used. In order to use the GMD decoding, the reliability information of the symbol is necessary. And in order to improve the decoding accuracy, it is important to obtain this reliability information with high accuracy. In the proposed method, the symbol luminance value and shape information are obtained from the reliability information in the symbol. The proposed method consists of the following three steps.

Step 1. From the luminance value of the symbol, the degree of noise of the entire symbol is derived to calculate the reliability information.

Step 2. From the extent of noise in the module, verify the degree of noise in the symbol and correct the reliability information.

Step 3. Perform the GMD decoding by using the reliability information.

In Step 1, when calculating the luminance value of the entire symbol, check the number of light and dark modules in the 8 modules that make up the symbol, and deal with only the light or dark modules with the larger number. Let the average value of the brightness values of the module with the larger number of light and dark modules be the brightness value of that symbol. When the number of light and dark modules is equal, the average value of brightness values of dark modules is used. The luminance value of each module is calculated from the RGB values of the center of the module and the surrounding pixels. Letting the RGB values be r, g, b (0 ≤ r, g, b ≤ 255), the luminance value L (0 ≤ L ≤ 255) can be obtained by Eq. (5) [8]. In this paper, the average of the luminance values of the center of the module and its surroundings, nine pixels in total, is taken as the luminance value of the module.

\[ L = 0.298912r + 0.586611g + 0.114477b \]  

If the luminance value of the symbol is close to 255 or 0 which is equal to the value at the time of generation, it can be considered that the probability of noise being added is low. On the other hand, in the case of a value close to 128, it can be considered that it is difficult to accurately identify light and dark. Utilizing this feature, a reliability function corresponding to the luminance value of the symbol is set. The reliability function defined in Eq. (6). In Eq. (6), L is a luminance value, and s is a threshold value for discriminating brightness when decoding.

\[ \theta = \begin{cases} \frac{s - L}{s} & (L < s) \\ \frac{L - s}{255 - s} & (L \geq s) \end{cases} \]  

Figure 6 shows Eq. (6). When the luminance value is close to the threshold value s, the reliability function is defined so that the reliability information becomes the lowest. When the luminance value is close to 255 or 0, the reliability function is defined so that the reliability information is highest. In this paper, the QR code in the image is divided into 5 × 5 blocks, and the average value of the luminance values of all the pixels in each block is set as the threshold value s.

In Step 2, the reliability information is corrected according to the shape of the symbol. The modification uses the degree of noise in each module constituting the symbol. The degree of noise in the module is calculated using the RGB value of the pixel near the center of the module. If the RGB values of pixels near the center of the module contain values that are significantly different, it is considered that the boundary between light and dark is unclear as the shape of the module deforms. In this paper, we assume that the shape of the module is deformed when there is even one pixel whose luminance value differs by 50 or more in the luminance value of the pixel. The reliability information of the symbol obtained in step 1 is divided by the number of modules whose shape is deformed among the modules constituting the symbol. The reliability information before modification is \( \theta \), the number of modules whose shape is deformed is \( c \), and the formula for obtaining the corrected reliability information is Eq. (7).

\[ \theta' = \frac{\theta}{c + 1} \]  

Using the corrected reliability information \( \theta' \) as the reliability information of the symbol, the GMD decoding of Step 3 is performed.

4. Recognition Experiment of Obfuscated QR Code

In this section, we describe the experiment applying our
method to an obfuscated QR code. In the experiment, we compare our method, the conventional method using the Euclidean decoding, and the conventional research [7] using the erasure correction. In order to compare the decoding accuracy, an obfuscated QR code added with noise only to the encoding region is used.

The 3-M type, 6-M type and 9-M type QR code is used in the experiment. Each QR code is generated with a size composed of 13 pixels on one side of one module. The experimental environment is shown in Table 1. In the experiments, photographing is carried out excluding factors that disturb recognition, such as avoiding shadows and light on the obfuscated QR code. As an evaluation criterion, recognition for each obfuscated QR code is performed 50 times.

