Identification of Unknown Parameters of Partially Observed Discrete-time Stochastic Systems by Using Pseudomeasurements*

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We consider partially observed discrete-time linear stochastic systems and assume that some entries of the system matrices are unknown. We propose a new method which identifies these unknown entries and the state vectors of these systems simultaneously. The key idea of the proposed method is utilization of the pseudomeasurement which is a fictitious and additional observation process on the unknown entries and will be modified so as to work for the partially observed systems. Augmenting the pseudomeasurement with the original observation process, we derive the new identification method by applying the extended Kalman filter. The proposed method is consistent with the conventional method (without pseudomeasurement).

1. Introduction

Identification of stochastic dynamical systems from input and output data is a fundamental and important problem, as recognized as a kind of inverse problems. Over the last two decades, one of the most popular methods to identify these systems has been the subspace-based identification method and a considerable number of research papers using this method have been published (see [8,19]). To fix the idea of the subspace-based method, consider the linear stochastic system described by a couple of equations

$$x(t+1) = Ax(t) + Bu(t) + w(t), \quad (1)$$

$$y(t) = Cx(t) + v(t), \quad (2)$$

where $w(t)$ and $v(t)$ are random noise sequences. In the subspace-based method, identification of the system matrices (A, B, C) is conducted from the input and output data $\{u(t), y(t)\}$ within a similarity transformation such as (A_T, B_T, C_T) where $A_T = T^{-1}AT$, $B_T = T^{-1}B$, $C_T = CT$ and T is a transformation matrix. Thus, the matrices (A, B, C) cannot be identified individually by the subspace-based method because the transformation matrix T cannot be determined.

In this paper, we assume that some entries of the system matrix A are unknown and denote the known part (resp. the unknown part) of A by A_k (resp. A_u)

(see Section 3 for the detail). We propose a new iterative method which identifies those unknown parameters and the state vector of the system simultaneously by making use of A_k explicitly. By the subspace-based method, we can have an estimate of the matrix A from the estimate of (A_T, B_T, C_T) , but it is impossible to utilize the information about the known part A_k of A explicitly. However, in Section 5, we will propose the iterative method which identifies both the states of the system and the unknown part A_u of A by using A_k explicitly. This is one of the advantages of the the proposed method in this paper over the subspace-based method.

The main idea of the proposed method is to use pseudomeasurement which is a fictitious observation process on the unknown entries of the stochastic dynamical systems. Firstly, the pseudomeasurement was utilized for various tracking problems by several researchers (see e.g., [1,15,16,20]). Later, Ohsumi and his colleagues applied the idea of pseudomeasurement to tracking problems of maneuvering ships ([13,14]) and some identification problems in environmental pollution problems ([11,12]), and also to identification problems of unknown exogenous inputs and related quantities ([5,6,7,9,10]).

Recently, Kameyama and Ohsumi applied it to identification problems of unknown entries of the system matrices ([2,3]). In our previous paper [18], we studied the same kind of identification problems for discrete-time linear stochastic systems while Ohsumi and his colleagues dealt with continuous-time linear stochastic systems. We proposed a new identification method in [18] only for the case where the dimension of the observation process is the same as that of the dynamical system and the coefficient matrix H is nonsingular (see Section 2 for the detail). However,

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in this paper, we consider the case where the dimension of the observation process is smaller than that of the dynamical system. Namely, we investigate a new identification method for partially observed (discrete-time) linear stochastic systems.

After introducing the pseudomeasurement for the given stochastic system, we augment it with the original observation process and derive the new identification method by applying the extended Kalman filter. Due to the additional observation process data (i.e., pseudomeasurement), we can expect better performance of the new identification method than that of the conventional approach. It can be seen in the simulation results in Section 6.

