Noise–Induced Phenomena in the Kaldor Business Cycle Model*

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The periodic behavior of macroeconomic indicators or business cycles is a common observation in the economic system. In the past, business cycle modeling often involved the use of non-linear economic dynamic theory, general equilibrium theory, and methods to analyze complex systems, such as agent-based modeling and complex network theory. Several studies have shown that crisis and synchronization in business cycles can be modeled using threshold characteristics in the economic systems. In a non-linear system, it is well known that additive noise leads to qualitative transformations in system behavior, such as coherence resonance, stochastic resonance, and noise-induced transitions (referred to as noise-induced phenomena in this study). Recent studies have shown that noise-induced transitions arise in the non-linear economic business cycle model with multiple attractors around the sub-critical Hopf bifurcation. However, the relationship between the threshold characteristics in the economic dynamics and noise-induced phenomena has not been studied. In this study, we focus on the threshold characteristic of the Kaldor business cycle model under the bias of income, and show that coherence resonance and stochastic resonance can be produced as an effect of this threshold characteristic.

1. Introduction

The periodic behavior of macroeconomic indicators or business cycles, such as the Kitchen cycle, Kondratiev cycle, and Kuznets cycle, has been observed in economic systems[1–3]. Since the 1920s, business cycle modeling has been used to formulate and evaluate economic policies. The earliest approach was based on non-linear economic dynamic theory, in which the major elements of a business cycle are modeled using bifurcation theory[4,5]. Several studies have also used general equilibrium theory[6–10] and methods for analyzing complex systems, such as agent-based modeling and complex network theory[11–14]. Focusing on threshold characteristics in economic systems, that is, external inputs over certain levels induce transition of the system state, several studies have also shown that crisis and synchronization in business cycles can be well-modeled[15,16].

However, the business cycle mechanism remains highly controversial.

Qualitative transformations in system behavior due to additive noise are widely observed in many kinds of non-linear systems, such as electrical circuits and biological systems[17–20]. (These phenomena, which include coherence resonance, stochastic resonance, and noise-induced transitions, are called noise-induced phenomena in this study.) Recent studies have examined the effects of additive noise on business cycles based on non-linear economic dynamic theory[21,22]. Especially, Bashkirtseva *et al.* have shown that noise-induced transitions[17] arise in non-linear economic dynamic models, such as the Kaldor business cycle model and the Goodwin business cycle model, with multiple attractors around the sub-critical Hopf bifurcation[23,24]. These studies analyze the influence of stochastic fluctuations on the business cycle from the perspective of noise-induced phenomena. In addition, the relationship between the threshold characteristics in the economic dynamics and the noise-induced phenomena has not been studied.

Previously, we focused on the threshold characteristic of the Kaldor business cycle model under income bias and demonstrated that coherence resonance[19,25], in which additive noise with appropriate strength makes the system behavior coherent, can be produced as an effect of this threshold characteristic[26]. In this study, based on our research outcomes, we evaluate this coherence resonance and confirm that stochastic resonance[19], in which the appropriate noise en-

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hances the response against the weak input signal, can arise in this system.

2. Model and Methods

2.1 The Kaldor Business Cycle Model

Kaldor proposed a business cycle model focusing on the dynamics of income Y and capital stock K , and the dependence of investment I and saving S on Y and K as follows[4]:

$$\dot{Y}(t) = \alpha(I(Y(t), K(t)) - S(Y(t))), \quad (1)$$

$$\dot{K}(t) = I(Y(t), K(t)) - qK(t). \quad (2)$$

Here, α is the adjustment coefficient in the goods market and q is the depreciation rate of capital stock, respectively. Functions of S and I are given by

$$S(Y(t)) = pY(t), \quad (3)$$

$$I(Y, K) = pu + r\left(\frac{pu}{q} - K\right) + f(Y - u), \quad (4)$$

$$f(x) = \frac{1}{1 + \exp(-4x)} - 0.5, \quad (5)$$

where $(u, \frac{pu}{q})$ corresponds to a fixed point (Y, K) , and r is an adjustment cost. In our simulation, we use $(p, q, r, u) = (0.3, 0.2, 1.0, 3.0)$ [22].

