社会シミュレーション手法による “Do-It-Yourself Lottery” の分析*

山田 隆志[†]・寺野 隆雄[†]

A Computational Study of “Do-It-Yourself Lottery” with Incomplete Information*

Takashi YAMADA[†] and Takao TERANO[†]

This study investigates what kind of dynamics players lead to in Barrow’s “do-it-yourself lottery” by agent-based computational economics approach. The lottery is such that players choose a positive integer which is expected to be the smallest but anyone else does not choose. For this purpose, we consider a simple game form in which players know the preceding result, not plays of others, for their decision making and then pursue computer simulations by changing the number of players and the upper limit. Our main finding is threefold: First, both the distributions of submitted and winning integer are different from those in steady states when the value of sensitivity parameter is small. Second, game efficiencies are higher than those in theory. Besides, in contrast to the theory, as the value of sensitivity parameter becomes larger, they rise. Third, there is a tendency of splitting the players into a winner and the losers.

1. Introduction

J.D. Barrow introduces a kind of guessing game called “Do-It-Yourself Lottery” (DIY-L) where players choose and submit one positive integer and the one choosing the smallest number that is not chosen by anyone else is the winner[1]. For instance, suppose a game with three players, A, B, and C. If the submission set is $(A, B, C) = (1, 2, 3)$, then player A wins the game. Or, if the submission set is $(A, B, C) = (1, 1, 2)$, then player C wins. But, if all of them choose the same integer, then there is no winner.

When one rebuilds this lottery so that it has an upper limit and the number of players is fixed, s/he can express it as a game. More precisely, DIY-L is a multi-player, multi-strategy and non-cooperative general-sum game with unique mixed strategy equilibrium. Besides, since this is a guessing game, it is usually iterated several or many times and players are

expected to think how others will choose an integer not given foregone choices of others.

Multi-player and multi-strategy game situations are quite common in socio-economic systems. But, they have not been studied as deeply as 2×2 ones because multi-player games are considered more complex[3,16,21]. Or, one may have to take into account the possibility of coalition in such situations[25]. Indeed, DIY-L has at least three players and usually three or more strategies. Moreover, it can be applied to several socio-economic systems such as lotteries and auctions although it has not been studied so much (see next section).

This study is to analyze how players knowing only the game results, not behaviors of others, behave and learn in DIY-L and consequently the game outcomes by agent-based computational economics approach. For this purpose, we postulate that the players are not perfectly rational but learn based on the game results and then analyze individual behavior and aggregate results by changing the game setup and the learning parameter. Here we employ a reinforcement learning model developed in behavioral economic literature because players in guessing games usually make their decisions with game results only, not their past choices. After giving the analytical results of DIY-L and the learning model, we provide with the computational setup and results.

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[†] 東京工業大学 大学院総合理工学研究科 知能システム科学専攻 Department of Computational Intelligence and Systems Science, Tokyo Institute of Technology; 4259 Nagatsuta-cho, Midori-ku, Yokohama city, Kanagawa 226-8502, JAPAN

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Table 1 Mixed strategy equilibrium in simplified DIY-L

N	M	1	2	3	4
3	2	0.5	0.5		
3	3	0.464	0.268	0.268	
3	4	0.458	0.252	0.145	0.145
4	2	0.5	0.5		
4	3	0.449	0.426	0.125	
4	4	0.448	0.425	0.126	0.002

The rest of this paper is organised as follows: The next section explains the design of DIY-L and gives a brief comparison between it and other similar games. Section 3. presents results of computational experiments. Section 4. discusses the implications of this study. Finally, this study ends with a couple of concluding remarks in Section 5..

2. “Do-It-Yourself Lottery”

2.1 Game Design

There are N players each of who chooses one positive integer from 1 to M (> 1). All of them know this setup. The player who submits the smallest integer that is not chosen by anyone else is a winner. The winner receives a positive payoff, usually normalized 1, and the losers do zero. If there is no uniquely chosen integer, all players become losers.

Here we consider DIY-L with $N \geq 3$ and $M \geq 3$. In case of bi-matrix game, there are three equilibria: (1) Both players choose 1 and (2) one player chooses 1 and the other does 2. But, since one never makes one's opponent a winner so long as (s)he keeps on choosing 1[19], this kind of game is not worth investigating.

Then, each game form has a unique mixed strategy equilibrium as in Table 1 in cases of $(N, M) = (3, 2), (3, 3), (3, 4), (4, 2), (4, 3)$, and $(4, 4)$ ^{1, 2}.

