Stochastic Graph Minor Generation via Dynamic Reference to Geographics Annotation*

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A stochastic-computational scheme is presented for generating *as-is* graph structure spanning complex local geographics. In this schematics, GPS tracks uploaded by a multitude of probe vehicles are exploited to expand a network of feasible roadway patterns. By associating the GPS tracks with a multitude of Brownian motions conditioned by landmark allocation, the graph structure is successively augmented in terms of feasible roadway segments. In reference to the augmented information, symbolic design of over-the-horizon maneuvering process can be interactively substantiated into a chain of topological minors grounded on the local geographics.

1. Introductory Remarks

Despite rapid advancement of vehicle information systems for localizing[11], planning[3], regulating[12] and operating[10] maneuvering processes, the scope of decision making in naturally complex scenes is still confined by on-vehicle control systems to be finally grounded on physical-geometric perspective. Recently, it has been demonstrated that vehicle control systems can be networked and deployed as a multitude of remote-visions for over-the-horizon coordination of the maneuvering processes[8]; images of distant scenes are localized and analyzed by probe vehicles to augment geographic annotations to a cut of satellite image. Such augmented annotations provide logical basis for multi-participant cooperation over the physical-geometric perspective[7]. For instance, future visitors to the scene can retrieve the annotations to anticipatively adapt the on-board vision systems *prior* to physical access. To this end, a symbolic plan should be designed in reference to the landmark allocation annotated by probe vehicles as illustrated in Fig. 1 where a future visitor is required to compile a set of landmarks to specify a path connecting the origin and a possible destination. To substantiate the symbolic plan, in this schematics, a multitude of GPS tracks are convened from probe vehicles and matched with the landmark connection within the visualization of the local terrain. Through the symbol-to-trajectory association, a multitude of anticipative and/or graph-based road following systems[6,1] can be collectively organized towards an *in-situ* instantiation of vehicle

Fig. 1 Schematics of multi-participant design

specific goals.

In this design process, on the other hand, the decision makers are required to manipulate an ‘open graph’ spanning intentional selection of annotated landmarks; a GPS track should be associated with specific parts of on-going graph structure as the generator of entire symbolic plans to be designed. To support practical decision making, at the first visit[9] in particular, the scope and the granularity of the graph structure should be interactively adjusted to the transition of the maneuvering context. In conventional symbolic planning systems, the totality of landmark connections has been compiled to visualize partially consistent systems of primitive graphs. Due to the context dependence of the graph structure, however, the organization of primitive graphs is still left in charge of human’s logical-geometric capability. As the result of nondeterministic scope selection, however, it is not easy even for human’s intelligence to maintain collective consistency in the integrated description of the geographics annotations. Thus, the cooperative design of maneuvering processes within naturally complex scenes is still an open problem.

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Based on the state-of-the-art space technologies, the cooperative augmentation process is essentially identified with a decentralized decision making on the multi-scope visualization of a common cut of satellite image. Noticing this, in this paper, we consider probing-based graph generation scheme; the problem is to identify GPS tracks with landmark connections through dynamic reference to the on-going geographics annotations.

2. Topological Minors within Local Geographics

As the computational basis of the multi-scope decision making, let Ω be an image plane coordinated in terms of (longitude, latitude). Within physical-geometric aspect, any maneuvering processes must be parsed into a chain of primitive motions. To this end, we should identify a possible primitive motion with a sequence of random vectors in Ω . Noticing this, the computational complexity due to the context dependence should be solved via the matching of the graph structure with a feasible class of random vectors.

Consider geographic information $\mathfrak{M}$ spanning a bird's eye view Ω . The multitude of the annotation processes generate a graph structure $G(N, C|\mathfrak{M})$; N is the totality of landmarks n with localization $n_{\perp} \in \Omega$; C denotes the set of symbolic connections on N , i.e.,