2. Problem Statement

Consider the discrete-time linear stochastic system for $t = 0, 1, 2, \dots$:

$$x(t+1) = Ax(t) + Cu(t) + Gw(t), \quad (3)$$

$$y(t) = Hx(t) + Sv(t), \quad (4)$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^\ell$ denote the state vector of the system, the observation vector and the input vector, respectively; $A \in \mathbb{R}^{n \times n}$ (i.e., a set of $n \times n$ matrices), $C \in \mathbb{R}^{n \times \ell}$, $G \in \mathbb{R}^{n \times d_1}$, $H \in \mathbb{R}^{m \times n}$ and $S \in \mathbb{R}^{m \times d_2}$. Here, we assume that the matrices C, G, H and S are known but the matrix A is partially unknown (see Section 3 for the detail). Moreover, $w(t) (\in \mathbb{R}^{d_1})$ and $v(t) (\in \mathbb{R}^{d_2})$ are independent zero mean white Gaussian noise sequences with the covariance matrices Q and R , respectively, i.e., $\mathcal{E}\{w(t)w(t)^T\} = Q$ and $\mathcal{E}\{v(t)v(t)^T\} = R$, where $\mathcal{E}\{\cdot\}$ denotes the mathematical expectation operator and T denotes transposition of a matrix. In the previous paper[18], we dealt with the case where $m = n$ holds and the matrix H is nonsingular, whereas we here assume $m < n$ and that the rank of H is m . The pair (A, H) is assumed tacitly to be observable. Let Y_t be the σ -field $\sigma\{y(s), 0 \leq s \leq t\}$ generated by the observation data $y(s)$ up to the time t .

3. Pseudomeasurement Approach

Let $A = (a_{ij})$ be an $n \times n$ matrix in which some of the elements are assumed to be unknown. We define the $n \times n$ matrices A_k and A_u , the known part of A and the unknown part of A respectively as follows:

$$\begin{aligned} (A_k)_{ij} &= a_{ij}, & (A_u)_{ij} &= 0 & \text{if } a_{ij} \text{ is known,} \\ (A_k)_{ij} &= 0, & (A_u)_{ij} &= a_{ij} & \text{if } a_{ij} \text{ is unknown,} \end{aligned}$$

where $(B)_{ij}$ denotes the (i, j) -element of the matrix B . Then, we immediately have

$$A = A_k + A_u. \quad (5)$$

Assuming that the number of unknown entries is p , we define the p -dimensional vector a by rearranging p unknown entries $\{a_{ij}\}$ in the order from upper to lower in the first column and so on. We also denote the matrix A_u as $A_u(a)$ to indicate the unknown parameter vector a explicitly in the following.

It is easy to find the $n \times p$ matrix $X(t)$ satisfying

$$A_u(a)x(t) = X(t)a, \quad (6)$$

where $X(t)$ can be written as $X(t) = \sum_{i=1}^n x_i(t) M_i$ for some $n \times p$ matrices $M_1, M_2, \dots, M_n$ depending on $A_u(a)$ (see[3] for the detail). We also have

$$X(t) = M_0(x(t) \otimes I_p), \quad (7)$$

where M_0 denotes the $n \times np$ matrix $[M_1 M_2 \dots M_n]$. Here, the symbol $\otimes$ denotes the Kronecker's product of matrices and I_p denotes the $p \times p$ identity matrix.

Denoting the pseudoinverse of H by $H^\dagger$, we define the $n \times n$ matrix L by $L := H^\dagger H$ and assume the following condition.

[Condition A] The following condition holds:

$$D(d) := LM_0[(I - L)d \otimes I_p] = O \quad \text{for any } d \in \mathbb{R}^n. \quad (8)$$

□

From (6)-(7), we can rewrite (3) as

$$\begin{aligned} x(t+1) &= A_k x(t) + M_0(x(t) \otimes I_p) a \\ &\quad + Cu(t) + Gw(t). \end{aligned} \quad (9)$$

We now seek a relation between the unknown vector a and the observation process $\{y(t)\}$. Firstly, it immediately follows from (4) that

$$Lx(t) = H^\dagger Hx(t) = H^\dagger y(t) - H^\dagger Sv(t). \quad (10)$$

Defining the matrix B by

$$B := LA_k(I - L), \quad (11)$$

we have

$$\begin{aligned} H^\dagger y(t+1) &= H^\dagger Hx(t+1) + H^\dagger Sv(t+1) \\ &= L\{A_k x(t) + M_0(x(t) \otimes I_p) a + Cu(t) + Gw(t)\} \\ &\quad + H^\dagger Sv(t+1) \\ &= LA_k H^\dagger y(t) + Bx(t) + LM_0(H^\dagger y(t) \otimes I_p) a \\ &\quad + LCu(t) + LGw(t) - LA_k H^\dagger Sv(t) \\ &\quad - LM_0(H^\dagger Sv(t) \otimes I_p) a + H^\dagger Sv(t+1) \end{aligned} \quad (12)$$

from (9), (10) and Condition A.

