PDS Based Bilateral Control System of Flexible Master-Slave Arms with Random Delay Using Kalman Filter*

Masaharu YAGI†, Kengo KIMURA‡ and Yuichi SAWADA‡

In this paper, a PDS (Proportional-Derivative-Strain) based bilateral control system for flexible master–slave arms with random delay is discussed. The proposed PDS based bilateral control system consists of a rigid master and flexible slave arms. The flexible slave arm is actuated by a high-gearved servomotor. Furthermore, the communication network which causes random delay connects both arms. The random delay is defined as the sum of the average time delay and a white Gaussian noise. A Kalman filter is designed to estimate the signals affected by the random delay. The controllers for the rigid master arm and the flexible slave arm are designed as the PD- and PDS-controllers, respectively. Numerical simulations are accomplished to confirm the performance of the proposed Kalman filter for the proposed PDS based bilateral control system.

1. Introduction

In recent years, teleoperation technologies draw much attention as technologies for robotic systems to provide human skills for remote locations. The aim of teleoperation technologies is useful in general environments such as offices and houses without special equipments. Therefore, these technologies should be realized by using existing communication networks, such as local area networks (LAN), wide area networks (WAN), and wireless LAN. In this research, a PDS based control system of flexible master-slave arms is discussed as one type of teleoperation technology. This system is constructed by a rigid master and a flexible slave arm, and a communication network. The flexible slave arm tracks the rigid master arm and the communication network causes random delay. State signals and observation signals of this system are interacted through the communication network. Thus, the PDS based control system with the communication network can be regarded as a type of feedback system with a communication network. Furthermore, since the communication network causes random delay, the signals transmitted through this network become noisy. Then, the random delay might cause instability of this system.

A lot of researchers have studied bilateral control system with time delay and flexible manipulators. Hoshino et al. and Mori et al. studied a symmetric bilateral system of flexible master-slave manipulators and proved the passivity of this system[1],[2]. Namerikawa researched a control scheme for teleoperation systems with time-varying delay and a simple PD-type control method[3],[4]. Matsuno et al. proposed a PDS feedback controller to control the two-link flexible beams and discussed the stability of the closed-loop system[5]. The flexible beams was modeled as the two interconnected flexible beams which was regarded as an element of the truss structure in the study [5]. However, these researches did not consider random delay.

Furthermore, state estimation problems for a networked system with random delay have been discussed by many researchers. Guo et al. researches the exponential stability for a type of discrete-time system with random delay[6], and Wu et al. proved the mean-square exponential stability for a networked system[7]. Liu et al. discussed the model predictive control problem for a closed-loop networked control system which causes random delay defined as a Markov chain[8]. Schenato studied the design of optimal estimators with random delay and packet loss[9],[10]. However, it was considered that these studies of random delay did not well model actual random delay because the random delay in these researches was not defined stochastically (e.g., as Gaussian noise). To reduce the effect of the time delay which is occurred by the communication network, the time delay should be modeled
and explicitly treated in the observation system.

In our previous researches, a bilateral control system with a communication network which causes time-varying delay was discussed. The system consisted of a rigid master arm, a flexible slave arm and a communication network. The internal stability and the passivity were proved by using the Lyapunov theorem, and the performance of the proposed bilateral control system was evaluated through numerical simulations [11]. However, the time delay was modeled as the sinusoidal wave in the study [11]. Furthermore, state estimation problems for a networked system with random delay were investigated. A Kalman filter was designed as a state estimator for a time-invariant linear system and the observation system that was affected by the random delay. The effect of the proposed Kalman filter was confirmed through numerical simulations [12]. However, the system was defined as the general time-invariant system in the study [12].