$$C = \{c(n^o, n^d) \mid n^o, n^d \in N\}.$$

In many practical situations, C is consisting of sufficiently rich 'gateway' networks and random 'turnpikes' between the gateways as illustrated in Fig. 2 where a route graph including 'downtown' and 'campus' gateways is visualized. To design a feasible plan in such a small world[13], the preassigned landmark connections should be re-organized in accordance with decision steps towards an *in-situ* rendezvous point. Consider a family of restrictions $\mathcal{F}[\mathfrak{M}] = \{\mathfrak{m}_i \mid \Omega_{\mathfrak{m}_i} \subset \Omega\}$ where the subdescription $\mathfrak{m}_i$ is supported by the local section $\Omega_{\mathfrak{m}_i} \subset \Omega$. On such a multi-scope representation, the re-organization process in cooperation with a 'first visitor' is formalized by

$$G(\{n^o, n^d\}, \{c(n^o, n^d)\}|\mathfrak{M}) \quad (1a)$$

$$\rightarrow \{G(\{n^o, (\cdot)\}, \{c(n^o, (\cdot))\}|\mathfrak{M}), \dots,$$

$$G(\{(\cdot), n^d\}, \{c((\cdot), n^d)\}|\mathfrak{M})\} \quad (1b)$$

$$\rightarrow \{G(N_{\mathfrak{m}_a}, C_{\mathfrak{m}_a}|\mathfrak{m}_a), \dots, G(N_{\mathfrak{m}_b}, C_{\mathfrak{m}_b}|\mathfrak{m}_b)\}, \quad (1c)$$

as shown in Fig. 3 where the route connecting the landmark symbols (n^o, n^d) (a) is subdivided into a chain of stopovers (b) on the class $G(N_{\mathfrak{m}_{(\cdot)}}, C_{\mathfrak{m}_{(\cdot)}}|\mathfrak{m}_{(\cdot)})$ (c). In other words, the symbolic connection $c(n^o, n^d)$ by path planner should be identified with a topological minor[2] of a feasible path. Due to such a non-deterministic association (1), the GPS track should be matched with a graph structure to be generated in ac-

Fig. 2 Gateway-turnpike structure of route graph

Fig. 3 Multi-scope geographics annotation

cordance with the planning and/or navigation steps.

3. Graph Minor Generation via Random Probing

Let a GPS track be identified with a sampling of a 2D stochastic process $\omega_t \in \Omega$, $t \geq 0$, to be grounded on a really existing roadway. Suppose that a sample path ω_t specifies a physical process uniquely observed within nonlinear filtering residual[4] and consider the articulation of the sample path with respect to a landmark set N . Logically, N can be partitioned with respect to the localization $n_{\perp}$ to selectively visualize on a intentional restriction $\mathfrak{m} \in \mathcal{F}[\mathfrak{M}]$ with $\Omega_{\mathfrak{m}} \subset \Omega$. By associating sample points (ω_1, ω_L) with the groundings of landmarks (n^o, n^d) , $n^{(\cdot)} \in N$, we have a substantiation of symbolic connection $c(n^o, n^d) \in C$ in terms of a sequence $\ell = \omega_1 \omega_2 \dots \omega_L$, $\omega_k \in \Omega$ where $n_{\perp}^o = \omega_1$ and $n_{\perp}^d = \omega_L$ designate the grounding of n^o and n^d , respectively. To this end, first, we should evaluate the GPS residual $\varepsilon_{\mathfrak{m}}^n$ and relocate each micro connection $c(\omega_k, \omega_{k+1})$ on a segment of the roadway pattern[6]:

$$\mathfrak{V} = \{\vec{v}_k, k = 1, 2, \dots\}, \quad \vec{v}_k = (\hat{\omega}_k, \hat{\theta}(k|\mathfrak{m})),$$

with origin $\hat{\omega}_k$ and direction $\hat{\theta}(k|\mathfrak{m})$ [5]. Next, the GPS trajectory $\hat{\ell} = \{\hat{\omega}_k, k = 1, 2, \dots\}$ should be partitioned

into the class of ordered subsets $\{\dots, \hat{\ell}_n, \dots, \hat{\ell}_m, \dots\}$ satisfying the following constraint:

$$\forall \hat{\omega}_t \in \hat{\ell}_n, \hat{\omega}_k \in \hat{\ell}_m : t \leq k \rightarrow \hat{\ell}_n \leq \hat{\ell}_m. \quad (2)$$

