周期的通信制約をもつネットワーク化制御システムの 可到達性 — 出力ホールド回路がない場合*

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Reachability of Networked Control Systems with Periodic Communication Constraints — No Output Holder Case*

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1. Introduction

This paper deals with networked control systems in which periodic data transmitting occurs according to some predetermined flow, and only data related to a limited number of input and output signals are transmitted due to the inputs and outputs of the plant being greater than communication bus. Yamamoto et al. took as the state space a model comprised of holders at both plant and controller input side, switches and network. They proposed the stabilization by a controller composed from a periodic observer and periodic state feedback [1]. Since the dynamics of the controller fulfils the role of the output holders, we can choose to forgo its use. Fujioka et al. proposed a model comprised of holders in only a plant input side, switches and a network, and gave a design method of a dynamical controller based on sampled-data control theory [2]. In the previous paper, we have already presented a modification against the model proposed by Yamamoto et al. and shown that for our modified model, controllability is preserved [3]. As we have already mentioned in [3], introducing holders achieves some role against the blackout of communication, however, it makes the dimension of the statespace higher. While, even if the holders of the output side are removed, it can be expected that the defect of output response will be relaxed by the controller dynamics such as [4,5]. In this paper, we shall propose a state space model comprised of holders only in a plant input side, switches, a network, a plant and a controller, and show that the reachability of the plant is preserved in the NCS as similar to previous work [3].

2. Preliminaries

Let the plant be a linear time invariant discrete-time system, whose state space representation is written as:

$$\begin{aligned} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t), \end{aligned} \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{R}^n$, $\mathbf{u}(t) \in \mathbb{R}^m$ and $\mathbf{y}(t) \in \mathbb{R}^p$ represent state vector, input vector and output vector, respectively. The controller can only communicate with b_c actuators through b_c input channels, where $b_c < m$. At any one time t , only b_c signals can be sent to the actuators. The m -dimensional ω -periodic vector $\boldsymbol{\sigma}_c(t)$ whose elements of 1 or 0 denote the access or non-access of the corresponding actuator inputs. For the non-access case, the actuator input is kept the previous value until the next access. On the other hand, the communication medium between the sensors and the controller has b_o output channels, where $b_o < p$. At any one time t , only b_o signals can be sent to the controller. The m -dimensional ω -periodic vector $\boldsymbol{\sigma}_o(t)$ whose elements of 1 or 0 denote the access or non-access of the corresponding controller inputs. For the non-access case, the controller input value becomes zero.

In this paper, $\|\mathbf{A}\|$ represents the Euclidean norm of matrix $\mathbf{A}$, and $\text{diag}(\mathbf{x})$ denotes a diagonal matrix whose diagonal elements consist of elements of vector $\mathbf{x}$, $\mathbf{I}_n$ and $\mathbf{0}_n$ denote an n -dimensional identity matrix and zero matrix, respectively.

【Definition 1】 Assume a limit of input (output) bus connections $b_c < m$ ($b_o < p$). The communication sequence $\boldsymbol{\sigma}_c(t)$ ($\boldsymbol{\sigma}_o(t)$) is said to be admissible if the following two conditions are met:

- i) $\|\boldsymbol{\sigma}_c(t)\|^2 < b_c$ ($\|\boldsymbol{\sigma}_o(t)\|^2 < b_o$), $t = 0, 1, \dots, \omega - 1$
- ii) $\text{Im}[\text{diag}(\boldsymbol{\sigma}_c(0), \text{diag}(\boldsymbol{\sigma}_c(1)), \dots, \text{diag}(\boldsymbol{\sigma}_c(\omega - 2)), \text{diag}(\boldsymbol{\sigma}_c(\omega - 1)))] = \mathbb{R}^m$
($\text{Im}[\text{diag}(\boldsymbol{\sigma}_o(0), \text{diag}(\boldsymbol{\sigma}_o(1)), \dots, \text{diag}(\boldsymbol{\sigma}_o(\omega - 2)), \text{diag}(\boldsymbol{\sigma}_o(\omega - 1)))] = \mathbb{R}^p$).

When $t \geq s$, the following matrix is composed from the communication sequence.