We here define the new process

$$y_p(t) = H^\dagger y(t+1) - LA_k H^\dagger y(t) - LCu(t) \quad (13)$$

which is obviously a simple known function of the input vector $u(t)$ and the observation vectors $y(t)$ and $y(t+1)$. Defining the new matrix

$$H_p(t) = LM_0(H^\dagger y(t) \otimes I_p), \quad (14)$$

we have the equality

$$y_p(t) = Bx(t) + H_p(t)a + (\text{noise terms}), \quad (15)$$

due to (12)-(13).

Since the noise terms in (15) are a linear combination of the mutually independent Gaussian white noises $w(t), v(t)$ and $v(t+1)$ and a is a constant vector, they can be written as a single Gaussian noise.

Thus, the equality (15) can be rewritten as

$$y_p(t) = Bx(t) + H_p(t)a(t) + S_p v_p(t), \quad (16)$$

with $S_p \in \mathbb{R}^{n \times d_4}$, where $\{v_p(t) (\in \mathbb{R}^{d_4})\}$ is a zero-mean Gaussian (but non-white) noise sequence with a covariance matrix R_p which has a correlation with the previously given Gaussian white noise sequences $\{v(t)\}$ and $\{w(t)\}$. Thus, $\{y_p(t)\}$ can be regarded as the observation process of the unknown vector a . We discuss when the pair (A_u, H) of the matrices satisfy Condition A in Appendix. We will present the modified pseudomeasurement method which can be applied to the case where Condition A does not hold in the forthcoming paper[17].

4. Augmented System

To identify the unknown vector a , we usually consider it as a function $a(t)$ of t and treat it as a random vector-valued process

$$a(t+1) = a(t) + G_a w_a(t) \quad (17)$$

by allowing some ambiguity. Here, $\{w_a(t) (\in \mathbb{R}^{d_3})\}$ is a newly introduced zero-mean Gaussian white noise sequence with covariance matrix Q_a which is independent with previously given random sequences. Notice that the matrices G_a and Q_a can be preassigned by the users. Actually, we may choose $G_a = O$ if we need no ambiguity in (17).

Let us introduce a new state vector defined by $z(t) := [x(t)^T, a(t)^T]^T$. The following augmented system can be obtained from (9) and (17):

$$z(t+1) = f(z(t)) + C_0 u(t) + G_0 w_0(t), \quad (18)$$

where

$$\begin{aligned} f(z(t)) &= (f_1(z(t)), f_2(z(t)), \dots, f_{n+p}(z(t)))^T \\ &= \begin{bmatrix} A_k & X(t) \\ O & I \end{bmatrix} z(t) \in \mathbb{R}^{n+p}, \\ C_0 &= \begin{bmatrix} C \\ O \end{bmatrix} \in \mathbb{R}^{(n+p) \times \ell}, \\ G_0 &= \begin{bmatrix} G & O \\ O & G_a \end{bmatrix} \in \mathbb{R}^{(n+p) \times (d_1+d_3)}, \\ w_0(t) &= \begin{bmatrix} w(t) \\ w_a(t) \end{bmatrix} \in \mathbb{R}^{d_1+d_3}. \end{aligned}$$

Defining the new $(m+n)$ -dimensional vector $y_0(t) := [y(t)^T, y_p(t)^T]^T$ and the new (d_2+d_4) -dimensional vector $v_0(t) := [v(t)^T, v_p(t)^T]^T$, we can obtain the augmented observation process

$$y_0(t) = H_0(t)z(t) + S_0 v_0(t), \quad (19)$$

from (4) and (16), where we used the notation

$$H_0(t) = \begin{bmatrix} H & O \\ B & H_p(t) \end{bmatrix} \in \mathbb{R}^{(m+n) \times (n+p)} \quad (20)$$

and

$$S_0 = \begin{bmatrix} S & O \\ O & S_p \end{bmatrix} \in \mathbb{R}^{(m+n) \times (d_2+d_4)}.$$