### 4.1 Recognition Experiment of Obfuscated QR Code Shadowing by the Circular Shape

An obfuscated QR code shadowing by the circular shape to the center part of the QR code is evaluated in this section. This obfuscated QR is used in the previous research [7]. Figure 7 shows an example of an obfuscated QR code (3-M type) shadowing by the circular shape of an area corresponding to 20%, 30%, 36% and 40% of the encoding region. The number of pixels per module in the 3-M type, 6-M type and 9-M type QR Code image in this section is about 50, 30, and 20.

Figure 8 shows the reading results of the conventional method and our method. In our method, it is possible to read the obfuscated QR code (3-M type, 6-M type) with noise added up to 36% of the entire QR code, and the obfuscated QR code (9-M type) with noise added up to 32% of the entire QR code. In the conventional method, it is possible to read the obfuscated QR code (3-M type, 6-M type) with noise added up to 22% of the entire QR code, and the obfuscated QR code (9-M type) with noise added up to 20% of the entire QR code. In the previous research [7], although there is no description about the photographing equipment and the printing environment, evaluated the QR code shadowing by the circular shape of an area corresponding to 10%, 20%, and 30% of the encoding region, while changing the version and error correction level. In the previous research [7], when the area of the noise is 20%, multiple versions have been successfully decoded at the error correction level M. However, when the noise area is 30%, all versions in the error correction level M fail to decode. Therefore, compared with the Euclidean decoding and the method using the erasure correction [7], the decoding accuracy of our method is improved.

Next, we evaluate the reliability information on the reading result of the obfuscated QR code shadowing by the circular shape. To evaluate the reliability information, we assume that the symbol is erased when the reliability information is less than 0.5, and not erased when the reliability information is 0.5 or more (When actually doing the GMD decoding, pieces of received symbols are handled as erasures from the smaller reliability, rather than estimating erasure by setting a threshold value in the reliability information). When evaluating the reliability information for each symbol, use the following equation.
Fig. 11 Experimental results of recognizing obfuscated QR code with blurred noise added to the encoding region.

\[ P = \frac{\text{All} - \text{Miss} - \text{Pass}}{\text{All} + \text{Pass}} \times 100 \] (8)

\text{All} is the number of symbols that were estimated to be erased and actually containing errors, \text{Miss} is the number of symbols that were estimated to be erased but actually without errors, and \text{Pass} is the number of symbols that were not estimated to be erased even though errors were included.

The evaluation result on the reliability information is shown in Fig. 9. Figure 9 shows the estimation accuracy of the erasure position based on the reliability information of the symbol in the reading of the obfuscated QR code. The estimation accuracy of the erasure position in the QR code of each parameter (3-M type, 6-M type and 9-M type) was calculated using Eq. (8). It was repeated 50 times and the average value was graphed. From the results, it can be seen that estimation accuracy is about 80% when reading obfuscated QR code (3-M type) with noise level up to 36% of the entire QR code. As the version is increased, the estimation accuracy decreases. When reading obfuscated QR code (9-M type) with noise level up to 36% of the entire QR code, estimation accuracy is about 20%, and also fails to read using the GMD decoding (Fig. 8).

4.2 Recognition Experiment of Obfuscated QR Code Added with Blurred Noise

An obfuscated QR code with blurred noise added to the encoding region is evaluated in this section. When creating an obfuscated QR code, we used “Pick” [11] which is one of the filter functions of GIMP [12]. The “Pick” function randomly replaces the luminance value of each pixel in the image from 9 pixels in the surrounding $3 \times 3$ pixels range. There are two parameters of “Randomization” and “Repeat,” and as both values become larger, more noise is added. In this section, we created an obfuscated QR code by fixing “Randomization” to 50 and increasing “Repeat” from 30 to 50 by 2. Figure 10 shows a sample in the case where “Repeat” is set to 30, 34, 38 and 42 (3-M type). The number of pixels per module in the 3-M type, 6-M type and 9-M type QR Code image in this section is about 50, 30, and 20. Thereafter, the parameter of “Repeat” is expressed as “noise level.”

Figure 11 shows the reading results of the conventional method and our method. In our method, it is possible to read the obfuscated QR code (3-M type) with noise level up to 44, the obfuscated QR code (6-M type) with noise level up to 46, and the obfuscated QR code (9-M type) with noise level up to 42. In the conventional method, it is possible to read the obfuscated QR code (3-M type) with noise level up to 38, and the obfuscated QR code (6-M type, 9-M type) with noise level up to 36. Therefore,
compared with the Euclidean decoding, the decoding accuracy of
our method is improved even if more blurred noise is added.