To evaluate the system behavior under an income bias, weak signal, and noise, we consider b as the bias term of $\dot{Y}$, a weak sinusoidal signal $I_{\text{ext}}(t) = A \sin 2\pi f_S t$, and a Gaussian white noise $D\xi(t)$ ($\langle \xi(t) \rangle = 0, \langle \xi(t), \xi(t') \rangle = \delta_{tt'}$, $\langle \cdot \rangle$ indicates average in t) for Eq. (1) as follows:

$$\begin{aligned} \dot{Y}(t) = & \alpha(I(Y(t), K(t)) - S(Y(t)) + b) \\ & + I_{\text{ext}}(t) + D\xi(t). \end{aligned} \quad (6)$$

Here, $I_{\text{ext}}(t)$ and $D\xi(t)$ are typical examples of external influence, such as government intervention and global factors[5]. The initial condition of $(Y(t), K(t))$ is set to $(Y(0), K(0)) = (1.0, 1.0)$.

2.2 Index for Evaluation

To measure periodicity of income $Y(t)$, we adopt the correlation time[25]

$$\tau_c = \int_0^\infty C_{yy}(\tau)^2 d\tau, \quad (7)$$

$$\begin{aligned} C_{yy}(\tau) &= \frac{\langle (Y(t+\tau) - \langle Y(t) \rangle)(Y(t) - \langle Y(t) \rangle) \rangle}{\langle (Y(t) - \langle Y(t) \rangle)^2 \rangle}. \end{aligned} \quad (8)$$

Here, $C_{yy}(\tau)$ indicates the auto-correlation of $Y(t)$ on time delay τ . $\tau_c \approx 0$ and $\tau_c \gg 0$ correspond to cases in which the time series of $Y(t)$ becomes irregular and periodic, respectively.

The response of $Y(t)$ against the signal $I_{\text{ext}}(t)$ is measured by the signal to noise ratio:

$$SNR = 10 \log_{10} \frac{P_S}{P_N}, \quad (9)$$

$$P_S = \frac{1}{2\Delta f_S} \int_{f_S - \Delta f_S}^{f_S + \Delta f_S} P(f) df, \quad (10)$$

$$\begin{aligned} P_N = & \frac{1}{2(\Delta f_N - \Delta f_S)} \left(\int_{f_S - \Delta f_N}^{f_S - \Delta f_S} P(f) df \right. \\ & \left. + \int_{f_S + \Delta f_S}^{f_S + \Delta f_N} P(f) df \right). \end{aligned} \quad (11)$$

Here, $P(f)$ indicates the power spectrum of the binarized time series of $Y(t)$ with the threshold of $Y(t) = 3.0$. P_S and P_N indicate the powers of the signal component and the noise component, respectively. In our simulation, we set $\Delta f_S = 2.04 \times 10^{-4}$, $\Delta f_N = 0.03$.

For the bifurcation analysis in the system, we utilize the eigenvalues l_j ($j = 1, 2$) of the Jacobian J for $(\dot{Y}, \dot{K})$ around the fixed point (Y_f, K_f) as follows:

$$J = \begin{pmatrix} \alpha(4e^{-4u_0}/((1+e^{-4u_0})^2 - p)) & -\alpha r \\ 4e^{-4u_0}/(1+e^{-4u_0})^2 & -r - q \end{pmatrix}, \quad (12)$$

where u_0 indicates $Y_f - u$.

3. Results and Evaluations

3.1 System Behavior of the Kaldor Business Cycle Model

At first, we demonstrate the appearance of a limit cycle in the absence of bias, noise, and signal ($b = 0, D = 0, A = 0$). Fig. 1 shows the maximum real part of eigenvalue l_j given by Eq.(12) around the fixed point $(u, \frac{pu}{q})$: $\max_j \text{Re}(l_j)$. In $0 \leq \alpha \lesssim 1.7$, the fixed point is stable ($\max_j \text{Re}(l_j) < 0$). In this case, the orbit of (Y, K) stays at $(u, \frac{pu}{q})$, which is the intersection between the Y -nullcline ($\dot{Y} = 0$) and the K -nullcline ($\dot{K} = 0$) as shown in Fig.2 (a). In $1.7 \lesssim \alpha \lesssim 3.0$ (Fig.1), the fixed point becomes unstable ($\max_j \text{Re}(l_j) > 0$). Furthermore, considering the imaginary parts of l_1 and l_2 , these complex conjugates intersect with the real axis at $\alpha \approx 1.7$; this bifurcation is specified as the Hopf bifurcation. This bifurcation produces the limit cycle along the vector field, as shown in Fig.2 (b).