5. Extended Kalman Filter

To estimate the state vector $x(t)$ and identify the unknown entries a simultaneously, we propose to apply the extended Kalman filter (EKF) to the augmented nonlinear system (18) with (19) although the noise sequence $\{v_p(t)\}$ is not white and has a correlation with the previously given Gaussian white noise sequences $\{w(t)\}$ and $\{v(t)\}$. (We will consider a modification of the EKF-based method in a forthcoming paper when the correlation between these noises are not negligible.) Thus, supposing that the estimate $\hat{z}(t|t)$ of $z(t)$ has been obtained, we use the linear approximation of f around $\hat{z}(t|t)$

$$\begin{aligned} f(z(t)) &= f(\hat{z}(t|t)) + \hat{F}_t(z(t) - \hat{z}(t|t)) \\ &\quad + (\text{higher order terms}), \end{aligned}$$

and obtain the linearized system of (18)

$$\begin{aligned} z(t+1) &= \hat{F}_t z(t) + C_0 u(t) + G_0 w_0(t) \\ &\quad + f(\hat{z}(t|t)) - \hat{F}_t \hat{z}(t|t), \end{aligned} \quad (21)$$

where

$$\begin{aligned} \hat{F}_t &= \left(\frac{\partial f(z)}{\partial z^T} \right)_{z=\hat{z}(t|t)} = \left[\left(\frac{\partial f^T(z)}{\partial z} \right)_{z=\hat{z}(t|t)} \right]^T \\ &= \begin{bmatrix} \frac{\partial f_1(\hat{z})}{\partial z_1} & \frac{\partial f_1(\hat{z})}{\partial z_2} & \dots & \frac{\partial f_1(\hat{z})}{\partial z_{n+p}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{n+p}(\hat{z})}{\partial z_1} & \frac{\partial f_{n+p}(\hat{z})}{\partial z_2} & \dots & \frac{\partial f_{n+p}(\hat{z})}{\partial z_{n+p}} \end{bmatrix}. \end{aligned}$$

Applying the Kalman filter to the linearized system (21) with (19), we have the following results. Here, $\hat{z}(t+1|t)$ and $\hat{z}(t+1|t+1)$ denote the optimal estimates of $z(t+1)$ given by the Kalman filter for the linearized system (21) with (19) respectively with respect to Y_t and Y_{t+1} . We also define the matrices $P(t|t)$ and $P(t+1|t)$ as follows:

$$P(t|t) = \mathcal{E} \left[\{z(t) - \hat{z}(t|t)\} \{z(t) - \hat{z}(t|t)\}^T \right] \quad (22)$$

and

$$\begin{aligned} P(t+1|t) &= \mathcal{E} \left[\{z(t+1) - \hat{z}(t+1|t)\} \right. \\ &\quad \left. \{z(t+1) - \hat{z}(t+1|t)\}^T \right]. \end{aligned} \quad (23)$$

[Proposition 1] The optimal estimates $\hat{z}(t+1|t)$ and $\hat{z}(t+1|t+1)$ obtained by the Kalman filter applied to the linearized system (21) with (19) have the following form:

$$\hat{z}(t+1|t) = f(\hat{z}(t|t)) + C_0 u(t), \quad (24)$$

$$\begin{aligned} \hat{z}(t+1|t+1) &= \hat{z}(t+1|t) \\ &\quad + K(t+1)[y_0(t+1) - H_0(t+1)\hat{z}(t+1|t)], \end{aligned} \quad (25)$$

where

$$\begin{aligned}
K(t+1) &= P(t+1|t)H_0^T(t+1) \\
&\times [H_0(t+1)P(t+1|t)H_0^T(t+1) + S_0R_0S_0^T]^{-1}, \quad (26) \\
R_0 &= \begin{bmatrix} R & O \\ O & R_p \end{bmatrix} \in \mathbb{R}^{(d_2+d_4) \times (d_2+d_4)}.
\end{aligned}$$

