A Proposal of Mathematical Approach to Formulate Emotional Desensitization to Novelty

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Abstract: Novelty is an important factor of creativity in product design. Acceptance of novelty, however, depends on one’s emotions. Raymond Loewy, a pioneer of industrial design, defined a broader term between attraction of novelty and fear of the unknown MAYA (Most advanced, yet acceptable), which is important for new designs that are widely accepted in society. Yanagisawa et al. (2019) developed a mathematical model of emotion dimensions associated with novelty such as arousal (surprise) and valence (positivity and negativity). The model formalized arousal as Bayesian information gain and valence as a function of arousal based on Berlyne’s arousal potential theory. On the one hand, people get used to novelty by repeated exposure. This so-called desensitization to novelty is an important factor to consider in the design of long-term products experience. In this paper, we proposed a mathematical model of desensitization to novelty based on the emotion dimension model. We formalized the desensitization as a decrement of information gained from a novel event through Bayesian update. We derived the information gained from repeated exposure of a novel stimulus as a function of three parameters: initial prediction error, initial uncertainty and noise of sensory stimulus. With the proposed model, we found an interaction effect of initial prediction error and initial uncertainty on desensitization (decrement of information gain). Furthermore, we demonstrated that a range of positive emotions on prediction errors shifted towards more novel by repeated novelty exposure. The experimental results of previous studies supported this simulation results.

Keywords: Novelty, Emotion, Valence, Desensitization, Bayesian update

1. INTRODUCTION

Novelty is an important factor of creativity in product design. Acceptance of novelty, however, depends on one’s emotions. Raymond Loewy, a pioneer of industrial design, defined a broader term between attraction of novelty and fear of the unknown MAYA (Most advanced, yet acceptable), which is important for new designs that are widely accepted in society (Loewy, 1951). Berlyne et al. (1970) advocated the theory that there is a range that maximizes pleasant feelings between familiar and novel, simple and complex. Silvia et al. (2005) showed the validity of Berlyne’s hypothesis through experiments with shapes and works of art. They revealed that emotions for novelty differ according to characteristics such as the individual’s knowledge. Yanagisawa et al. (2019) developed a mathematical model of emotional dimensions associated with novelty that considers personal characteristics using information theory and Bayes model. The model formalized arousal (primary emotional dimension) as a function of prediction error, uncertainty and external noise. The function model was supported by experimental results using the event related potential (ERP) P300 of human participants as an index of arousal. Furthermore, they formalized valence as a function of arousal based on Berlyne's arousal potential theory.

On the one hand, people get used to novelty through its repeated exposure. Psychologists define the diminished emotional and physiological responsiveness to a stimulus after repeated exposure to it as desensitization (Murug et al., 2015). Desensitization to novelty is an important factor to consider in the design of long-term products experience. For example, Lévy et al. (2006) showed that the range of novelty that maximizes pleasant feelings, shifts to a more novel and more complicated direction by repeating stimuli. However, this study was not analyzed mathematically and biases from factors such as one’s prior knowledge and experience were not exhaustively investigated. In this paper, we applied and developed the model of Yanagisawa (2019) and described the framework of a mathematical model of desensitization using information theory. We then considered the effects of the initial prediction error and the uncertainty on the time series change of emotion as the main factors constituting novelty. Finally, we compared the experimental results of previous studies with the simulation results of this study and showed its validity.
2. MODELING DESENSITIZATION TO NOVELTY

2.1 Modeling based on information theory

In this chapter, we describe a method for mathematically modeling desensitization. Since this method is based on the model of Yanagisawa (2019), we briefly explain essential parts and focus on original parts.

We defined novelty as the amount of information a person acquires by experiencing an event. The information content given by an event (self-information) is consistent with its uncertainty before experiencing it. Considering a transition before and after experiencing an event, the expected value of self-information (information entropy) of the prior distribution represents the uncertainty of the prior expectation. The information entropy decreases by experiencing an event. This decrease is proportional to the information acquired from the event, which is called the information gain. An event with large novelty brings us large information gain and causes salient emotions such as surprise. Generally, emotions are spatially arranged by mainly two dimensions: arousal and valence. Since surprise is positioned as an emotional state with high arousal, information gain is considered to correspond to arousal.

We express desensitization using information theory. Repeating the same stimulus decreases the information gain, as the prior distribution gradually approaches the likelihood distribution. Therefore, we consider desensitization as a decrement of information gain.