Within the fluctuation of GPS track ε_m^n , we can exploit the ordered set $\hat{\ell}$ as a feasible path connecting the landmarks in $\mathbf{m}$. Along such a micro graph, we can confine the space of origin-destination-landmarks; a landmark $n \in N$ with grounding $n_\perp \in \Omega_m$ is said to be a $\mathbf{m}$ -contact with respect to $\hat{\ell}$ if the following condition is satisfied:

$$\forall \eta(n_\perp, \hat{\ell}) < \varepsilon_m^n, \quad (3)$$

where $\forall \eta(n_\perp, \hat{\ell}) = \min_{\hat{\omega} \in \hat{\ell}} |n_\perp - \hat{\omega}|$. For a set of landmarks satisfying the $\mathbf{m}$ -contact condition (3), i.e.,

$$\tilde{N} = \{\tilde{n} \in N \mid \forall \eta(\tilde{n}_\perp, \hat{\ell}) < \varepsilon_m^n\},$$

hence, we have the following ordered partitioning of the set $\hat{\ell}$ with respect to $\tilde{N}$:

$$\begin{aligned} \hat{\ell} &= \bigcup_k \hat{\ell}_{n_k}, & \hat{\ell}_{n_i} \cap \hat{\ell}_{n_j} &= \emptyset, n_i, n_j \in \tilde{N}, \\ & & \hat{\ell}_{n_1} &\leq \hat{\ell}_{n_2} \leq \dots \end{aligned} \quad (4)$$

Consider a $\mathbf{m}$ -connected landmark $\tilde{n}_t^* \in \tilde{N}$ satisfying

$$\forall \eta(\tilde{n}_t^*, \hat{\ell}_t) \leq \forall \eta(\tilde{n}_\perp, \hat{\ell}_t), \quad \tilde{n}^*, \tilde{n} \in \tilde{N}. \quad (5)$$

By chaining such landmarks along $\hat{\ell}$, we can induce a feasible version of the topological minors $c(\tilde{n}_t^*, \tilde{n}_{t+1}^*)$, $t=1, 2, \dots$ within the representation of local geographics $\mathbf{m}$. To localize the origin of the first connection, we activate the following

Initial setup: For given $\hat{\ell} = \{\hat{\omega}_k, k=1, 2, \dots\}$ and a set of $\mathbf{m}$ -contact landmarks $\tilde{N}$, detect $\mathbf{m}$ -connected landmark $\tilde{n}^* \in \tilde{N}$ satisfying

$$|\hat{\omega}_1 - \tilde{n}_\perp^*| = \forall \eta(\hat{\omega}_1, \tilde{N}). \quad (6)$$

The first landmark $\tilde{n}^*$ nearest to the origin of the GPS trajectory $\hat{\omega}_1$ is associated with the destination of the first connection via the following

One-step Extension: Select $\hat{\ell}_{\tilde{n}^*}$ satisfying

$$\hat{\ell}_{\tilde{n}^*} = \left\{ \hat{\omega}_k \in \hat{\ell} \mid |\hat{\omega}_k - \tilde{n}_\perp^*| = \forall \eta(\hat{\omega}_k, \tilde{N}) \right\}, \quad (7a)$$

and reset $\tilde{N} \leftarrow \tilde{N} - \tilde{n}^*$, $\hat{\ell} \leftarrow \hat{\ell} - \hat{\ell}_{\tilde{n}^*}$, then update $\tilde{n}^*$ by a $\mathbf{m}$ -connected landmark in $\tilde{N}$ such that

$$\forall \eta(\tilde{n}_\perp^*, \hat{\ell}_{\tilde{n}^*}) \leq \forall \eta(\tilde{n}_\perp, \hat{\ell}_{\tilde{n}^*}), \quad \tilde{n}^*, \tilde{n} \in \tilde{N}, \quad (7b)$$

respectively. The iteration of the expansion step is halted by the following

Halting condition: $(\tilde{N} = \emptyset) \vee (\hat{\ell} = \emptyset)$.

Thus we have a computational scheme for mechanical

generation of graph structure to be grounded on the local geographics $\mathbf{m}$ via observations $\hat{\ell}$.