Here, $\hat{z}(0|-1) = \bar{z}_0$ is the initial estimate and $P(0|-1)$ is chosen to be cI for some $c(>0)$. Moreover, the matrices $P(t+1|t)$ and $P(t|t)$ satisfy the recursive relations

$$P(t+1|t) = \hat{F}_t P(t|t) \hat{F}_t^T + G_0 Q_0 G_0^T \quad (27)$$

and

$$P(t|t) = P(t|t-1) - K(t)H_0(t)P(t|t-1). \quad (28)$$

respectively, where

$$Q_0 = \begin{bmatrix} Q & O \\ O & Q_p \end{bmatrix} \in \mathbb{R}^{(d_1+d_3) \times (d_1+d_3)}.$$

(Proof) Applying the Kalman filter to the linear system (21) with (19), we have

$$\begin{aligned}
\hat{z}(t+1|t) &= \hat{F}_t \hat{z}(t|t) + C_0 u(t) + f(\hat{z}(t|t)) - \hat{F}_t \hat{z}(t|t) \\
&= f(\hat{z}(t|t)) + C_0 u(t)
\end{aligned}$$

and

$$\begin{aligned}
\hat{z}(t+1|t+1) &= \hat{z}(t+1|t) \\
&+ K(t+1)[y_0(t+1) - H_0(t+1)\hat{z}(t+1|t)].
\end{aligned}$$

The rest of the results can be obtained by the standard argument of the Kalman filter. $\square$

[Proposition 2] The equalities (24) and (25) can be written explicitly as follows:

$$\begin{aligned}
&\begin{bmatrix} \hat{x}(t+1|t) \\ \hat{a}(t+1|t) \end{bmatrix} \\
&= \begin{bmatrix} A_k \hat{x}(t|t) + M_0(\hat{x}(t|t) \otimes I_p) \hat{a}(t|t) + C u(t) \\ \hat{a}(t|t) \end{bmatrix} \quad (29)
\end{aligned}$$

and

$$\begin{aligned}
&\begin{bmatrix} \hat{x}(t+1|t+1) \\ \hat{a}(t+1|t+1) \end{bmatrix} = \begin{bmatrix} \hat{x}(t+1|t) \\ \hat{a}(t+1|t) \end{bmatrix} + K(t+1) \\
&\times \begin{bmatrix} y(t+1) - H\hat{x}(t+1|t) \\ y_p(t+1) - B\hat{x}(t+1|t) - H_p(t+1)\hat{a}(t+1|t) \end{bmatrix}. \quad (30)
\end{aligned}$$

(Proof) Since

$$\begin{aligned}
f(z(t)) &= \begin{bmatrix} A_k & X(t) \\ O & I \end{bmatrix} \begin{bmatrix} x(t) \\ a(t) \end{bmatrix} \\
&= \begin{bmatrix} \{A_k + A_u(a(t))\}x(t) \\ a(t) \end{bmatrix},
\end{aligned}$$

we have

$$\begin{aligned}
f(\hat{z}(t|t)) &= \begin{bmatrix} \{A_k + A_u(\hat{a}(t|t))\}\hat{x}(t|t) \\ \hat{a}(t|t) \end{bmatrix} \\
&= \begin{bmatrix} \hat{A}(t|t)\hat{x}(t|t) \\ \hat{a}(t|t) \end{bmatrix}, \quad (31)
\end{aligned}$$

where $\hat{A}(t|t) = A_k + A_u(\hat{a}(t|t))$. Thus we have (29). The equality (30) immediately follows from (25). $\square$

[Remark 3] We note that

$$\begin{aligned}
\frac{\partial f^T(z)}{\partial z} &= \frac{\partial}{\partial z} \begin{bmatrix} x^T \{A_k + A_u(a)\}^T & a^T \end{bmatrix} \\
&= \begin{bmatrix} \frac{\partial}{\partial x} x^T \{A_k + A_u(a)\}^T & \frac{\partial}{\partial x} a^T \\ \frac{\partial}{\partial a} x^T \{A_k + A_u(a)\}^T & \frac{\partial}{\partial a} a^T \end{bmatrix} \\
&= \begin{bmatrix} \{A_k + A_u(a)\}^T & 0 \\ X^T & I \end{bmatrix}
\end{aligned}$$

holds, where we used the equality

$$\frac{\partial}{\partial a} x^T A_u^T(a) = \frac{\partial}{\partial a} a^T X^T = X^T.$$