In this paper, a PDS based bilateral control system for flexible master–slave arms with random delay is investigated. By using Hamilton’s principle, the mathematical models of the master arm and the slave arm are derived. The random delay is defined as the sum of the average time delay and a Gaussian noise. A novel Kalman filter is designed to estimate the state signal and the observation signal that are affected by the random delay. The PD and PDS controllers are designed for generating the reaction torque for the rigid master arm and the reference signal for the flexible slave arm, respectively. The state estimates which are estimated by the novel Kalman filter are used for these controllers. Numerical simulations are demonstrated to confirm the effectiveness of the proposed Kalman filter for the proposed system. The proposed system is formulated as the suitable stochastic system since the random delay is defined as the stochastic process. This study is an unique research which means that previous researches are not done such this research.

2. Mathematical Models

The considered PDS based bilateral control system consists of a rigid master arm and a flexible slave arm, which are connected through a communication network (shown in Fig. 1). The flexible slave arm is actuated by a high-geared servomotor, and the communication network causes random delay.

In this section, mathematical models of the proposed control system are derived to demonstrate numerical simulations [13].

2.1 Rigid Master Arm

The rigid master arm is constructed by an uniform rigid rod and a hub that is rotated by a DC motor (shown in Fig. 2). \(O_m X_m Y_m\) denotes the inertial Cartesian coordinate system. \(m_m\) and \(\ell_m\) represent the mass and the length of the rigid master arm, respectively. The operating force, \(F_h(t)\) acts at the end of this arm and generates an operational torque, \(\tau_h(t)\), i.e., \(\tau_h(t) := F_h(t) \ell_m\). \(\theta_m(t)\) and \(\tau_m(t)\) represent the rotational angle and the reaction torque, respectively. The physical parameters of the rigid arm are
as follows: $J_h$ is the moment of inertia of the motor’s rotor and $J_m$ is the moment of inertia of the rigid master arm. $\mu_m$ is the coefficient of friction of the motor shaft. The mathematical model of the rigid master arm is obtained as follows:

$$\left(J_h + J_m + \frac{1}{4} m_c \ell^2 \right) \ddot{\theta}_m(t) = -\mu_m \dot{\theta}_m(t) + \tau_h(t) + \tau_m(t) + g_m \gamma_m(t), \quad (1)$$

where $\gamma_m(t)$ is the system noise which defined as a white Gaussian noise process. $g_m$ is the coefficient for the system noise.

Eq. (1) should be rewritten as a state space model to design a state estimator. $x_m(t) := [\theta_m(t) \; \dot{\theta}_m(t)]^T$ represents a state vector. The state space model of the rigid master arm can be written as

$$\dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) + C_m \gamma_m(t). \quad (2)$$

2.2 High-gear Servomotor

In this research, the flexible slave arm is rotated by a high-gear servomotor in the horizontal plane. The block diagram of the high-gear servomotor is shown in Fig. 3. As seen in this figure, $\Theta_{com}(s)$ and $\theta(s)$ are, respectively, the Laplace transforms of $\theta_{com}(t)$ and $\theta(t)$. $\theta_{com}(t)$ and $\theta(t)$ represent the reference angle and the output angle, respectively. The transfer function from $\Theta_{com}(s)$ to $\theta(s)$ can be calculated [14]. By applying the inverse Laplace transform, the control torque, $\tau(t)$, is obtained as follows:

$$\tau(t) = \frac{k_c K_P}{R} \{\theta_{com}(t) - \theta(t)\} + \frac{k_c K_D}{R} \{\dot{\theta}_{com}(t) - \dot{\theta}(t)\} - \frac{k_c k_r}{R} \ddot{\theta}(t), \quad (3)$$

where $k_r$ denotes the torque constant and $k_c$ denotes the back electromotive force constant. $R$ denotes the internal resistance of the coil in the DC motor. $K_P$ and $K_D$ are the proportional and derivative gains of the PD controller, respectively. This torque drives the flexible slave arm.

2.3 Flexible Slave Arm

The parallel-structured single-link flexible arm is employed as the slave arm in this study (shown in Fig. 4). This arm is the experimental device of our group. In this research, the mathematical model of the flexible arm is derived by considering this arm. This arm is constructed by a pair of uniform Euler-Bernoulli beams. Both ends of the flexible beam clamp an unit hub and a tip-mass. Since the mathematical model of this arm is represented by using the highly complex nonlinear differential equations, this arm is approximated as a single-link flexible arm in this research (shown in Fig. 5). This simplified flexible arm has the same boundary conditions as the original arm [15].