In the deterministic implementation the initial step, the order of computation cost in the $\tilde{N}$ -generation is evaluated in terms of $\|N\|$ and $\|\hat{\ell}\|$, i.e.,

$$C_m \sim \|N\| \times \|\hat{\ell}\|,$$

where $\|(\cdot)\|$ stands for the size of set $(\cdot)$; similarly, to localized the initial origin $\tilde{n}^*$, we need at most $\|N\|$ times of the evaluation $|\hat{\omega}_1 - (\cdot)|$, $(\cdot) \in \tilde{N}$. Since the set N must be regenerated at each minimization step in open graph structure, the *a priori* estimate for successive expansion (7) is finally indexed in terms of

$$C_{\hat{\ell}} \sim \|N\| \times \|\hat{\ell}\|.$$

Hence, we have the following evaluation of the deterministic landmark connection generation:

$$C_d \sim C^e \|N\| \cdot \|\hat{\ell}\|, \quad (8)$$

where C^e denotes the computation cost for the Euclidean minimization in (6) + (7).

4. Stochastic Graph Minor Generation

To reduce the computation cost C_d within the context of open graph generation, the initial- and extension-steps must be applied to the latest version of the set N with dynamic reference to the on-going annotation in $\mathbf{m}$; to this end, it is required to select the landmarks within a neighborhood of the trajectory $\hat{\ell}$ prior to the $\mathbf{m}$ -contact evaluation. To avoid the computational contradiction, we reformulate the graph minor generation process within the framework of stochastic contact evaluation.

Conditional Brownian Motion Model: Consider 2D flux along the totality of Brownian motion processes with origin $n_\perp \in \Omega_m$. The probability to capture a sample path at time indexed by σ is given by the following Gaussian distribution:

$$g_\sigma(\omega|n) = \frac{1}{2\pi\sigma} \exp \left[-\frac{|\omega - n_\perp|^2}{2\sigma} \right].$$

Noticing the multi-scale decomposition of Delta function $\delta_{n_\perp}$:

$$\begin{aligned} g_\sigma(\omega|n) - \delta_{n_\perp} &= \int_0^\sigma \frac{1}{2} \Delta g_s(\omega|n) ds \\ &\simeq \frac{\sigma}{2} \Delta g_\sigma(\omega|n), \end{aligned}$$

we have the probability distribution $\varphi_\sigma(\omega|N)$ to finally capture at least one Brownian motion shifted from somewhere in N as the solution to the following

$$\frac{\sigma}{2} \Delta \varphi_\sigma(\omega|N) + [\chi_N - \varphi_\sigma(\omega|N)] = 0, \quad (9)$$

where $\chi_N = \frac{1}{\|\bar{N}\|} \sum_{n \in N} \delta_{n_\perp}$. By identifying the information arising from the landmarks $N - n$ with independent noisy background, we have the following decomposition:

$$\varphi_\sigma(\omega|N) = g_\sigma(\omega|n) + \chi_\Omega, \quad (10)$$

where χ_Ω stands for the probability distribution of the background noise. Let σ_N be the variance based on φ_σ . Noticing

$$\sigma_N = \sigma + \sigma_\Omega,$$

we have the zero-cross boundary

$$\begin{aligned} \partial\varphi_\sigma^N &= \left\{ \partial\omega \in \Omega \mid \varphi_\sigma(\partial\omega|N) = \bar{p}_N \right\}, \\ \bar{p}_N &= \frac{1}{2\pi\sigma_N} \exp \left[-\frac{1}{2} \left(1 - \frac{\sigma}{\sigma_N} \right)^2 \right], \end{aligned} \quad (11)$$

under the conditioning by $n \in N$. Thus, we can finally detect a Brownian motion conditioned by a landmark n at $n_\perp$ somewhere on the boundary $\partial\varphi_\sigma^N$.

Dynamic Graph Generation: Let the origin n be randomly re-selected in each unit time interval. In such a situation, the probability to capture an iteratively conditioned Brownian motion is given by

$$\begin{aligned} \tilde{g}_t^n(\omega|N) &= \left(\frac{1}{\|\bar{N}\|} \right)^t g_t(\omega|n) \\ &= e^{-\rho t} g_t(\omega|n), \quad \rho = \log \|\bar{N}\|. \end{aligned} \quad (12)$$