2.2 Related Game

DIY-L is similar to other multi-player and multi-strategy games in some regards. Among these, Highest Unique Bid Auction (HUBA)[23], Lowest Unique Bid Auction (LUBA)[8,10,12,20,22] and Swedish Lottery (SL)[26] are natural extensions and employed in practice. But, there are several differences between DIY-L and these games: Firstly, DIY-L is not an auction meaning that *staying out* is not allowed and that there is not a good or participation fee when players submit an integer. Likewise, every player needs to submit only one positive integer at every round, *i.e.* s/he is not allowed not to participate (*staying out*) or to bid two or more integers. Secondly, the number of players is always fixed to N and every player knows

the upper limit M . Thirdly, the lottery is a strategic form game. Hence, no player is informed whether his/her submitted integer is unique (cf. Gallice[10]). Fourthly, there is no tie-breaking rule. Finally, this is a repetitive game.

On the other hand, the differences between DIY-L and other representative multi-player games are in the followings: The players in minority game (MG) are three or more odd-numbered and choose one of the two symmetric strategies to be on the minority side (e.g.[2,5,6]). As a result, the game has $\binom{N}{(N-1)/2}$ asymmetric Nash equilibria in pure strategies, a unique mixed strategy equilibrium and infinite asymmetric mixed strategy equilibria. Unlike MG, an even-numbered players are allowed and that of strategies is two or more in DIY-L. Besides, the mixed strategy equilibrium is unique but its value is different from the integer because there is only one winner if exists³ and there are ranks in the pure strategies⁴; p -beauty contest game (p -BCG) is another candidate to DIY-L in the senses that there are at least three players who guess which integers other players will choose and that there is usually only one winner (e.g.[7,11,17,18]). While p -BCG is dominance solvable, level- k reasoning in DIY-L does not lead to such a state because level- k ($k > 0$) thinkers always choose one of the two integers, 1 (k : odd) or 2 (k : even) (level-0 players behave randomly)[19].

3. Computational Experiments

In this section, we present results of computational experiments using a representative reinforcement learning model. The objectives of this section are to investigate mainly

- distributions of submitted and winning integers in each game form, and
- behaviors of players, especially the differences between the winning agent and others.

3.1 Setup

The reinforcement learning algorithm used in this study is Roth-Erev model with the exponential choice rule[4,9]:

Player i ($i = 1, \dots, N$) has a propensity $w_{i,k}(t)$ for each pure strategy k ($k = 1, \dots, M$) at round t . Before the game, she is assumed to have a non-negative propensity for all pure strategies, namely $w_{i,j}(0) = w_{i,k}(0) \geq 0$ for $j \neq k$.

At every round, she chooses one pure strategy in accordance with an exponential selection rule

³This also indicates that DIY-L is a strictly competitive game, not having an aspect of cooperative game.

⁴McCabe et al. have reported that the experimental results of three-person matching-pennies game where the unique and symmetric mixed strategy equilibrium exists show that the subjects can learn to behave rationally under appropriate condition[14].

¹Östling et al. have a succinct algorithm to calculate mixed strategy equilibrium in this setup[19].

²As in this table, mixed strategy equilibrium in DIY-L with $N \geq 3$ and $M = 2$, namely binary choice game, is independent of the number of players.

Fig. 1 Mixed strategies in DIY-L with exponential Roth-Erev model as a function of sensitivity parameter λ

$$p_{i,k}(t) = \frac{\exp(\lambda \cdot w_{i,k}(t))}{\sum_{j=1}^M \exp(\lambda \cdot w_{i,j}(t))}$$

where $p_{i,k}(t)$ is the selection probability for integer k and λ is a positive constant called sensitivity parameter.

After a round, she updates her propensities as

$$w_{i,k}(t+1) = (1 - \phi)w_{i,k}(t) + \phi R$$

where R is a digital payoff and ϕ is a positive constant called learning parameter ($0 < \phi < 1$).

Figure 1 shows the steady states as a function of λ for each game setup. As in this figure, when λ is small, such players behave almost randomly. Meanwhile, as λ becomes larger, they are most likely to choose only 1 (or 2).