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$$E_c(t, s) = \prod_{j=s}^t (\mathbf{I} - \text{diag}(\boldsymbol{\sigma}_c(j))). \quad (2)$$

Note that the communication sequence matrix is a diagonal matrix and $E_c(t + \omega, s + \omega) = E_c(t, s)$. Using this matrix, the communication sequence matrix polynomial with the indeterminate μ is defined as:

$$G_c(\mu) = \sum_{l=0}^{\omega-1} [\{ \mu^{\omega-1} \mathbf{I} + \mu^{\omega-2} E_c(l+1, l+1) + \mu^{\omega-3} E_c(l+2, l+1) + \cdots + \mu E_c(l+\omega-2, l+1) + E_c(l+\omega-1, l+1) \} \text{diag}(\boldsymbol{\sigma}_c(l))].$$

Naturally $G_c(\mu)$ is a diagonal matrix, and its diagonal elements $g_{ci}(\mu)$'s are polynomial at most degree $\omega-1$. $g_c(\mu) = \det G_c(\mu)$ is referred to as a communication sequence characteristic polynomial. The following lemma is clear.

[Lemma 1] Let the zero of $g_c(\mu)$ be μ_i . Then a communication sequence matrix polynomial $G_c(\mu)$ is nonsingular if and only if $\mu \neq \mu_i$.

Taking inputs and outputs updated at time t as $\mathbf{u}_l(t)$ and $\mathbf{y}_l(t)$, respectively, then inputs $\mathbf{u}(t)$ and $\mathbf{y}_l(t)$ at time t are represented as follows:

$$\begin{aligned} \mathbf{u}(t) &= \text{diag}(\boldsymbol{\sigma}_c(t)) \mathbf{u}_l(t) + (\mathbf{I}_m - \text{diag}(\boldsymbol{\sigma}_c(t))) \mathbf{u}_l(t-1) \\ \mathbf{y}_l(t) &= \text{diag}(\boldsymbol{\sigma}_o(t)) \mathbf{y}(t). \end{aligned}$$

The following equivalent expressions are obtained through successive substitution:

$$\mathbf{u}(t) = \sum_{s=t-\omega+1}^t D_c(t, s) \mathbf{u}_l(s) \quad (3)$$

where

$$D_c(t, s) = \begin{cases} \text{diag}(\boldsymbol{\sigma}_c(t)), & t = s \\ E_c(t, s+1) \text{diag}(\boldsymbol{\sigma}_c(s)), & t > s. \end{cases} \quad (4)$$

Note that $D_c(t, s) = \mathbf{0}$ ($s = t - \omega, t - \omega - 1, \dots$). Substituting (3) into (1), we have that

$$\begin{aligned} \mathbf{x}(t+1) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B} \sum_{s=t-\omega+1}^t D_c(t, s) \mathbf{u}_l(s) \\ &= \mathbf{A}\mathbf{x}(t) + \mathbf{B} D_c(t, t) \mathbf{u}_l(t) \\ &\quad + [\mathbf{B} D_c(t, t-1), \mathbf{B} D_c(t, t-2), \dots, \\ &\quad \mathbf{B} D_c(t, t-\omega+1)] \begin{bmatrix} \mathbf{u}_l(t-1) \\ \mathbf{u}_l(t-2) \\ \vdots \\ \mathbf{u}_l(t-\omega+1) \end{bmatrix}. \end{aligned} \quad (5)$$

Here, the expanded state vector is defined as follows:

$$\mathbf{x}_e(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}_l(t-1) \\ \mathbf{u}_l(t-2) \\ \vdots \\ \mathbf{u}_l(t-\omega+1) \end{bmatrix} \in \mathbb{R}^{n_e}, \quad n_e = n + m(\omega-1). \quad (6)$$

Here we define the extended output signal $\mathbf{y}_e(t)$ which includes usable measured value $\mathbf{y}_l(t)$ and input signal $\mathbf{u}_l(s)$ ($s = t-1, t-2, \dots, t-\omega+1$) obtained at past time:

$$\begin{aligned} \mathbf{y}_e(t) &= \begin{bmatrix} \mathbf{y}_l(t) \\ \mathbf{u}_l(t-1) \\ \mathbf{u}_l(t-2) \\ \vdots \\ \mathbf{u}_l(t-\omega+1) \end{bmatrix} \\ &= \begin{bmatrix} C_{e11}(t) & \mathbf{0}_{p \times m(\omega-1)} \\ \mathbf{0}_{m(\omega-1) \times n} & \mathbf{I}_{m(\omega-1)} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{u}_l(t-1) \\ \mathbf{u}_l(t-2) \\ \vdots \\ \mathbf{u}_l(t-\omega+1) \end{bmatrix} \end{aligned} \quad (7)$$

where, $C_{e11}(t) = \text{diag}(\boldsymbol{\sigma}_o(t))C$.