By matching the variation

$$d\varphi_t = \varphi_\sigma(\hat{\omega}_t + \vec{v}_t dt|N) - \varphi_\sigma(\hat{\omega}_t|N)$$

due to the segmentation with the $\tilde{g}_t^n(\omega|N)$ -distribution, we can identify the GPS segment $\vec{v}_t$ with the conditional Brownian motion process. To this end, first, the variation is evaluated as follows:

$$d\varphi_t = -\nabla\varphi_\sigma(\hat{\omega}_t|N)^T \vec{v}_t dt. \quad (13)$$

Noticing $\varphi_\sigma \simeq g_\sigma$ within the area $|\omega - \hat{n}_\perp| < \sqrt{\sigma_N}$ and

$$\begin{aligned} \nabla\varphi_\sigma(\omega|N) &\simeq \nabla g_\sigma(\omega|n) = -\frac{1}{\sigma}(\omega - \hat{n}_\perp)g_\sigma(\omega|n) \\ &\simeq -\frac{1}{\sigma}(\omega - \hat{n}_\perp)\varphi_\sigma(\omega|N), \end{aligned}$$

next, we have the following stochastic differential:

$$d\tilde{\varphi}_t = \frac{d\varphi_t}{\varphi_t} = \tilde{\phi}_t dt + \sqrt{\sigma_w} dw_t,$$

where $\tilde{\phi}_t = (\omega - \hat{n}_\perp)^T \vec{v}_t$ and the fluctuation due to the background distribution χ_Ω is simulated in terms of Wiener process w_t ; noticing the ambiguity due to digital imaging of the satellite image, we have an order estimate for the variance parameter σ_w as follows:

$$\sigma_w \sim \frac{1_\varphi}{\|\bar{N}\|_\Omega \cdot \|\bar{v}\|_\Omega}, \quad (14)$$

where 1_φ designates the maximum of the distribution φ ; $\|\bar{N}\|_\Omega$ and $\|\bar{v}\|_\Omega$ stand for the nominal distance between landmarks and average length of the $\vec{v}$ in pixel unit in Ω , respectively. By modifying the Wiener measure in terms of the iteration factor $e^{-\rho t}$ induced in (12), we have the likelihood ratio Λ_t for indexing an essential shift of the variation φ_t from w_t process as follows[14]:

$$\begin{aligned} \Lambda_t &= e^{-\rho t} e^{\zeta_t}, \\ d\zeta_t &= \frac{\tilde{\phi}_t d\tilde{\varphi}_t}{\sigma_w} - \frac{\tilde{\phi}_t^2 dt}{2\sigma_w}. \end{aligned} \quad (15)$$

Noticing the infinitesimal representation

$$\frac{d\Lambda_t}{\Lambda_t} = \frac{\tilde{\phi}_t}{\sigma_w} d\tilde{\varphi}_t - \rho dt, \quad (16)$$

finally, we have the dynamic detection scheme:

Start up: If $(\Lambda_t < 1) \wedge (\varphi_\sigma(\hat{\omega}_t|N) > \bar{p}_N)$ is satisfied, then

- (1) Reset the likelihood ratio by $\Lambda_t = 1$.
- (2) Estimate the nearest landmark $\hat{n}_\perp$ in a rectangle $d\hat{\Omega}_t$ with diagonal vertexes $\hat{\omega}_t$ and $\tilde{\omega}_t$ given by

$$\begin{aligned} \tilde{\omega}_t &= \hat{\omega}_t + \beta(\varphi_\sigma; \hat{\omega}_t|N), \\ \beta(\varphi_\sigma; \hat{\omega}_t|N) &= 2\sqrt{-2\sigma \log \left[\frac{\varphi_\sigma(\hat{\omega}_t|N)}{\varphi_{\max}} \right]} \\ &\quad \times \frac{\nabla\varphi_\sigma(\hat{\omega}_t|N)}{|\nabla\varphi_\sigma(\hat{\omega}_t|N)|}, \end{aligned} \quad (17)$$

where $\varphi_{\max} = \max_{\omega \in \Omega} \varphi_\sigma(\omega|N)$.

Tracking: If $(\Lambda_t > 1) \wedge (\varphi_\sigma(\omega|N) > \bar{p}_N)$ is satisfied, then update the likelihood ratio by (16).