3.2 Dependence of System Behavior on Bias of Income

Next, we investigate the influence of the bias of income b on the Y -nullcline and the K -nullcline in a noise/signal-free condition. Fig. 3 shows the vector field and the nullclines in $b = 0$ ((a)) and $b = -0.1$ cases ((b)). For $b = 0$, the nullclines intersect at $(Y, K) \approx (3.0, 4.5)$. If we apply the perturbation to this system, the orbit of (Y, K) moves from the fixed point, according to the strength of this perturbation. On the other hand, for $b = -0.1$, the intersection moves to $(Y, K) \approx (2.5, 4.2)$. If the perturbation above a certain level is input, the orbit of (Y, K) is drawn to the flow in $K \lesssim 4.2$ and reaches the region with large values of Y (≈ 4.0). This system behavior is the threshold characteristic.

Furthermore, the dependence of this threshold characteristic on the parameters α and b are evaluated.

Fig. 1 Maximum real part of eigenvalue $\max_j \text{Re}(l_j)$ of Jacobian for $(\dot{Y}, \dot{K})$ at fixed point $(u, \frac{pu}{q})$ between gross product $Y(t)$ and capital stock $K(t)$ as the function of the adjustment coefficient α . ($p=0.3, r=1.0, q=0.2, u=3.0, A=0, b=0, D=0$)

Fig. 2 Orbit of $(Y(t), K(t))$, Y -nullcline ($\dot{Y}=0$), K -nullcline ($\dot{K}=0$), and vector field $(\dot{Y}, \dot{K})$ on phase plane $Y-K$. (a) $\alpha=0.8$. (b) $\alpha=1.8$. ($p=0.3, r=1.0, q=0.2, u=3.0, A=0, b=0, D=0$)

Fig. 4 shows the maximum and minimum values of $Y(t)$ as a function of b in the $\alpha=2.0, 5.0, 8.0$ cases. In the $\alpha=2.0$ case, $Y(t)$ increases accompanied by b in $-0.2 \leq b \lesssim -0.02$ and begins to oscillate from $b \approx -0.02$. In the $\alpha=5.0, 8.0$ cases, the oscillations begin from $b \approx -0.08, -0.09$, respectively. The amplitude of oscillation becomes larger with increasing α .

Additionally, we examine the bifurcation as a func-

Fig. 3 Orbit of $(Y(t), K(t))$, Y -nullcline ($\dot{Y}=0$), K -nullcline ($\dot{K}=0$), and vector field $(\dot{Y}, \dot{K})$ on phase plane $Y-K$. (a) $b=0$. (b) $b=-0.1$. ($p=0.3, r=1.0, q=0.2, u=3.0, \alpha=5.0, A=0, D=0$)

Fig. 4 Dependence of maximum and minimum values of gross product $Y(t)$ on parameter b . ($p=0.3, r=1.0, q=0.2, u=3.0, A=0, D=0$)

tion of parameters α and b by l_j . The bifurcation diagram, as shown in Fig. 5 indicates the regions where the fixed point becomes stable (gray region) and where the limit cycle arises around the unstable fixed point (white region) through the Hopf bifurcation (red dashed line). Further, the stable fixed point and limit cycle co-exist at the outer edge of the white region given by the green dots. These results specify this Hopf bifurcation as a sub-critical Hopf bifurcation.

Fig. 5 Bifurcation diagram by eigenvalues of Jacobian J for (Y, K) around fixed point. Gray region: fixed point becomes stable. White region: limit cycle arises and fixed point becomes unstable. Green points: stable fixed point and limit cycle co-exist. Red dashed line: sub-critical Hopf bifurcation. ($p=0.3, r=1.0, q=0.2, u=3.0, A=0, D=0$)

3.3 Coherence Resonance of the Kaldor Business Cycle Model

In this section, we evaluate the influence of noise on system behavior in three types of conditions examined in the previous section. These are the conditions for the existence of a stable fixed point, limit cycle, and the co-existence of a stable fixed point and a limit cycle.

Fig. 6 shows the time series of $Y(t)$ (left) and its cross correlation $C_{yy}(\tau)$ given by Eq.(8) (right) in case of $D=5.0 \times 10^{-3}, 0.1, 0.3, 1.0, 5.0$ under the condition for the existence of a stable fixed point ($\alpha=5.0, b=-0.1$). At $D=5.0 \times 10^{-3}$ (a), $Y(t)$ stays around the fixed point. However, $Y(t)$ exhibits oscillation with the period of approximate 10 at $D=0.3$ (c). In condition of stronger noise strength ($D=1.0$ (d), 5.0 (e)), the random behavior of $Y(t)$ becomes dominant. Owing to this tendency of $Y(t)$, it is hard to decay the oscillation of $C_{yy}(\tau)$ at $D=0.1$ (b) in comparison with other noise strength cases.