Expressing (5) and (7) in terms of (6), we obtain the following ω -periodic discrete time system:

$$\begin{aligned} \mathbf{x}_e(t+1) &= \mathbf{A}_e(t) \mathbf{x}_e(t) + \mathbf{B}_e(t) \mathbf{u}_l(t) \\ \mathbf{y}_e(t) &= \mathbf{C}_e(t) \mathbf{x}_e(t). \end{aligned} \quad (8)$$

Here

$$\mathbf{A}_e(t) = \begin{bmatrix} \mathbf{A} & \mathbf{A}_{e12}(t) \\ \mathbf{0} & \mathbf{A}_{e22} \end{bmatrix} \quad (9)$$

$$\mathbf{A}_{e12}(t) = [\mathbf{B} D_c(t, t-1), \dots, \mathbf{B} D_c(t, t-\omega+1)] \quad (10)$$

$$\mathbf{A}_{e22} = \left[\begin{array}{cccc} \mathbf{0}_{m \times m} & \cdots & \cdots & \cdots & \mathbf{0}_{m \times m} \\ & \mathbf{I}_m & \ddots & & \vdots \\ & \mathbf{0}_{m \times m} & \ddots & \ddots & \vdots \\ & \vdots & \ddots & \ddots & \vdots \\ \mathbf{0}_{m \times m} & \cdots & \cdots & \mathbf{I}_m & \mathbf{0}_{m \times m} \end{array} \right] \Bigg\}^{\omega-1} \quad (11)$$

$$\mathbf{B}_e(t) = \left[\begin{array}{c} \mathbf{B} D_c(t, t) \\ \mathbf{I}_m \\ \mathbf{0}_{m(\omega-2) \times m} \end{array} \right] \Bigg\}^n_{m(\omega-1)} \quad (12)$$

$$\mathbf{C}_e(t) = \begin{bmatrix} C_{eu}(t) \\ C_{el}(t) \end{bmatrix} = \begin{bmatrix} C_{e11}(t) & \mathbf{0}_{p \times m(\omega-1)} \\ \mathbf{0}_{m(\omega-1) \times n} & \mathbf{I}_{m(\omega-1)} \end{bmatrix}. \quad (13)$$

Transition matrix for the augmented system (8) is defined as:

$$\Phi_e(t, t-j) = \begin{cases} \mathbf{I}_{n_e}, & j=0 \\ \mathbf{A}_e(t-1) \mathbf{A}_e(t-2) \cdots \mathbf{A}_e(t-j), & j \geq 1. \end{cases} \quad (14)$$

It follows from (9) that

$$\Phi_e(t, t-j) = \begin{bmatrix} \mathbf{A}^j & \Phi_{e12}(t, t-j) \\ \mathbf{0} & \mathbf{A}_{e22}^j \end{bmatrix}, \quad j=1, 2, \dots, \omega \quad (15)$$

where

$$\begin{aligned} \Phi_{e12}(t, t-j) &= \mathbf{A}^{j-1} \mathbf{A}_{e12}(t-j) + \mathbf{A}^{j-2} \mathbf{A}_{e12}(t-j+1) \cdot \\ &\quad \mathbf{A}_{e22} + \cdots + \mathbf{A} \cdot \mathbf{A}_{e12}(t-2) \mathbf{A}_{e22}^{j-2} + \mathbf{A}_{e12}(t-1) \mathbf{A}_{e22}^{j-1}. \end{aligned} \quad (16)$$

(Remark 1) If $\mathbf{A}$ is not nilpotent, since

$$\Phi_e(t, t-\omega) = \begin{bmatrix} A^\omega & \Phi_{e12}(t, t-\omega) \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

there exists a nonzero eigenvalue λ of $\Phi_e(t, t-\omega)$, which coincides with that of A^ω .

$\Phi_e(t) (= \Phi_e(t, t-\omega))$ is called a monodromy matrix. Let ν be the left eigenvector of $\Phi_e(t)$. It is well known [3] that the periodic system (8) is reachable, if and only if for any ν ,

$$\nu[B_e(t-1), \Phi_e(t, t-1)B_e(t-2), \dots, \Phi_e(t, t-\omega+1)B_e(t-\omega)] \neq \mathbf{0}. \quad (17)$$

3. Main Result

In this section, we shall show that reachability of system (1) are preserved in NCS. The following theorem shows that the reachability is preserved.