Thus we have

$$\begin{aligned}
\hat{F}_t &= \left(\frac{\partial f(z)}{\partial z^T} \right)_{z=\hat{z}(t|t)} \\
&= \begin{bmatrix} A_k + A_u(a) & X \\ O & I \end{bmatrix}_{x=\hat{x}, a=\hat{a}} \\
&= \begin{bmatrix} \hat{A}(t|t) & \hat{X}(t|t) \\ O & I \end{bmatrix}, \quad (32)
\end{aligned}$$

where

$$\hat{X}(t|t) = X(t) \Big|_{x=\hat{x}} = \sum_{i=1}^n \hat{x}_i(t|t) M_i.$$

$\square$

We state one advantage of the proposed method here. By choosing $H_0 = [H \ O_{m \times p}]$ without using $y_p(t)$ and $H_p(t)$, the proposed method reduces to the conventional method (without the pseudomeasurement). Thus, our method is consistent with the conventional method, and so we have the choice of the proposed method (by utilizing (20)) or the conventional method (by choosing $H_0 = [H \ O_{m \times p}]$). This is an advantage of the proposed method in practice.

6. Numerical Simulation

We applied the new identification method via pseudomeasurement approach and the conventional identification method (without pseudomeasurement) to a simple example and compared the results. We consider the single-input two-output three-dimensional linear system with

$$\begin{aligned}
A &= \begin{bmatrix} 0 & 0 & d_1 \\ 1 & 0 & d_2 \\ 0 & 1 & a \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad G = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}, \\
H &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = I_2, \quad Q = I_2, \quad R = 0.01I_2,
\end{aligned}$$

where the elements d_1 and d_2 of A are known and chosen as $d_1 = 0.7425$ and $d_2 = -2.4660$ respectively. On the other hand, the element a is unknown. We assign the true parameter a as follows: $a = 2.7200$ (see page 127 of [4]). In the simulation, the initial state of system (3) is given by $x(0) = [1, 1, 1]^T$, and the known input is given by $u(t) \equiv 1$. We applied the identification method via pseudomeasurement approach

starting from the initial estimates as follows: $\hat{x}(0|-1) = [0, 0, 0]^T$, $\hat{a}(0|-1) = 5$, $P(0|-1) = 0.9 \times I_4$. User-defined parameters are given by $G_a = I_1$, $S_p = I_3$, $Q_a = I_1$ and $R_p = 10I_3$.

The simulation results via pseudomeasurement approach are shown as Simulation 1 in Figs. 1-5. While Fig. 1 depicts the trajectory of stochastic system (3), the state estimate $\hat{x}(t|t)$ and the identified parameter $\hat{a}(t|t)$ via pseudomeasurement are shown in Fig. 2 and Fig. 3, respectively. The trajectories of the diagonal elements (1,1) – (3,3) (resp. (4,4)) of the estimation error covariance matrix $P(t|t)$ are given in Fig. 4 (resp. Fig. 5). For comparison, we also show the simulation results by the conventional identification method using the extended Kalman filter (without pseudomeasurement) as Simulation 2 in Figs. 6-10, i.e., Fig. 6 for the state trajectory $x(t)$ of (3), Fig. 7 for the state estimate $\hat{x}(t|t)$ of $x(t)$ and Fig. 8 for the identified parameter $\hat{a}(t|t)$, respectively. Similarly to Figs. 4 and 5, the trajectories of the diagonal elements of $P(t|t)$ are given in Figs. 9 and 10 for the conventional identification method without pseudomeasurement.

In view of Figs. 1, 2, 4 and Figs. 6, 7, 9, there is no big difference between the two methods on the state estimation. However, we can see the difference between Figs. 3, 5 and Figs. 8, 10. Namely, the identified parameter $\hat{a}(t|t)$ and the a -part of the covariance matrix $P(t|t)$ via pseudomeasurement in Figs. 3 and 5 are better than those by the conventional method in Figs. 8 and 10.