With these learning algorithms, we pursue the computational experiments under the following conditions:

- $(N, M) = (3, 3), (3, 4), (4, 3), (4, 4)$
- Each player uses the same learning model with the same initial and parameter values.
- Each game has 10,000 rounds, iterated 100 times.
- Each player knows current round, her previous submission, and the previous winning integer (if there is no winner, then this input is zero) for

her decision-making. Which means, she does not know the previous submission of others.

- $\lambda = 10.0, 20.0, 30.0, 40.0, 50.0,$

$\phi = 0.1, 0.3, 0.5, 0.7, 0.9$

Then, Table 2 presents the equilibrium values for $\lambda = 10.0, 20.0, 30.0, 40.0$ and 50.0 .

3.2 Results

First, Kolmogorov-Smirnov test was conducted for the distributions of submitted and winning integers for the last 5,000 rounds (Figure 2 illustrates the distributions for $\phi = 0.1$). The results are that most of them are not similar to those in the equilibrium points at the 1% significant level. The exceptions are as follows:

- $\phi = 0.1$
Nothing.
- $\phi = 0.3$
Submission: 37 runs in $(N, M) = (3, 3)$, 7 runs in $(N, M) = (3, 4)$, and 10 runs in $(N, M) = (4, 3)$.
- $\phi = 0.5$
Submission: 11 runs in $(N, M) = (3, 3)$, 1 run in $(N, M) = (3, 4)$, 98 runs in $(N, M) = (4, 3)$, and 94 runs in $(N, M) = (4, 4)$.
- $\phi = 0.7$
Submission: 1 run in $(N, M) = (4, 3)$ and 2 runs

Table 2 Steady states in DIY-L with exponential Roth-Erev model

a. $(N, M) = (3, 3)$				
λ	1	2	3	
10.0	0.466	0.267	0.267	
20.0	0.525	0.238	0.238	
30.0	0.555	0.223	0.223	
40.0	0.573	0.214	0.214	
50.0	0.585	0.207	0.207	

b. $(N, M) = (3, 4)$				
λ	1	2	3	4
10.0	0.437	0.214	0.175	0.175
20.0	0.524	0.276	0.100	0.100
30.0	0.509	0.459	0.016	0.016
40.0	0.502	0.490	0.004	0.004
50.0	0.501	0.498	0.001	0.001

c. $(N, M) = (4, 3)$			
λ	1	2	3
10.0	0.410	0.359	0.231
20.0	0.442	0.407	0.152
30.0	0.463	0.447	0.090
40.0	0.479	0.473	0.048
50.0	0.488	0.487	0.025

d. $(N, M) = (4, 4)$				
λ	1	2	3	4
10.0	0.363	0.274	0.202	0.161
20.0	0.417	0.341	0.141	0.101
30.0	0.447	0.411	0.080	0.062
40.0	0.467	0.454	0.041	0.037
50.0	0.481	0.477	0.022	0.021

in $(N, M) = (3, 4)$.

Winner: 79 runs in $(N, M) = (4, 3)$ and 1 run in $(N, M) = (4, 4)$.

- $\phi = 0.9$

Winner: 1 run in $(N, M) = (3, 3)$, 98 runs in $(N, M) = (4, 3)$, and 67 runs in $(N, M) = (4, 4)$

These results and Figure 2 indicate that many of simulation runs did not converge to the corresponding theoretical steady state and that the distributions are quite different from the game setup. In addition, these figures also imply that there are several game patterns, *i.e.* which integer(s) is/are frequently chosen and then becomes a winner.

Likewise, Figure 3 displays the game efficiency for each game setup and parameter. Note that the game efficiency means the ratio that there is a winner in the game. As in this figure, these results have a couple of characteristics: (1) The game efficiencies depend not only on the game setup but also on both the learning parameter and the sensitivity parameter. This is confirmed from the fact that the analysis of variance rejected the null hypothesis. (2) Also, although the benchmark efficiency is different from the sensitivity parameter not the learning parameter, the experimen-

tal efficiency declines as either the learning parameter becomes larger or the sensitivity parameter becomes smaller. They are contradictory to the benchmarks. In addition, the experimental efficiencies are always higher than theoretical ones. This means that each of the agents shows a different behavior from that in theory.