A simplified single-link flexible arm has a length, $\ell$. $OXY$ is defined as the inertial Cartesian coordinate system and $O_{x'y'}$ is defined as the rotating coordinate system. $u(t,x)$ denotes the transverse displacement of the flexible beam from the $x-$axis. $\theta(t)$ denotes the rotational angle between the $OX-$ and the $Ox-$axes. The physical parameters of the flexible arm are as follows: $\rho$ and $S$ are the uniform mass density and the cross section, respectively; $EI$ is the uniform flexible rigidity (where $E$ is Young’s modulus and $I$ is the second moment of the cross-sectional area); $\gamma_D$ is the coefficient of Kelvin–Voigt-type damping and $\mu$ is the coefficient of friction of the slave motor shaft; $J_0$ denotes the inertia moment of the motor shaft of the slave arm; and $m$ denotes the mass of the tip-mass, assuming that the tip-mass is a point mass.

The total kinetic energy $T(t)$ and the potential energy $U(t)$ are expressed as follows:

$$T(t) = T_h(t) + T_b(t) + T_m(t) \quad (4)$$

$$U(t) = \int_0^T \frac{1}{2} EI \{u''(t,x)\}^2 dx \quad (5)$$

The approximated random delay is represented as $T + \gamma_o(t)$, where $T$ and $\gamma_o(t)$ denote the average time delay and the random process, respectively. The random delay is greater than zero and is defined by

$$
\gamma_o(t) = \begin{cases} 
\gamma(t) & \text{for } \gamma(t) > -T \\
0 & \text{otherwise},
\end{cases}
$$

(15)

where $\gamma(t)$ is the white Gaussian noise. Its mean and covariance are $E \{ \gamma(t) \} = 0$ and $E \{ \gamma^2(t) \} =: R$, respectively. It is assumed that the covariance of $\gamma(t)$ holds $R \ll T^2$. Therefore, $\gamma_o(t)$ can be approximated by the white Gaussian noise, $\gamma(t)$, i.e., $\gamma(t) \approx \gamma_o(t)$.

4. Synthesis of Kalman Filter

A Kalman filter is designed as the state estimator to reduce the effect of the random delay in this research. An ordinary time-invariant linear stochastic system with random delay is discussed for designing the Kalman filter [13].

$$
dx(t) = Ax(t)dt + Bu(t)dt + Gdw_o(t) \tag{16}
$$

$$
dy(t) = Cx(t - (T + \gamma(t)))dt, \tag{17}
$$

where $x(t) \in \mathbb{R}^m$ is the state vector and $y(t) \in \mathbb{R}^n$ is the observation vector; $u(t) \in \mathbb{R}^n$ is a control input; and $dw_o(t)$ is an increment of the Wiener process with zero-mean and covariance $Q_o(\triangleq E \{ dw_o^2(t) \})$. $dw_o(t)$ is expressed as $\gamma_o(t)dt$, where $\gamma_o(t)$ denotes the system noise, which is defined as a white Gaussian noise.

To process the random delay clearly, the Taylor series expansion form is applied to the state vector $x(t - (T + \gamma(t)))$ around $t_s := t - T$ in the observation system. Thus, eq. (17) is rewritten as

$$
dy(t) \equiv Cx(t_s)dt - C \{ Ax(t_s) + Bu(t_s) \} dw_o(t) - CG \gamma_o(t) \gamma_o(t)dt. \tag{18}
$$

The high-order terms with respect to $dt$ are neglected and $\gamma_o(t_s)\gamma_o(t)$ can be ignored. Hence, the observation system can be rewritten as

$$
dy(t) = Cx(t_s)dt - C \{ Ax(t_s) + Bu(t_s) \} dw_o(t), \tag{19}
$$

where $dw_o(t)$ is defined as $\gamma(t)dt$ and denotes an increment of the Wiener process with zero-mean and covariance $R \triangleq E \{ dw_o^2(t) \}$. In eq. (19), the second term of the right-hand side expresses the multiplicative noise that is expressed as a product of the state, $x(t_s)$, and the random variable, $\gamma(t)$.