The estimation errors $\|\hat{a}(t|t) - a\|$ of the unknown parameter a by the two methods were computed respectively for 20 numerical experiments. The mean

Fig. 1 State trajectory of the linear stochastic system (Simulation 1)

Fig. 2 State estimation via pseudomeasurement (Simulation 1)

Fig. 3 Identified process of a via pseudomeasurement (Simulation 1)

Fig. 4 x -part of $P(t|t)$ via pseudomeasurement (Simulation 1)

and the standard deviation of the average of the estimation error $\|\hat{a}(t|t) - a\|$ for $10 < t \leq 100$ by the first method (with the pseudomeasurement) were 0.01411 and 0.00644 respectively. On the other hand, those by the second method (without the pseudomeasurement) were 0.02242 and 0.00983 respectively. Thus, the mean and the standard deviation of the error became less than two thirds respectively due to the utilization of the pseudomeasurement.

7. Conclusions

We consider partially observed discrete-time linear stochastic systems and assume that some entries of the system matrices are unknown. We present a new method which identifies unknown entries of the system matrix A and the state vector of the system simultaneously. The key idea of the proposed method is utilization of the pseudomeasurement which is a fictitious and additional observation process on the unknown entries and was modified so as to work for the partially observed systems. The proposed method is consistent with the conventional method (without pseudomeasurement) and so they can easily be unified to be a single iterative process for simultaneous identification and state estimation by switching the coefficient matrix of the augmented observation process. This is an advantage of the proposed method in practice. The simulation results show the effectiveness of the proposed method.

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Fig. 5 a -part of $P(t|t)$ via pseudomeasurement (Simulation 1)

Fig. 6 State trajectory of the linear stochastic system (Simulation 2)

Fig. 7 State estimation for without pseudomeasurement (Simulation 2)

Fig. 8 Identified process of a without pseudomeasurement (Simulation 2)

Fig. 9 x -part of $P(t|t)$ without pseudomeasurement (Simulation 2)

Fig. 10 a -part of $P(t|t)$ without pseudomeasurement (Simulation 2)

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Appendix

In Appendix, we discuss when the pair (A_u, H) of the matrices A_u and H satisfies Condition A. We consider the simple case $(n, m, p) = (3, 2, 1)$. Let the matrix H be given by one of the following specific forms for simplicity:

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Then, we immediately have

$$L_1 := H_1^\dagger H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad L_2 := H_2^\dagger H_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$L_3 := H_3^\dagger H_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Noting that $p = 1$, we denote the unknown parameter vector a by α (a real number) and the unknown part of A by $A_u(\alpha)$ in this section. Then, the matrix $A_u(\alpha)$ has the form

$$(A_u(\alpha))_{ij} = \begin{cases} \alpha & (i = i_0, j = j_0) \\ 0 & (i \neq i_0 \text{ or } j \neq j_0) \end{cases}$$

for some $i_0 (1 \leq i_0 \leq 3)$ and $j_0 (1 \leq j_0 \leq 3)$. The pair (i_0, j_0) is called the *location* of the unknown parameter α of the matrix $A_u(\alpha)$. For each location of α and each choice of H , we indicate whether the pair (A_u, H) of the matrices A_u and H satisfies Condition A or not in the following table.

the location of α	H_1	H_2	H_3
(1, 1)	S	O	S
(1, 2)	S	O	N
(1, 3)	N	O	S
(2, 1)	S	N	O
(2, 2)	S	S	O
(2, 3)	N	S	O
(3, 1)	O	N	S
(3, 2)	O	S	N
(3, 3)	O	S	S

In the table, "S" indicates that Condition A and $H_p(t) \neq O$ hold, whereas "O" indicates that Condition A holds but $LM_0 = O$ (and so $H_p(t) \equiv O$). On the other hand, "N" indicates that Condition A does not hold. In view of the table, for any location of α , we can find a matrix H_j such that the pair (A_u, H_j) satisfies Condition A and we can apply the pseudomeasurement approach to the system with (A_u, H_j) .