2.2 Bayesian update model

Let a prior be \( \pi(\theta) \) in term of a parameter \( \theta \) that one estimates. After one obtains continuous data \( x \in R \) by experiencing an event, the prior \( \pi(\theta) \) is updated to the posterior \( \pi(\theta|x) \) according to the following formula derived from Bayes’s theorem:

\[
\pi(\theta|x) = \frac{f(x|\theta)^{\alpha} \pi(\theta)}{\int_{\theta} f(x|\theta)^{\alpha} \pi(\theta) d\theta} \propto f(x|\theta)^{\alpha} \pi(\theta),
\]

where \( f(x|\theta) \) is a likelihood function of \( \theta \) when data \( x \) is obtained. \( \alpha \) is a parameter that adjusts the amount of change of the information gain, which is called the learning rate (Yano et al., 2017).

We consider the posterior when experiencing events \( k \) times \( \pi_k(\theta|x) \) to be the prior when experiencing events \( k+1 \) times \( \pi_{k+1}(\theta) \). Assuming that the likelihood functions are independent simultaneous distributions, the order of data collection is irrelevant. Thus, the posterior \( \pi_k(\theta|x) \) is obtained from the following formula:

\[
\pi_k(\theta|x) \propto \prod_{i=1}^{k} f(x_i|\theta)^{\alpha} \pi(\theta).
\]

\( \pi_k(\theta|x) \) is proportional to the product of each likelihood function and the initial prior.

Assume one obtains \( n \) samples of event \( x \) and encodes them as a Gaussian distribution \( N(\mu,\sigma^2) \) with a flat prior. Now assume a non-flat prior of \( \mu \) that follows a Gaussian distribution \( N(\eta,\tau^2) \). Using Bayes’s theorem, the prior when experiencing an event \( n \) times is updated to a Gaussian distribution \( N(\eta_n,\tau_n^2) \), where:

\[
\eta_n = \frac{\alpha S_{pl}\bar{x} + S_i \eta}{\alpha S_{pl} + S_i}; \tau_n^2 = \frac{S_{pl}S_i}{\alpha S_{pl} + S_i}.
\]

In these formulae, \( \bar{x} \) is the mean of the data, \( S_{pl} = \tau^2 \), and \( S_i = \sigma^2 \).

2.3 Arousal update model (desensitization)

The information gain from the prior to the posterior \( G \) can be derived from formula (3) as follows:

\[
G = \int_{-\infty}^{\infty} \pi(\mu|x) \ln \frac{\pi(\mu|x)}{\pi(\mu|x)} d\mu = \frac{1}{2} \left( A + B \delta^2 \right),
\]

where \( A = \frac{g_n}{S_n} - \ln \frac{g_n}{S_n} - 1 \), \( B = \frac{\alpha^2 S_{pl}S_i}{g_n S_{pl} + S_i} \), and \( g_n = \alpha S_{pl} x + S_i \).

2.4 Formalization of acceptable prediction error

Berlyne (1970) assumed that there is a range that maximizes pleasure between the novel and the familiar, and between simple and complex stimuli. He also assumed that this hedonic qualities of stimuli arise from separate biological incentivization systems: the reward
system and the aversion system, each of which is represented by a sigmoid function. The joint operation of these two systems create an inverted U-shaped curve called Wundt curve. The valence of a stimulus changes from neutral to positive as the arousal increases but shifts from positive to negative once the arousal passes the peak positive valence. The range of pleasant feeling is obtained from the following formula:

\[ \delta_s^2 = \frac{1}{B} \ln \frac{h_e^G - h_e^G}{h_e - h_e} - A, \] (5)

In this formula, \( G_\alpha \), \( h_\alpha \), and \( c \) represent the thresholds of the information gain that activate the reward systems, the maxima of positive valence levels, and the gradients, respectively. \( G_\alpha \) and \( h_\alpha \) represent these in aversion systems.

3. THE EFFECT OF INITIAL PREDICTION ERRORS AND INITIAL UNCERTAINTIES ON EMOTIONAL DIMENSIONS

3.1 Desensitization (update of arousal)
We first analyzed how initial prediction errors, initial uncertainties, and external noise affect the updates of prediction errors and uncertainties. From formula (3), the partial derivatives of the prediction error and the uncertainty with respect to updating times \( n \) are the following:

\[ \frac{\partial \delta_s}{\partial n} = -\frac{\alpha \delta_s S_{\rho}/S_i}{(\alpha S_{\rho}/S_i + 1)^2}, \] (6)

\[ \frac{\partial S_{\rho m}}{\partial n} = -\frac{\alpha /S_i}{(\alpha n/S_i + 1/S_{\rho})}. \] (7)

Formula (6) implies that the prediction error decreases as \( n \) increases, and the decay rate of prediction error increases as the ratio of initial uncertainty to noise \( (S_{\rho}/S_i) \) increases. On the other hand, formula (7) implies that uncertainty decreases as \( n \) increases and the decay rate of uncertainty decreases as the initial uncertainty \( S_{\rho} \) or the noise \( S_i \) increases.