To reduce the effect of the multiplicative noise, the Kalman filter should be designed for the stochastic system described by eqs. (16), (19). An innovation process, $dv(t)$, is given as

$$
dv(t) = dy(t) - C \frac{d}{dt} x(t_s) dt = Ce(t_s)dt - C \{ Ax(t_s) + Bu(t_s) \} dw_o(t). \tag{20}
$$

where $e(t_s)$ denotes the state estimate. $e(t_s)$ is represented as the error between $x(t_s)$ and $\hat{x}(t_s)$.

The correlation function

$\ldots$

3. Modeling of Random Delay

The communication network causes the random delay. It is known that the actual random delay has log-normal distribution. However, the random delay is approximated by the sum of an average time delay and a random process in this research since a function with log-normal distribution has not been constructed. The approximated random delay is represented as $T + \gamma_o(t)$, where $T$ and $\gamma_o(t)$ denote the average time delay and the random process, respectively. The random delay is greater than zero and is defined by

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dy(t) = Cx(t - (T + \gamma(t)))dt, \tag{17}
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where $x(t) \in \mathbb{R}^m$ is the state vector and $y(t) \in \mathbb{R}^n$ is the observation vector; $u(t) \in \mathbb{R}^n$ is a control input; and $dw_o(t)$ is an increment of the Wiener process with zero-mean and covariance $Q_o(\triangleq E \{ dw_o^2(t) \})$. $dw_o(t)$ is expressed as $\gamma_o(t)dt$, where $\gamma_o(t)$ denotes the system noise, which is defined as a white Gaussian noise.

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where $e(t_s)$ denotes the state estimate. $e(t_s)$ is represented as the error between $x(t_s)$ and $\hat{x}(t_s)$, i.e., $e(t_s) := x(t_s) - \hat{x}(t_s)$. The correlation function

$\ldots$
of \(dv(t)\) is given as
\[
E\{dv(t)dv^T(t)\} = C [AH(t_s)A^T + BQ_u B^T] R C^T, \tag{21}
\]
where \(H(t_s) := E\{x(t_s)x^T(t_s)\}\) is the autocorrelation matrix value of the state and \(Q_u := E\{u^2(t_s)\}\) is the autocorrelation value of the input. Assuming that the state, \(x(t_s)\), and the input, \(u(t_s)\), are uncorrelated, the correlation functions of these variables can be considered zero, i.e., \(E\{x(t_s)u^T(t_s)\} = E\{u(t_s)x^T(t_s)\} = 0\). The state estimate, \(\hat{x}(t_s)\), can be calculated by applying the orthogonal projection[16]. The estimation error covariance, \(P(t_s|t_s)\), is calculated by the Riccati differential equation and the autocorrelation function of the state, \(H(t_s)\), is calculated by the Lyapunov differential equation. Eventually, the equations of \(x(t_s|t_s), P(t_s|t_s)\), and \(H(t_s)\) are given as follows:
\[
d\hat{x}(t_s|t_s) = A\hat{x}(t_s|t_s)dt + Bu(t - (T + \gamma(t)))dt
\]
\[
+ P(t_s|t_s)C^T [C \{AH(t_s)A^T + BQ_u B^T\} RC^T]^{-1}
\]
\[
\times \{dy(t) - C\hat{x}(t_s|t_s)dt\} \tag{22}
\]
\[
\dot{P}(t_s|t_s) = AP(t_s|t_s) + P(t_s|t_s)A^T + GQ_u G^T
\]
\[
- P(t_s|t_s)C^T [C \{AH(t_s)A^T + BQ_u B^T\} RC^T]^{-1}
\]
\[
\times CP(t_s|t_s) \tag{23}
\]
\[
\dot{H}(t_s) = AH(t_s) + PH(t_s)A^T + GQ_u G^T. \tag{24}
\]
The control input should be used as \(u(t - (T + \gamma(t)))\) because the control input, \(u(t_s)\), cannot be obtained.