Next, Figures 4 ($\lambda = 10.0$) and 5 ($\lambda = 50.0$) present how each ranked agent played and how often she won the game for the last 5,000 rounds on average ($\phi = 0.1$). The ranked agent is determined by the number of wins during that period. What we can observe from these figures are as follows:

- (1) The 1st ranked agent chooses the smallest integer more often than other players. As a result, she wins more by sticking this strategy. This is confirmed from Figures 6 and 7, for instance¹. They display the average time series of relative submission probabilities which are calculated from the propensity and the exponential selection rule. The 1st ranked agent puts more weight on the propensity for the smallest integer when she wins by submitting this number. Then the positive feedback works in accordance with the sensitivity parameter λ ; The more, the faster.
- (2) The last ranked player, on the other hand, seems to choose every integer with equal probabilities. This can be also explained from Figures 6 and 7. Since the last ranked player rarely wins the game, her propensities do not increase at all. More concretely, when she submits one integer and loses the game, her propensity for such an integer decreases. Then, the other integers are likely to be chosen. But, if she loses the game once again, the propensity for the chosen integer also decreases. This is repeated. After all, no propensity is reinforced and then all integers are almost equally chosen and submitted.
- (3) The behaviors of other players are different from λ ; Submitting 1 more and more provides her with more winning chances for a smaller λ whereas avoiding from choosing 1 is better for a larger λ .

When sensitivity parameter λ is small, the exponential choice rule sometimes mistakenly or unconsciously picks up one strategy with relatively smaller propensity. With respect to DIY-L, this corresponds to the fact that the 1st ranked player submits 2 or larger from time to time. As a result, there are more chances for other players to win the game by choosing the smallest integer instead. Figures 4 and 6 explain this.

On the other hand, when λ is large enough, the 1st ranked player almost always chooses 1.

¹Due to the limited space, we provide with the results of $(N, M) = (4, 3)$ DIY-L only. But the same is true for the other settings.

Fig. 2 Mean frequencies of submitted and winning integers in simulated DIY-L for the last 5,000 rounds with $\phi = 0.1$ (left panel: submission, right panel: winner, w0: ‘no-winner’)

Fig. 3 Game efficiencies in simulated DIY-L. Game efficiency means that there is a winner in DIY-L and the benchmark ones are presented in each panel.

Hence, for the other players, choosing such this pure strategy has no meaning any more. Instead, they are required to find a second-best strategy as early as possible to be the 2nd ranked player on average: The 2nd ranked player sticks to one integer other than 1 in three-person DIY-L. By doing so, the two integers, 1 and 2(or 3), are occupied. It is only when the 3rd ranked player chooses 1 that the 2nd ranked player wins meanwhile the 1st ranked player wins in the rest of the cases (Figure 7a, b)¹. In four-person DIY-L, it suffices that the 2nd ranked player occupies 2 only. This is because an undesirable situation in which one of the rest two players submits 1 and the other does 2 can happen if she chooses 3 (or 4) (Figure 7c, d). To this end, keeping on choosing 1 or 2 is a best or better way to dominate the game in this case.

- (4) As the upper limit M becomes larger while the number of players N is fixed, the 1st ranked

¹The 2nd ranked player seemingly chooses each integer with almost equal probabilities in these panels but she sticks to one of the two or three integers at most of the rounds.

agent can win more.

This is because the under-dog players need to take into account all the pure strategies and then choose one integer with (rapidly decreasing) probability $1/M$.

- (5) As N increases while M is fixed, all the players lose the possibilities to win the game.

This fact is the same as the game efficiencies.

The reason why the computational results did not reproduce the theoretical benchmark lies in both the characteristics of DIY-L and the learning model employed itself². As explained in Sections 2. and 3., DIY-L has only one winner who receives a positive

²One of the two anonymous referees questioned about whether the results were different from those of neutral cases, namely the cases that the players behaved randomly ($\lambda = 0.0$). According to the additional experiments and analyses based on this, the original distributions ($\lambda > 0.0$) were found to be different from the new ones ($\lambda = 0.0$) when ϕ was small. But, some of them were not statistically different in cases of $\phi = 0.5, 0.7$, and 0.9 . On the other hand, the behavior of each ranked player was almost the same. The authors thank this comment.