Out of $\partial\varphi_\sigma^N$ -boundary: If $\varphi_\sigma(\omega|N) \leq \bar{p}_N$, then diminish Λ_t as follows:

$$\frac{d\Lambda_t}{\Lambda_t} = -\rho dt. \quad (18)$$

In this scheme, the first contact with the boundary $\partial\varphi_\sigma^N$ is detected at the GPS track point $\hat{\omega}_t$ where the start up procedure is activated to select an estimate of the origin landmark $\hat{n}^o$. The consistency of the origin with subsequent $\vec{v}_t$ -sequence is continually tested by the tracking process. After successive verification of the tracking- and subsequent out of boundary conditions, the GPS track reaches a boundary point; at which the start up procedure is activated, again, to detect an estimate of the destination landmark $\hat{n}^d$ satisfying (17); a new origin is identified with the destination, i.e., $\hat{n}^o \leftarrow \hat{n}^d$, to restart the graph generation process. Thus, we can exploit the scheme for the generation of a topological minor $G(\tilde{N}, \{c(\tilde{n}^o, \tilde{n}^d)\})$ spanning the micro graph structures $\mathfrak{V}$. Since the distribution $\varphi_\sigma(\omega|N)$ is generated as the steady state solution to the dynamical system

$$\frac{\partial}{\partial t} \varphi_t(\omega|N) = \frac{\sigma}{2} \Delta \varphi_t(\omega|N) + [\chi_N - \varphi_t(\omega|N)], \quad (19)$$

we can apply the graph generation scheme (16) – (18) to open graph structure where the landmark distribution χ_N is updated in accordance with asynchronous landmark uploading by a multitude of probe vehicles.

An example of detection results is illustrated in Fig. 4 where a series of GPS data (plotted by small circles in the main window) is matched with landmark set; in this case, 26 landmarks are allocated in a cut of satellite image of 640×480 resolution to yield the field information φ_σ in subwindow (a) of 320×240 resolution where the GPS track is converted to a set of segments consisting of 989 $\vec{v}_t$ -vectors and matched with φ_σ . By combining the definition (12) with the estimate (14), the complexity factor ρ and the variance parameter σ_w are adjusted to 4.6 and 2.8×10^{-4} respectively. Along the segment set, the zero-cross condition (11) is tested to mark the trajectory on the φ_σ -field in (a); when the start up condition is satisfied, the estimator (17) was activated to visualize $d\hat{\Omega}_t$ in the main window. The transition of Λ_t is marked by bold line in (b) where the level $\bar{p}_N$ and the start up time are indicated by thin line and large circle respectively. The detection of new landmarks is emphasized by the $d\hat{\Omega}_t$ -box with thick contours; simultaneously, the scene image of the new landmark is displayed in the subwindow (c). Throughout the session of the GPS track, 24, 43 and 380 of effective origin-destination pairs, boundary contacts and in-the-boundaries were detected, respectively; from total 988 of the micro graph structures, thus, an essential graph minor consisting of 23 connections was extracted via 38% of tracking- and 2.3% of start up computations, respectively; the resulted symbolic path, which is specified by human planners in conventional schematics as illustrated in Fig. 1, is indicated by bold lines connecting m-contact landmarks in the main window. The stochastic graph generation scheme was applied to various scenes shown in Figs. 5 and 6 to yield symbolic paths going through one of the gateway and mountain country successfully.

By invoking the evaluation (17), we can confine the search space of possible landmarks; this implies that the computation cost for the Euclidean minimization in the deterministic scheme including (3) – (7) can be significantly reduced. Furthermore, we can substitute the computation of φ_σ -field generation and likelihood test for the $\|\hat{\ell}\|$ -times iteration of Euclidean minimization in N . Let the φ_σ -field be dynamically generated via successive approximation scheme. Suppose that the cost for φ_σ -field generation on entire Ω and likelihood test at each GPS access can be indexed by $m_\varphi \|\Omega\| C^\varphi$ and C^φ , respectively, where m_φ denotes the iteration of dynamic scheme (19). Noticing the *a priori* estimate $C_s \sim C^\varphi m_\varphi \|\Omega\|$, we have the following relative evaluation

Fig. 4 Graph minor generation based on GPS track

Fig. 5 Graph minor generation: Gateway scale

Fig. 6 Graph minor generation: Interurban scale

$$C_s \sim m_\varphi \frac{\|\Omega\|}{\|\hat{\ell}\|} \frac{C^\varphi}{C^e} C_d. \quad (20)$$

Since $m_\varphi < 10^2$ and $\|\hat{\ell}\| > 10^2$, in many practical situations, it follows that

$$C_s \sim \sigma \frac{C^\varphi}{C^e} C_d,$$

where σ is the range of the Brownian motion specify-

ing the $\varphi_\sigma(\omega|N)$ -distribution in (9) and adjusted to the granularity of the landmark distribution in symbolic planning of the over-the-horizon maneuvering processes, i.e., $\sigma = \|\Omega\|/\|N\|$.