【Theorem 1】 Suppose that $\sigma_c(t)$ is admissible and assume that A is not nilpotent and its eigenvalues $\lambda^{1/\omega}$'s do not coincide with the zeros of $g_c(\mu)$. Then, if (A, B) is reachable, the augmented system (8) is reachable.

Let ν be the left eigenvector associated with a eigenvalue $\lambda \neq 0$ of $\Phi_e(t)$. Partitioning ν corresponding to the structure of state $x_e(t)$ as:

$$\nu = [\nu_1, \nu_2, \dots, \nu_\omega] \quad (18)$$

To prove the theorem, we need the following facts.

【Lemma 2】 For vector ν_j defined by (18), the following relations hold.

$$\begin{aligned} \text{i) } & \nu_1 A^j = \lambda^{j/\omega} \nu_1, \quad j = 0, 1, \dots, \omega-1 \quad (19) \\ \text{ii) } & \nu_1 A^{\omega-1} B D_c(t, t-j) + \nu_1 A^{\omega-2} B D_c(t+1, t-j) + \\ & \dots + \nu_1 A^j B D_c(t+\omega-j-1, t-j) = \lambda \nu_{j+1}, \\ & j = 1, 2, \dots, \omega-1. \quad (20) \end{aligned}$$

(Proof) (19) is clear. We shall prove (20). Let $(X)_j$ denote the j -th column block of X . It follows from (10), (11) and (16) that

$$(\Phi_{e12}(t, t-\omega))_j = \sum_{k=j}^{\omega-1} A^k B D_c(t+\omega-k-1, t-j). \quad (21)$$

Since

$$\nu_1 \Phi_{e12}(t, t-\omega) = \lambda [\nu_2, \nu_3, \dots, \nu_\omega],$$

pre-multiplying (21) by ν_1 yields (20). **

(Proof of Theorem 1) We shall show that if the system (8) is not reachable, (A, B) is not reachable. It follows from (12), (15) and (16) that for $j = 1, 2, \dots, \omega-2$,

$$\begin{aligned} & \Phi_e(t, t-j+1)B_e(t-j) \\ &= \begin{bmatrix} \sum_{k=0}^{j-1} A^k B D_c(t-k-1, t-j) \\ \mathbf{0} \\ \vdots \\ \mathbf{I}_m \\ \vdots \\ \mathbf{0} \end{bmatrix} \leftarrow *. \quad (22) \end{aligned}$$

The pointed row is $(j+1)$ -th block. Suppose that the system (8) is not reachable. Then it follows from (17) that there exists the left eigenvector ν associated with a nonzero eigenvalue λ of $\Phi_e(t)$ such that

$$\nu \Phi_e(t, t-j+1)B_e(t-j) = \mathbf{0}, \quad j = 1, 2, \dots, \omega-1. \quad (23)$$

Pre-multiplying ν to (22) and applying (23), we have

$$\begin{aligned} \nu_{j+1} &= -\nu_1 \sum_{k=0}^{j-1} A^k B D_c(t-k-1, t-j) \\ & \quad j = 1, 2, \dots, \omega-1. \end{aligned} \quad (24)$$

By substituting (24) into (20), we have

$$\begin{aligned} & \nu_1 A^{\omega-1} B D_c(t, t-j) + \nu_1 A^{\omega-2} B D_c(t+1, t-j) \\ & + \dots + \nu_1 A^j B D_c(t+\omega-j-1, t-j) \\ & + \lambda(\nu_1 A^{j-1} B D_c(t-j, t-j) \\ & + \nu_1 A^{j-2} B D_c(t-j+1, t-j) \\ & + \dots + \nu_1 B D_c(t-j, t-j)) = \mathbf{0}. \end{aligned} \quad (25)$$

This equation is the same as equation (42) of [3] (p. 103). Thus, applying the same procedure as the proof of Theorem 1 in [3], results in

$$\nu_1 B G_c(\lambda^{1/\omega}) = \mathbf{0}.$$

Since $G_c(\lambda^{1/\omega})$ is nonsingular, in virtue of the assumption, we have $\nu_1 B = \mathbf{0}$. This implies that (A, B) is not reachable. **

4. Conclusion

It has been shown that reachability is preserved in NCS without output holders. The problem for future study is to show that sample detectability is preserved and to design a controller [4].

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