We then analyzed how the initial prediction error and the initial uncertainty affect updates of information gain. Figure 1 shows updates of information gain when the initial uncertainty is fixed. At any \( n \), the information gain increases as the initial prediction error increases and converges to 0 by updating. The decay rate of information gain increases as initial prediction error increases since the partial derivative of the decay rate of information gain with respect to the initial prediction error is always less than zero.

Figure 2 and Figure 3 show updates of information gain when the initial prediction errors are fixed in two ways. In the case of \( n=1 \), larger initial uncertainties result in larger information gains at initial prediction error = 1. On the other hand, at initial prediction error = 10, larger initial uncertainties result in smaller information gains. That is, the two functions of different uncertainties have an intersection and the information gains change as the initial prediction error increases. As the number of updates increases, the difference in information gains between the two functions also changes at both of initial prediction errors. That is, lower initial uncertainty tends to lead to a larger information gain by repeating the stimulus.

![Figure 1: Updates of information gain for different initial prediction errors (initial uncertainty = 1.0)](image1)

![Figure 2: Updates of information gain for different initial prediction errors (initial uncertainty = 1.0)](image2)
uncertainties (initial prediction error = 1)

![Graph showing updates of information gain for different initial uncertainties.](image)

**Figure 3**: Updates of information gain for different initial uncertainties (initial prediction error = 10)

The initial decay rate of information gain peaks at a certain uncertainty. At initial prediction error = 1, the decay rate is maximized at initial uncertainty = 20 (Fig. 2). However, at initial prediction error = 10, the decay rate peaks at initial uncertainty = 10 (Fig. 3). We assume that the peak of decay rate also shifted toward smaller uncertainties by updating.

3.2 Updates of acceptable prediction error

We investigated how the initial uncertainty affects the updates of acceptable prediction error range, shown in formula (5), by updating. Figure 4 shows the initial prediction error increase by updating, where the valence shifts from positive to negative. Furthermore, larger initial uncertainties result in larger increase rate of acceptable prediction error range.

![Graph showing updates of acceptable prediction error range for different initial uncertainties.](image)

**Figure 4**: Updates of acceptable prediction error range for different initial uncertainties

4. DISCUSSION

We showed an interaction effect between the initial prediction error and the initial uncertainty on the decrement of information gain. The larger the initial information gain, the sooner the information gain decreases and converges to 0. The total value of the information gain increases as the uncertainty increases at initial prediction error = 1 but decreases at initial prediction error = 10. This means that one is surprised for a long period even if the uncertainty of the event is not particularly large. This phenomenon tends to occur as the initial prediction error gets larger. As Silvia et al. (2005) showed in their study, we assume that the uncertainty of an event depends on the knowledge and experience that the person possesses, the attribute of the object, such as the affinity that comes from the typicality of the object, and the relationship between the person and the object. This result can be useful for designing products one enjoys for a long period. Designers can maximize the surprise experienced while using the product by designing the appearance of it, even if it is familiar.

Furthermore, we showed that the positive range of the Wundt curve shifted to a more novel direction by updating. This means that people, by repeating stimuli, gradually prefer more novel and complicated things. The experimental result of a previous study (Lévy, 2006) support the validity of this result. Furthermore, we showed that the rate of acceptable prediction error range becomes larger as the initial uncertainty increases. This means that an event with high uncertainty tends to change the acceptable prediction error range less dynamically. This result matches our intuition. For example, children tend to be interested in things (i.e., how to play instruments or how to speak a foreign language) earlier than adults. We suggest that the uncertainty of an event can explain the difference of the experience of the event between children and adults.

In this paper, we proposed a mathematical model of the time series change of emotional dimensions (arousal and valence) associated with novelty. We formalized the desensitization as a decrement of information gain, which represents arousal, from a novel event using the idea of Bayesian update. We derived the information gain from repeated exposure of a novel stimulus as a function of three parameters: initial prediction error, initial uncertainty and noise of sensory stimuli. With the proposed model, we found an interaction effect between the initial prediction error and the initial uncertainty on...
desensitization (decrement of information gain). Furthermore, we showed that the range of positive emotion shifted toward a more novel experience by repeated novelty exposure. This simulation results support the experimental results of the previous study where Lévy et al. (2006) showed that exposure to a stimulus with a little higher novelty than one’s preferred level of perceived novelty caused a shift toward more novel experience level.

This model suggests that the easiness of desensitization to novelty (easiness to get used to novelty) is different depending on the initial uncertainty, which is affected by contextual factors such as prior information, personal knowledge and experience, and familiarity of an event even if the deviation (prediction error) from the prior expectation is the same. We expect this model be used as a mathematical index to design products that provide people a longer-lasting novel impression.

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