As shown in the open triangles of Fig. 7 (a), the dependence of τ_c given by Eq.(7) that corresponds to this behavior at $b=-0.1$ on noise strength D is demonstrated. τ_c indicates approximately 0.9 in $5.0 \times 10^{-3} \leq D \lesssim 5.0 \times 10^{-2}$; however, τ_c rises with increasing D . Then, at $D \approx 0.3$, τ_c achieves its peak value ($\tau_c \approx 1.6$). That is, coherence resonance arises in the Kaldor business cycle model with the bias of income. In condition of $b=-0.15, -0.2$ indicated by the open circle and square, the fixed point moves away from the threshold in the negative direction. Therefore, it needs stronger noise strength to achieve the peak of τ_c and deteriorates the peak value. Fig. 7 (b) shows the existence of a limit cycle ($b=-0.05$) and co-existence of stable fixed point and limit cycle ($b=-0.072$). Note that the location of the initial condition $(Y(0), K(0)) = (1.0, 1.0)$ is outside the orbit of limit cycle in ($b=-0.072$). In both cases the val-

Fig. 6 Time series of gross product $Y(t)$ (left) and its cross correlation $C_{yy}(\tau)$ (right). (a) $D=5 \times 10^{-3}$. (b) $D=0.1$. (c) $D=0.3$. (d) $D=1.0$. (e) $D=5.0$. ($p=0.3, r=1.0, q=0.2, u=3.0, \alpha=5.0, b=-0.1, A=0$)

ues of τ_c decreases according to the noise strength D due to disturbing autonomous oscillations; therefore, the peak of τ_c is not confirmed. In the positive bias ($b > 0$) cases of the existence of a stable fixed point (see Fig. 7 (c)), limit cycle ($b=0.05$ in Fig. 7 (d)) and the co-existence of stable fixed point and limit cycle ($b=0.072$ in Fig. 7 (d)), we confirm the same tendency with negative bias cases due to the symmetric structure of $\dot{Y}$ and $\dot{K}$ against $(u, \frac{pu}{q})$.

3.4 Stochastic Resonance of Kaldor Business Cycle Model

In this section, the response against the input signal $I_{\text{ext}}(t)$ under the noise is evaluated in the three types of conditions mentioned above. Fig. 8 shows the time series of $Y(t)$ (left) and its power spectrum $P(f)$ (right) in the case of $D=4.0 \times 10^{-3}, 0.06, 0.1, 0.5, 5.0$ under the existence of a stable fixed point ($\alpha=5.0, b=-0.1$) and applying $I_{\text{ext}}(t)$ ($A=0.1, f_s=0.1$). At $D=4.0 \times 10^{-3}$ (a), $Y(t)$ stays around the fixed point. However, the oscillation with the period of approximate $1/f_s \approx 10$ appears at $D=0.06$ (b) and $D=0.1$ (c). This periodicity can be also observed by the peak of the power spectrum $P(f)$ at $f \approx f_s=0.1$. This oscillation disappears when increasing noise strength further, as shown in Figs. 8 (d) and (e).

The dependence of SNR given by Eq.(9) corresponding to this behavior at $b=-0.1$ on noise strength D is demonstrated by the open triangles of Fig. 9 (a).

Fig. 7 Dependence of characteristic correlation time τ_c on noise strength D . (a) Condition for the existence of stable fixed point in negative bias. (b) Condition for the existence of limit cycle ($b = -0.05$) and for the co-existence of stable fixed point and limit cycle ($b = -0.072$) in negative bias. (c) Condition for the existence of stable fixed point in positive bias. (d) Condition for the existence of limit cycle ($b = 0.05$) and for the co-existence of stable fixed point and limit cycle ($b = 0.072$) in positive bias. ($p = 0.3, r = 1.0, q = 0.2, u = 3.0, \alpha = 5.0, A = 0$)

Fig. 8 Time series of gross product $Y(t)$ (left) and the power spectrum $P(f)$ of the binarized time series of $Y(t)$ (right). (a) $D = 4.0 \times 10^{-3}$. (b) $D = 0.06$. (c) $D = 0.1$. (d) $D = 0.5$. (e) $D = 5.0$. ($p = 0.3, r = 1.0, q = 0.2, u = 3.0, \alpha = 5.0, b = -0.1, A = 0.1, f_S = 0.1$)