As in the Section 4.1, the evaluation result on reliability in-
formation is shown in Fig. 12. From the results, it can be seen
that estimation accuracy is about 80% when reading obfuscated
QR code (3-M type, 6-M type) with noise level up to 36. As the
version is increased, the estimation accuracy decreases. When
reading obfuscated QR code (9-M type) with noise level up to 36,
estimation accuracy is about 60%. In particular, as in Section 4.1,
since the shape of the graph is similar to the transition of the read-
ing results in Fig. 11, it is considered that the calculated reliability
information greatly contributes to improving reading accuracy.

4.3 Discussion

Experimental results show that compared with the conven-
tional decode method and the method using the erasure correc-
tion [7], our method can read the obfuscated QR code. In
the previous research [7], some symbols are regarded as an erasure,
and if the one attempt fails, reading also fails. On the other hand,
in our method, if the trial has failed, the trial is repeated while
changing the number of erasures. Besides, in the GMD decoding
method, it is difficult to improve the decoding accuracy unless
the reliability information can be acquired with high accuracy.
As shown in Figs. 9 and 11, it is considered that the calculated reliability information greatly contributes to improving reading accuracy.

In the conventional QR code application, the reading process is
performed with a plurality of times while the user is not aware.
Hence, it can be considered that if the success rate is 10% or
more, the reading process can be performed with a practical
number of trials. Our method can read the obfuscated QR code with
noise added up to 46% of the entire QR code or the obfuscated
QR code (3-M type) with noise up to 38% of the entire QR code.
Besides, the processing time required for successful reading in
the experiment is about 1.5–2.5 seconds when using the method of
Section 3. If the success rate is about 40%, it can be read in
about 10 seconds and it is considered that the usefulness is not
impaired in the QR code application.

5. Conclusion

In this paper, we propose a method using reliability informa-
tion and GMD decoding for the purpose of improving recogni-
tion accuracy of an obfuscated QR code. In Section 3, we pro-
pose a method for calculating reliability information of symbols
suitable for QR code, making it possible to use GMD decoding.
We evaluated obfuscated QR code and showed that the decoding
accuracy of our method is superior to the conventional research
and the conventional decoding method. Since our method is an
improvement of only the decoding method, it is possible to im-
prove recognition accuracy without affecting the use of existing
QR codes. In this paper, the reliability function that correspond-
ing to generic noise is used. When there is a specific tendency
to the noise added to the QR code, recognition accuracy can be
expected to be improved by newly defining a reliability function
that takes the tendency into account. Evaluation using other reli-
ability functions is the subject of future study. Furthermore, we
will develop a method that can read the QR code with high accu-
racies from a remote place using the proposed method in this paper.
QR codes are increasingly used for display of information such
as exhibition halls of companies. Therefore, if the QR code can
be read from a remote place, the convenience of the QR code is
considered to be higher. We will improve the convenience of the
QR code by improving the proposed method so that it can cope
with the case where the number of pixels of the QR code is small.

References

[1] Japanese Industrial Standards Committee: JIS, X0510, Two dimen-
[2] DENSO WAVE INCORPORATED, Qrcode.com, available from
(http://www.denso-wave.com/qrcode/).
QR Code Using Pad Code Words, FIT2013, Vol.12, No.4, pp.509–514
method for solving key equation for decoding Goppa codes, Infor-
Correcting Capability: Decoding Method using Erasure Error Cor-
rection and the Ability, IEICE Tech Rep., LOIS, Vol.111, No.286,
[8] RECOMMENDATION ITU-R BT.601-7 Studio encoding parameters
of digital television for standard 4:3 and wide-screen 16:9 aspect ratios
(2011).
asus.com/gp/Tables/ASUS_MeMO_Pad_HD7_ME173X/).
[10] Kamuya, N.: On Multisequence Shift Register Synthesis and
Generalized-Minimum-Distance Decoding of Reed-Solomon Codes,
plug-in-randomize-pick.html).
(https://www.gimp.org/).

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Fig. 12 Experimental results of estimation accuracy based on reliability
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