We now turn to the case: $p = 2$. Let the unknown parameter vector a be given by $a = [\alpha_1, \alpha_2]^T$. Using the notations $M_i(\alpha_j)$ for the matrix M_i corresponding to $A_u(\alpha_j)$ and $M_i(a) = M_i(\alpha_1, \alpha_2)$ for the matrix M_i corresponding to $A_u(a)$ respectively for $i = 0, 1, 2, 3$ and $j = 1, 2$, we immediately have

$$M_0(\alpha_1, \alpha_2) = \begin{bmatrix} M_1(\alpha_1) & M_1(\alpha_2) & M_2(\alpha_1) & M_2(\alpha_2) \\ & & M_3(\alpha_1) & M_3(\alpha_2) \end{bmatrix}.$$

Then, we can show the following lemma easily.

[Lemma 1] If for some $k (1 \leq k \leq 3)$ the equalities

$$L_k M_0(\alpha_j) [(I - L_k)d] = 0 \quad (\text{the zero vector}) \quad (\text{A1})$$

hold for $j = 1, 2$ and for any $d (\in \mathbb{R}^3)$, then we have

$$L_k M_0(\alpha_1, \alpha_2)[(I - L_k)d \otimes I_2] = O \quad (\text{A2})$$

for any $d \in \mathbb{R}^3$.

(Proof) Writing $(I - L_k)d = [\tilde{d}^{(1)}, \tilde{d}^{(2)}, \tilde{d}^{(3)}]^T$, we have

$$\begin{aligned} & L_k M_0(\alpha_1, \alpha_2)[(I - L_k)d \otimes I_2] \\ &= L_k [M_1(\alpha_1) \ M_2(\alpha_1) \ M_3(\alpha_1) \ M_2(\alpha_2) \\ &\quad M_3(\alpha_1) \ M_3(\alpha_2)] \begin{bmatrix} \tilde{d}^{(1)} & 0 \\ 0 & \tilde{d}^{(1)} \\ \tilde{d}^{(2)} & 0 \\ 0 & \tilde{d}^{(2)} \\ \tilde{d}^{(3)} & 0 \\ 0 & \tilde{d}^{(3)} \end{bmatrix} \\ &= \begin{bmatrix} L_k [M_1(\alpha_1) \ M_2(\alpha_1) \ M_3(\alpha_1)] \begin{bmatrix} \tilde{d}^{(1)} \\ \tilde{d}^{(2)} \\ \tilde{d}^{(3)} \end{bmatrix} \\ L_k [M_1(\alpha_2) \ M_2(\alpha_2) \ M_3(\alpha_2)] \begin{bmatrix} \tilde{d}^{(1)} \\ \tilde{d}^{(2)} \\ \tilde{d}^{(3)} \end{bmatrix} \end{bmatrix} \\ &= [L_k M_0(\alpha_1)(I - L_k)d \quad L_k M_0(\alpha_2)(I - L_k)d] \\ &= O \quad \square \end{aligned}$$

When $a = [\alpha_1, \alpha_2, \dots, \alpha_p]^T$ and $p \geq 3$, the notation $M_i(a) = M_i(\alpha_1, \alpha_2, \dots, \alpha_p)$ is used for the matrix M_i corresponding to $A_u(a)$ respectively for $i = 0, 1, 2, 3$. Lemma 1 can be easily extended to the case where $p \geq 3$. Thus, we obtain the following result.

[Corollary 4] If $a = [\alpha_1, \alpha_2, \dots, \alpha_p]^T$ and $M_0(\alpha_\ell)$ satisfies (A1) for any $\ell (\ell = 1, 2, \dots, p)$ for some $k (1 \leq k \leq 3)$, then $M_0(a)$ satisfies the equality

$$L_k M_0(\alpha_1, \alpha_2, \dots, \alpha_p)[(I - L_k)d \otimes I_p] = O \quad (\text{A3})$$

for any $d \in \mathbb{R}^3$. Hence, the pair $(A_u(a), H_k)$ satisfies Condition A. $\square$

Choosing p unknown parameters $\alpha_\ell (\ell = 1, 2, \dots, p)$ whose locations indicate “S” for some H_k in the table and putting $a = [\alpha_1, \alpha_2, \dots, \alpha_p]^T$, we can obtain the system with the pair $(A_u(a), H_k)$ which satisfies Condition A. We note that some of the indications at the locations of α_ℓ ’s can be “O” also.

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