SNR indicates approximately 0 at $D = 3.0 \times 10^{-2}$;

however, SNR rises with increasing D . Then, at $D \approx 0.5$, SNR achieves its peak value ($SNR \approx 22$). We can interpret this enhancement of signal response by noise with the appropriate strength as stochastic resonance. In conditions of $b = -0.15$ (open circle) and -0.2 (open square), the peak of SNR decreases due to the shift in the fixed point in the negative direction, as in the coherence resonance case. Fig. 9 (b) shows the cases with existence of a limit cycle ($b = -0.05$) and co-existence of the stable fixed point and limit cycle ($b = -0.072$). Note that the location of $(Y(0), K(0))$ is outside the orbit of limit cycle in the case of ($b = -0.072$), as well as the coherence resonance case in section 3.3. In these cases, the peak of SNR does not exist as it does in the coherence resonance case. In the positive bias ($b > 0$) cases of the existence of a stable fixed point, we confirm the same tendency with negative bias cases due to the symmetric structure of $\dot{Y}$ and $\dot{K}$, as shown in Fig.9 (c) and (d).

We also examine dependence on signal strength A in stochastic resonance. Fig. 10 shows the dependence of A and D on SNR under the condition of fixing $b = -0.1$. In $10^{-3} \leq A \lesssim 10^{-2}$, the signal response efficiency is low ($SNR \approx 0$), but increases accompanied with signal strength A , the value of SNR exhibits its peak against the changing value of D at $D \approx 0.1$ in $10^{-2} \lesssim A \leq 0.2$. However, further enhancing signal strength ($A \gtrsim 0.2$), decreases SNR according to the noise strength D ; therefore, we cannot confirm the

Fig. 9 Dependence of SNR on noise strength D . (a) Condition for the existence of a stable fixed point in negative bias. (b) Condition for the existence of a limit cycle ($b = -0.05$) and for the co-existence of stable fixed point and limit cycle ($b = -0.072$) in negative bias. (c) Condition for the existence of a stable fixed point in positive bias. (d) Condition for the existence of limit cycle ($b = 0.05$) and for the co-existence of stable fixed point and limit cycle ($b = 0.072$) in positive bias. ($p = 0.3, r = 1.0, q = 0.2, u = 3.0, \alpha = 5.0, A = 0.1, f_s = 0.1$)

Fig. 10 Dependence of SNR on noise strength D and signal strength A . ($p = 0.3, r = 1.0, q = 0.2, u = 3.0, \alpha = 5.0, b = -0.1, f_s = 0.1$)

Fig. 11 Dependence of SNR on signal frequency f_s . ($p = 0.3, r = 1.0, q = 0.2, u = 3.0, \alpha = 5.0, b = -0.1, A = 0.1, D = 0.07$)

peak of SNR .

Next, we evaluate dependence on the signal frequency in stochastic resonance under the condition ($A = 0.1, D = 0.07, b = -0.1$). As shown in Fig. 11, the signal frequency f_s dependence of SNR peaks at $f_s \approx 0.15$. Thus, stochastic resonance has the resonance frequency.

4. Conclusion

In this study, we examined system behavior in the Kaldor business cycle model under the bias of income. We found that a negative/positive bias led to the threshold characteristic of the system. In addition, through bifurcation analysis, we confirmed three kinds of distinctive parameter regions. The first was the existence of a stable fixed point. The second was

the existence of a limit cycle and the third was the co-existence of a stable fixed point and a limit cycle. Furthermore, we investigated coherence resonance in these regions using τ_c . Consequently, we confirmed that coherence resonance is induced by noise with an appropriate strength in the first region. However, in other regions, the additive noise disturbs the orbit of autonomous oscillations based on its strength. Finally, under weak input signal, we evaluated stochastic resonance in the above three regions using SNR . The result showed that stochastic resonance is produced by noise with an appropriate strength, and a signal with weak strength and specific frequency only in the first region, as in the coherence resonance case. That is, the business cycle rests under the condition

of decreasing/increasing income; however, the additive noise induces regeneration in the business cycle due to coherence resonance and stochastic resonance. If the business cycle exists in the noise-free condition, the noise only perturbs the business cycle based on its strength.

In future studies, we plan to examine noise-induced phenomena in more realistic models for dynamic economies to evaluate the economic validity of the mechanisms dealt with in this paper.

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