5. Synthesis of Controllers

The controllers to generate the reaction torque for the rigid master arm and the reference angle for the high-geared servomotor are designed in reference to our previous studies in this research[11]. The PD controller is derived to provide the reaction torque and the PDS controller is derived to calculate the reference angle, respectively. These controllers are expressed as follows:
\[
\tau_m(t) = K_{Dm}\left\{\hat{\theta}(t_s|t_s) - \theta_m(t)\right\}
\]
\[
+ K_{Dm}\left\{\hat{\theta}(t_s|t_s) - \hat{\theta}_m(t)\right\} \tag{25}
\]
\[
f(t) = \frac{R}{k_c K_D}\left[K_{Ps}\left\{\hat{\theta}_m(t_s|t_s) - \theta(t)\right\}
\right.
\]
\[
+ K_{Ds}\left\{\hat{\theta}_m(t_s|t_s) - \hat{\theta}(t)\right\}
\]
\[
+ K_S\left\{\hat{x} - \hat{x}^T(t)\right\}
\]
\[
+ E\{u''(t,0) - u''(t,\ell)\}
\]
\[
- \frac{k_c K_P}{R}\left\{\theta_{com}(t) - \theta(t)\right\}
\]
\[
+ \frac{(k_c + K_D)k_c}{R}\left\{\hat{\theta}(t)\right\}, \tag{26}
\]
where \(K_{Ps}(i = m,s)\) is the proportional gain, \(K_{Ds}(i = m,s)\) is the derivative gain, and \(K_S\) is the strain gain. The control input, \(f(t)\), is defined as \(\theta_{com}(t)\), i.e., \(f(t) := \dot{\theta}_{com}(t)\). Therefore, by integrating \(f(t)\), the reference angle, \(\theta_{com}(t)\), can be calculated.

6. Numerical Simulations

The performance of the proposed system is demonstrated through numerical simulations in this research. It is confirm that the angle of the flexible slave arm tracks that of the rigid master arm and the proposed Kalman filter can reduce the effect of the random delay.

6.1 Setup

It is considered that the flexible slave arm is made of phosphor bronze in the numerical simulations. The length, thickness, and width of the flexible slave arm is \(l = 0.3[m], 1.0 \times 10^{-3}[m]\), and \(4.0 \times 10^{-2}[m]\), respectively. Tables 1-3 list the physical parameters of the rigid master arm, the flexible slave arm, and the high-geared servomotor.

The number of modes of the proposed system is set as ten. Matlab and Simulink are used to demonstrate the numerical simulations. The initial conditions of the proposed system are set as \(\theta_m(0) = 0[rad], \dot{\theta}_m(0) = 0[rad/s], \theta(0) = 0[rad], \dot{\theta}(0) = 0[rad/s], u(0,x) = 0[m], \) \(\dot{u}(0,x) = 0[m/s]\). Furthermore, the average time delay of the random delay, \(T\), is set as \(T = 0.25, 0.50[s]\) (shown in Fig. 6(a),(b)). Fig. 7 shows the sample simulation results of the tip’s displacement.
of the slave arm without the system noise. As seen in this figure, the tip’s displacement does not vibrate. Since states of the master and slave arms are only affected by the random delay, this figure indicates the validity of the proposed Kalman filter. The simulation results in next subsection are affected by the random delay and the system noise. In this paper, the filter for reducing the effect of the system noise does not designed. However, because this filter is the usual filter, it is considered that the effect of the system noise can be reduced.

6.2 Results

Figs. 8-11 show the results of the numerical simulations. Figs. 8, 9 show the results when the average time delay, $T$, is 0.25[s] and Figs. 10, 11 show the results when the average time delay, $T$, is 0.50[s]. As seen in each figures, the top figures depict the angle of the rigid master arm, $\theta_m(t)$, and the estimated angle of the rigid master arm, $\hat{\theta}_m(t_s|t_s)$, and the angle of the flexible slave arm, $\theta(t)$; the middle figures depict the tip displacement of the flexible slave arm, $u(t,\ell)$; and the bottom figures depict the reaction torque, $\tau_m(t)$, and the control torque, $\tau(t)$.