Fig. 4 Average numbers of wins for the ranked agents in the second half of DIY-L with $\phi = 0.1$ and $\lambda = 10.0$ Fig. 5 Average numbers of wins for the ranked agents in the second half of DIY-L with $\phi = 0.1$ and $\lambda = 50.0$

Fig. 6 Time series plot of relative submission probabilities of each ranked agent in $(N, M) = (4, 3)$ DIY-L with $\phi = 0.1$ and $\lambda = 10.0$

Fig. 7 Time series plot of relative submission probabilities of each ranked agent in $(N, M) = (4, 3)$ DIY-L with $\phi = 0.1$ and $\lambda = 50.0$

payoff if exists and the agents in this study behave and learn adaptively. Thus, it is only when she wins at a round that she can learn from the past. This means that a higher sensitivity parameter easily divides them into a winner and the losers. As pointed out by Camerer[4], since the losers learn nothing while they lose, the gap between the two groups immediately broadens if the sensitivity parameter is large enough. Consequently, the steady-state is hard to be realized and the asymmetric pure strategy Nash equilibria are achieved.

4. Discussion

In this section, the implications of this study are discussed from the perspectives of socio-economic systems.

As mentioned in Section 2.2, DIY-L is considered as a minimum version of HUBA, LUBA and SL. With respect to HUBA and LUBA as auctions (that is to say, they are one-shot games), neither of them has been theoretically established and well-designed for practical use. For instance, it is not an optimal way for participants to submit their private values for a good or there has been no robust mechanisms to maximize the income to hold (e.g[12]). Likewise, in terms of SL (that is to say, it is a repetitive game where the number of players is not fixed), the relations between how participants should choose an integer and the setting of prize and participation fee have not been yet studied. Although the setup of DIY-L in this study has been limited, it can be extended in various ways.

In addition to it, there are several possible applications of DIY-L to real socio-economic systems. For instance, Östling et al. have introduced “choices of traffic routes and research topics, or buyers and sellers choosing among multiple markets” (p. 3)[19]. Other than these including HUBA, LUBA and SL, the draft in Nippon Professional Baseball (NPB)¹ and lotteries for apartment houses and parking spots are possible extensions. In the draft, every team wishes to acquire a best player but the chances become smaller if many other teams think so. On the other hand, there are more winning chances to have better or basic player avoiding the competition. The same is true for the rest applications; Suppose that players commute by car everyday and they wish to park their cars as close as possible to their company or station and that all renew the contract of parking spots at the same time every year/month. What is the best strategy to have a best, not better or basic, place to comfortably commute for the next term? These examples are explained by extended DIY-L. In designing such socio-economic systems, DIY-L as a game is (theoretically) suitable and (both theoretically and computationally) extensible[15].

¹Unlike that in the professional sports in the United States, the 1st round picks are determined by a lottery if two or more teams select a prospect amateur player.

Besides, the mixed strategy equilibrium in DIY-L as a (behavioral) game depends on the numbers of players and strategies as in Table 1. This implies that the players may need to change or modify their behavior when they face an environment within which the setup changes over time. Indeed, choices of traffic routes may be influenced by weather, days of the week, time slot (peak or off-peak) and with or without accidents or constructions. Although this study has investigated DIY-L with the fixed numbers of players and strategies and the main result is something like “on a first-come-first-served basis,” DIY-L may be worth investigating to understand and design several socio-economic systems because actual systems have changing numbers of players and options and, in most cases, mixed strategy equilibrium which is difficult or almost impossible to learn[13,24].

5. Concluding Remarks

We study individual behavior and learning, and the whole game outcomes in Barrow’s DIY-L with incomplete information by agent-based computational economic approach. Players know their own submissions and the winning integer at the previous round for decision-making. In this setup, reinforcement learning is usually considered and used. For this purpose, we employ one representative learning algorithm and pursue computer simulations by changing the game setup and the parameters of the learning model. We observe that the distributions of submitted and winning integers are often different from those in theory and that the game efficiency is always higher than theoretical one. Moreover, when learning speed is high, there is a tendency of splitting the players into a winner and the losers.

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Authors

Takashi YAMADA (Member)

He belongs to Department of Computational Intelligence and Systems Science, Tokyo Institute of Technology. His research interests include (Agent-based) Computational Economics and Finance, Behavioral and Experimental Economics, and Empirical Finance.

Takao TERANO

Takao Terano is a professor at Department of Computational Intelligence and System Science, Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology. He received BA degree in Mathematical Engineering in 1976 from University of Tokyo, M. A. degree in Information Engineering in 1978 from University of Tokyo, and Doctor of Engineering Degree in 1991 from Tokyo Institute of Technology. His research interests include Agent-based Modeling, Knowledge Systems, Evolutionary Computation, and Service Sciences. He is a member of the editorial board of major artificial intelligence and system science-related academic societies in Japan and a member of IEEE, AAAI, and ACM. He is also the president of PAAA.