5. Interactive Rendezvous

The effectiveness of the stochastic scheme was demonstrated through experimental studies within the context of interactive rendezvous. Consider co-operative design of maneuvering process in the scene spanning $5.05\text{km} \times 3.11\text{km}$ area in a cut of satellite image as indicated in Fig. 2; the problem is to successively augment the *a priori* graph to establish dynamic connection between the localization of the first visitor and a meeting point to be determined within the campus area. For designed route to reach a destination, the connected graph $G(N, C|\mathfrak{M})$ should include a spanning tree. Following graph theory, hence, the connections should satisfy the following evaluation:

$$\|C\| \geq C_{\min} = \|N\| - 1. \quad (21a)$$

Noticing Euler-Poincaré's formula, on the other hand, the totality of planar connections in a sufficiently complex geographics should be bounded by

$$\|C\| \leq C_{\max} = 3\|N\| - 6. \quad (\|N\| \geq 3) \quad (21b)$$

In the initial graph structure with $\|N\| = 26$ and $\|C\| = 59$, as indicated in Fig. 2, there have been defined a set of connections satisfying

$$\|C\| \simeq 2.4C_{\min} \text{ and } \|C\| \simeq 0.82C_{\max}.$$

Furthermore, an average number of connections at a landmark is estimated by $2\|C\|/\|N\| \simeq 4.5$. Thus, we can expect the existence of a feasible origin-destination connection in downtown- and campus-gateways, respectively. By applying a probed GPS track as demonstrated in Fig. 4, a set of feasible connections were augmented to specify an initial design from somewhere in the downtown towards the campus area as illustrated in Fig. 7.

GPS Track Based Graph Generation: To substantiate departure plan from downtown gateway, a GPS track consisting of 433 segments was matched with landmark distribution on the local geographics shown in Fig. 5. In this figure, total 7 origin-destination connections were detected to augment the initial graph in terms of a sequence of feasible connections along the GPS track. Similarly, the probed tracks were applied to the satellite image one after another to generate the graph representation of downtown area as displayed in Fig. 8.

On-Line Graph Tracking Due to partial inconsistency of the local roadway graph (Fig. 8) with the initial design (Fig. 7), it is not easy to depart from

Fig. 7 An initial design of route graph

Fig. 8 Route graph on feasible connections

Fig. 9 On-line graph matching

first visit gateway without any confusion and/or illusion; to go-through the gateway area, the visitor is required to localize encountered scenes within a partial observation of the complex roadway pattern; simultaneously, the planned route should be identified with a version of topological minor of the entire roadway pattern to be followed through the subsequent maneuvering process. To anticipatively mitigate the illusion essentially ineluctable in first visit scenes and to support the restoration from resulted confusion as well, the stochastic graph augmentation scheme can be applied to on-going GPS track. An example of on-line graph augmentation is illustrated in Fig. 9 where

a local section of the initial plan is re-organized within the GPS track based graph $G(N, C)$ (Fig. 8). In this case, the connections in the route graph was directly matched with the vector $\vec{v}_k$ without reference to the φ_σ -field (a); through the re-organization, the localization of the visitor is visualized as an substantiation of the planned route shown in (b); in contrast with conventional GPS mapping, the visitor has the access path to ‘future scenes’ along the subsequent route as indicated in (c). As a result of the on-line augmentation process, the part of the initial plan is finally grounded to the local geographics interactively confined along the maneuvering process successfully.