In Fig. 8(a)-Fig. 11(a), the estimated angle of the rigid master arm, $\hat{\theta}_m(t_s|t_s)$, is well estimated by using the proposed Kalman filter because the effect of the random delay is reduced. Furthermore, the angle of the flexible slave arm, $\theta(t)$, tracks the angle of the rigid master arm well. In Fig. 8(b), Fig. 10(b), the overshoot appears to reach the desired state. However, due to the PDS controller, it quickly converges...
Fig. 9 Numerical simulation results \((T = 0.25[s])\)

(a) Angles, \(\theta_m(t), \hat{\theta}_m(t_s | t_s)\) and \(\theta(t)\)

(b) The tip’s displacement, \(u(t, \ell)\)

(c) Torque, \(\tau_m(t)\) and \(\tau(t)\)

Fig. 10 Numerical simulation results \((T = 0.50[s])\)

(a) Angles, \(\theta_m(t), \hat{\theta}_m(t_s | t_s)\) and \(\theta(t)\)

(b) The tip’s displacement, \(u(t, \ell)\)

(c) Torque, \(\tau_m(t)\) and \(\tau(t)\)

Table 4 Improvement rates of the rotational angle of the flexible slave arm [%]

<table>
<thead>
<tr>
<th>Input signal</th>
<th>Average delay</th>
<th>0.25</th>
<th>0.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step function</td>
<td>98.9</td>
<td>99.3</td>
<td></td>
</tr>
<tr>
<td>Sinusoidal wave</td>
<td>95.9</td>
<td>97.5</td>
<td></td>
</tr>
</tbody>
</table>

Measurement values are defined as the states which are affected by the random delay. Moreover, these values are measured by using the proposed Kalman filter or not. The mean squared errors are calculated by using these values. Furthermore, the improvement rates are calculated as follow:

\[
\text{Improvement rate} = \frac{MSE_1 - MSE_2}{MSE_1} \times 100[\%]
\]

where \(MSE_1\) is the mean squared error without the proposed Kalman filter and \(MSE_2\) is the mean squared error with the proposed Kalman filter. The improvement rate indicates the reduced effect for the random delay. It is found that the proposed Kalman filter can reduce the effect of the random delay.

7. Conclusions

A PDS based bilateral control system for flexible master-slave arms with random delay was investigated in this paper. By using Hamilton’s principle, the mathematical model of the rigid master and flexible slave arms were derived. Because the flexible
slave arm was actuated by the high-gear ed servomotor, the transfer function from the reference input to the torque was calculated through the proper model. The random delay was defined as a sum of the average time delay and a white Gaussian noise. The average time delay was assumed to be known in this research. The state space models including the system noise of the rigid master arm and the flexible slave arm were derived to design the novel Kalman filter. The observation system included the random delay. To treat the random delay explicitly, the Taylor series expansion form was applied to the observation system. By these procedures, the approximated observation system showed multiplicative noise that consisted of the product of the states and the randomness of the random delay. The proposed Kalman filter had three equations to estimate the state and to reduce the effect of the random delay: the estimate state system, the Riccati equation for the estimation error covariance, and the Lyapunov differential equation for the autocorrelation of the states. The PD controller was designed to generate the reaction torque for the rigid master arm and the PDS controller was designed to generate the control input for the flexible slave arm. Numerical simulations were accomplished to confirm the performance of the proposed control system and the proposed Kalman filter. As seen in the numerical simulation results, the proposed Kalman filter was a well-designed filter since the effect of the random delay was reduced. Furthermore, the PD and PDS controllers were also well-designed since the reaction torque and control input were generated, and the angle of the flexible slave arm tracks the estimated angle of the rigid master arm well.

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**References**


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