Route Graph Integration As illustrated in Fig. 3, the turnpike route may connect nominal groundings of the landmarks; for instance, the symbolic path (selected in Fig. 7) is connected to notified grounding of the campus. To substantiate the arrival phase of the maneuvering process, it is required to integrate the turnpike route with the destination gateway displayed in Fig. 10(a); the route graph on the campus area is consisting of local connections (bold lines) and turnpike routes (thin lines). As demonstrated in (b) of this figure, we can exploit the algorithm (9) + (16) + (17) to generate three connections to be annotated to the local geographics; a new route graph is marked by bold lines along a probed GPS track (plotted by small circles) to connect a bus stop and an entrance (emphasized by bold boxes) *prior to* physical access. By using the augmented version of the annotations to the destination geographics, the first visitor can self-navigate towards a physical gateway; simultaneously, through the broad casting of the on-going geographics annotation to meeting organizers and other participants as well, a multitude of the first visit processes are cooperatively organized on a common cut of satellite images. Since the dynamic graph generation scheme is essentially decentralized process on on-going landmark set N to be broad cached, the over-the-horizon cooperation is essentially implemented in terms of the φ_σ -field based computation. Noticing the stability of φ_σ generation scheme (19) with respect to partial variation of landmark allocation, thus, we can exploit the stochastic scheme as an efficient support throughout a over-the-horizon maneuvering process.

6. Discussions

The performance of the graph generation scheme is summarized in Table 1 where $d\|C\|$ means the new connections; β and $\partial\varphi_\sigma^N$ designate the number of start up computations and zero-cross boundary detections, respectively. As shown in this table, we can substitute φ_σ -based evaluation (16) – (18) for 86%–99% of Euclidean minimization procedure; via simple boundary detection (11), 26%–38% of GPS segment were selected to test the consistency with the conditional Brownian motion in the start up condition; finally,

(a) Destination gateway

(b) Graph minor generation: Block scale

Fig. 10 Trajectory-based graph integration

Table 1 Performance of stochastic scheme

Fig.	$\ \ell\ $	ρ	$\sigma_w^\dagger$	$d\ C\ $	β	$\partial\varphi_\sigma^N$
4	989	4.6	2.8	23	40	380
5	433	19.4	0.62	7	11	114
6	2,837	4.6	2.8	6	17	800
10	242	97.5	0.03	3	7	80

$\dagger: \times 10^{-4}$

3–22 symbolic connections were detected in confined search space $d\hat{\Omega}_t$ to augment the graph structure. Adding to the computational efficiency, the stochastic scheme exhibits significant robustness in a variety of practical situations; throughout the experiments indicated in Figs. 4 – 10, the process parameter (σ, ρ, σ_w) were specified in terms of *a priori* information of the geographics complexity, i.e., ($\|\ell\|, \|N\|, \|\Omega\|$) within local geographics captured through satellite images spanning 600m×400m to 5km×3km areas.

The computation cost for graph minor generation is discussed in Table 2 where the computation time C^φ is compared with the cost for Euclidean minimization C^e ; in this table, the costs C_f^e and C_s^e are measured in the implementation of landmark set by using Linux file system and main memory expansion of object compositions, respectively. As demonstrated

Table 2 CPU Time for graph minor generation

cpu core		×8	×1	×1
clock	[GHz]	1.2	2.3	0.3
C^φ	[μsec]	0.050	0.056	0.24
$C_f^{e\dagger}$	[μsec]	1,996	2,557	4,915
$C_s^{e\dagger}$	[μsec]	106	156	415
$C^\varphi/C^{e\dagger}$		0.25–4.41	0.22–3.59	0.49–5.78

†: $\|N\| = 100$, ‡: $\times 10^{-4}$

in this table, the cost factor C^φ/C^e is evaluated by 10^{-4} in various types of hardware and/or software implementations. In other words, the reduction of search space is crucial for the implementation of graph generation systems. In current implementation, however, the identification of a symbolic connection requires 1.7–4 times landmark detection within controlled space $d\hat{\Omega}_t$. The reduction of computational redundancy is left to future investigations.

7. Concluding Remarks

A stochastic-computational scheme was presented for successive augmentation of roadway graph spanning complex local geographics. In this scheme, GPS tracks are successively matched with a multitude of Brownian motion processes conditioned by nearest landmarks. By using the stochastic detection scheme, the computation cost for Euclidean minimization is significantly reduced. Due to intrinsic dynamism governing the scheme, in addition, the route graph can be interactively augmented in accordance with the design steps on on-going geographics annotations.

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